CS380: Computer Graphics
Triangle Rasterization

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Course URL:
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Class Objectives (Ch. 7)

- **Understand triangle rasterization using edge-equations**
- **Understand mechanics for parameter interpolations**
- **Realize benefits of incremental algorithms**

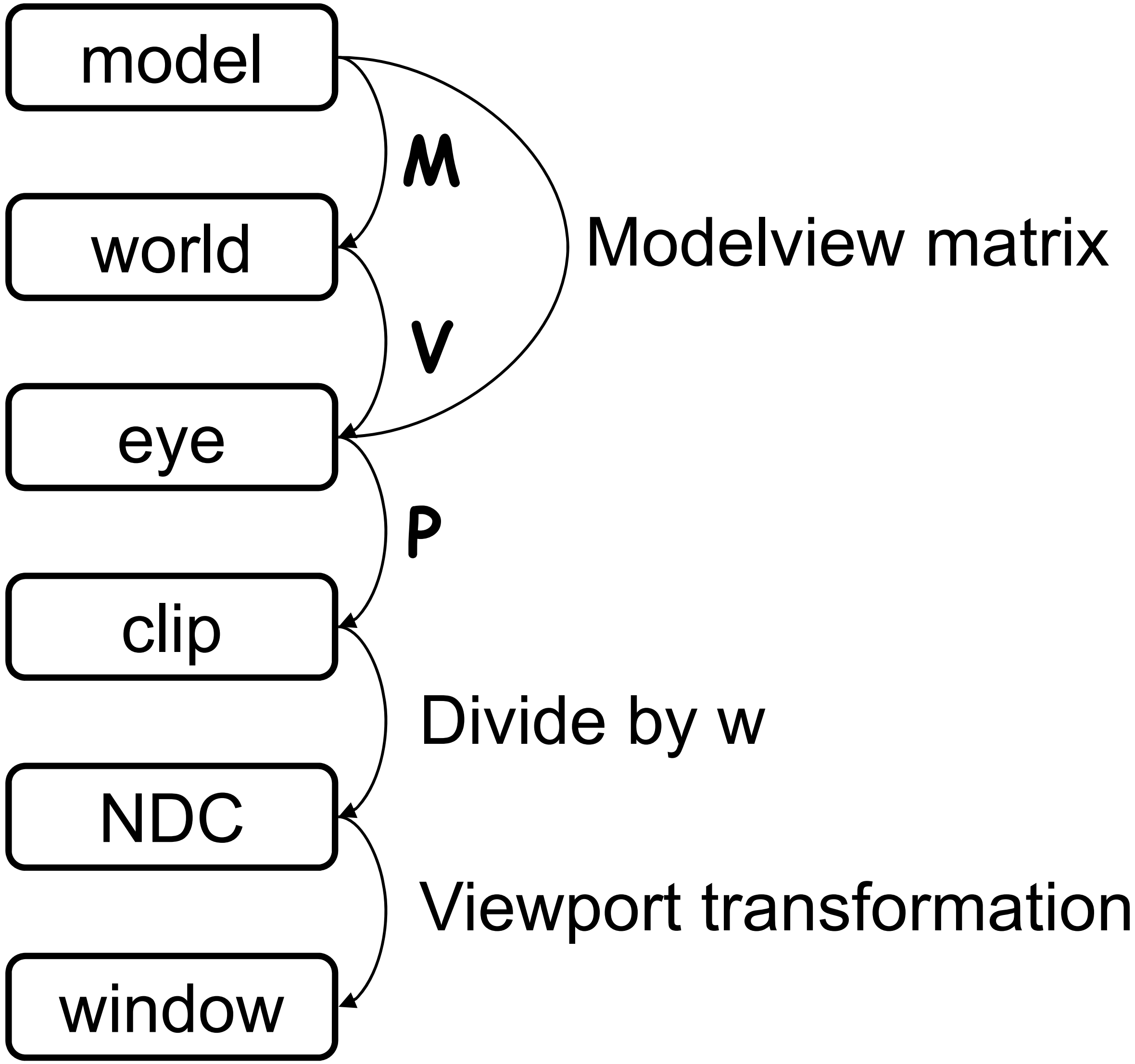
- **At the last class:**
  - **Discussed clipping and culling methods of view-frustum, back-face, and hierarchical culling methods**

Questions

- **How do we apply clipping and culling when there are transparent parts are included in objects? .. virtual optical lenses to obtain realistic view?**

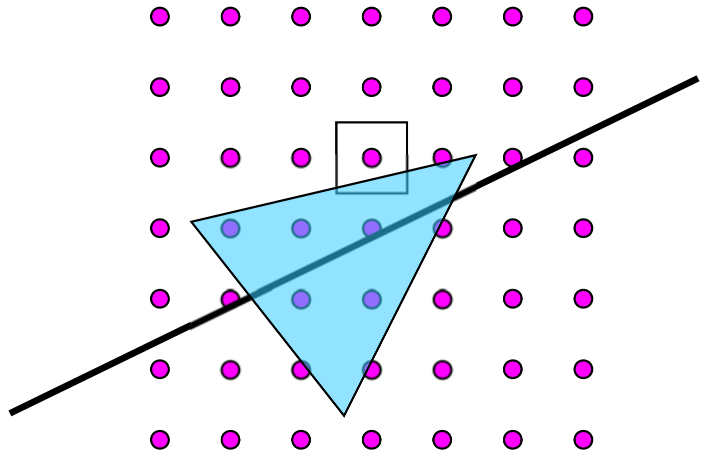
- **I thought GPUs are exploited most only when extensive multi-threading is used. But up till now there doesn't seem to be any multithreading in the src codes in the lecture/homework materials.**

Coordinate Systems



Primitive Rasterization

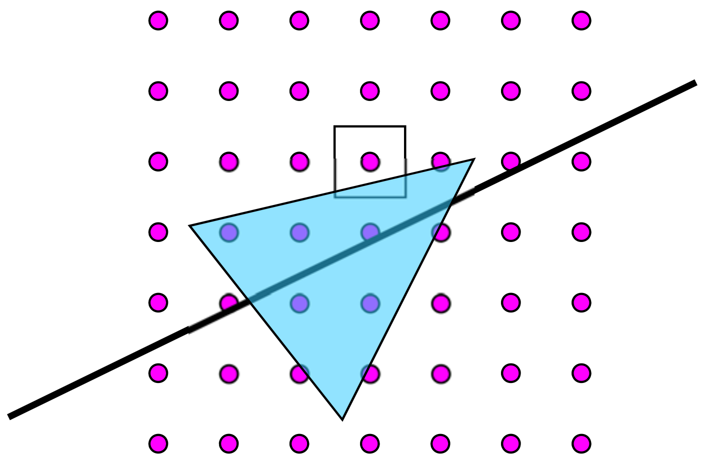
- Rasterization converts vertex representation to pixel representation



- Coverage determination
  - Computes which pixels (samples) belong to a primitive
- Parameter interpolation
  - Computes parameters at covered pixels from parameters associated with primitive vertices

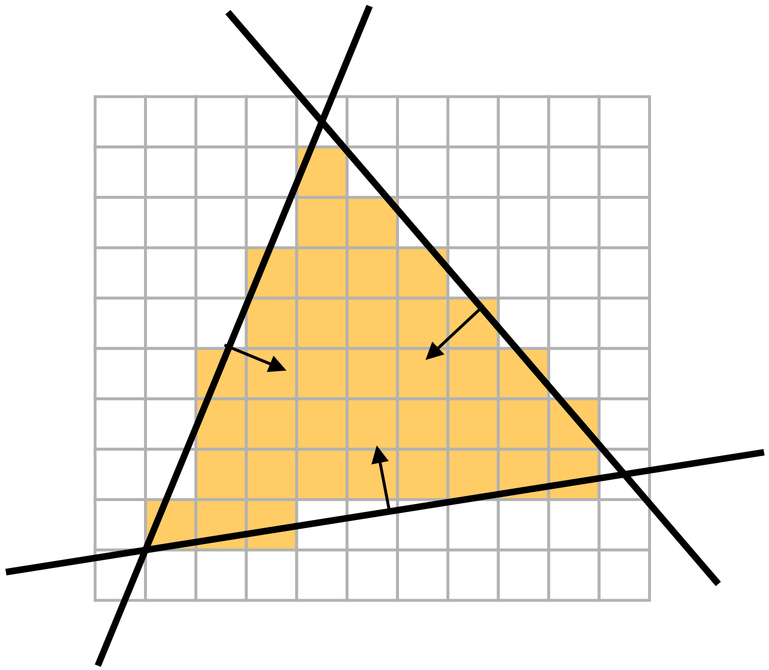
Coverage Determination

- **Coverage is a 2D sampling problem**
  - **Commonly reduced to 1D problem of checking a sample point**
- **Possible coverage criteria:**
  - **Distance of the primitive to sample point (often used with lines)**
  - **Percent coverage of a pixel (used to be popular)**
  - **Sample is inside the primitive (assuming it is closed)**



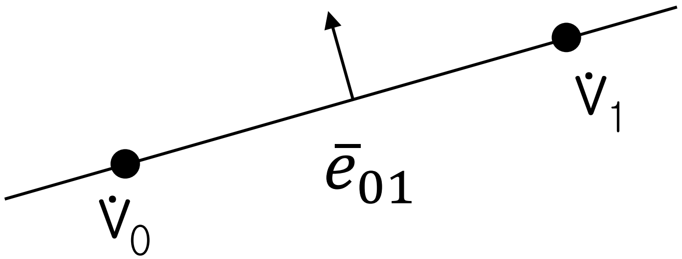
Rasterizing with Edge Equations

- Compute edge equations from vertices
- Compute interpolation equations from vertex parameters
- Traverse pixels evaluating the edge equations
- Draw pixels for which all edge equations are positive
- Interpolate parameters at pixels



Edge Equation Coefficients

- **The cross product between 2 homogeneous points generates the line between them**



$$\bar{e} = \dot{v}_0 \times \dot{v}_1$$
$$= [x_0 \quad y_0 \quad 1]^t \times [x_1 \quad y_1 \quad 1]^t$$
$$= [(y_0 - y_1) \quad (x_1 - x_0) \quad (x_0 y_1 - x_1 y_0)]$$

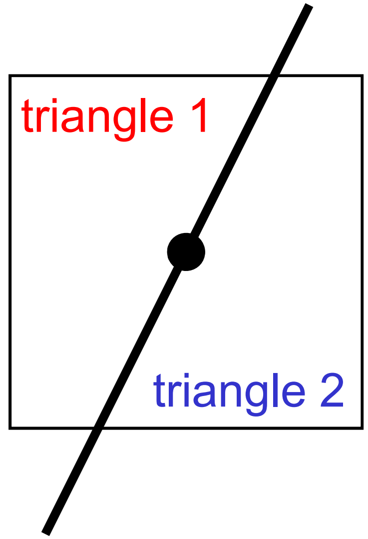
$$A_{01} \qquad B_{01} \qquad C_{01}$$

$$E(x,y) = \bar{e}_{01}(x,y) = A_{01}x + B_{01}y + C_{01}$$

- **A pixel at (x,y) is "inside" an edge if E(x,y)>0**

Shared Edges

- **Suppose two triangles share an edge. Which covers the pixel when the edge passes through the sample (E(x,y)=0)?**

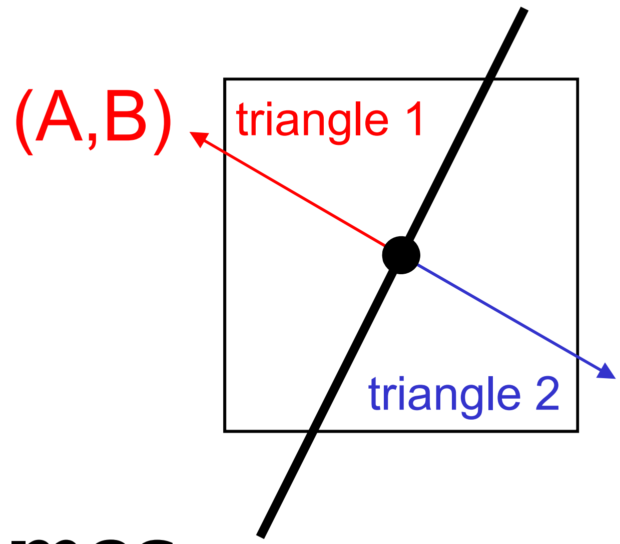


- **Both**
  - **Pixel color becomes dependent on order of triangle rendering**
  - **Creates problems when rendering transparent objects - "double hitting"**
- **Neither**
  - **Missing pixels create holes in otherwise solid surface**
- **We need a consistent tie-breaker!**

Shared Edges

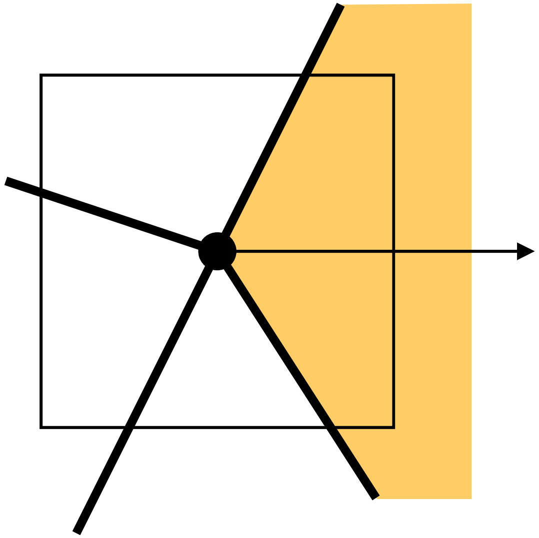
- **A common tie-breaker:**

$$\text{bool } t = \begin{cases} A > 0 & \text{if } A \neq 0 \\ B > 0 & \text{otherwise} \end{cases}$$

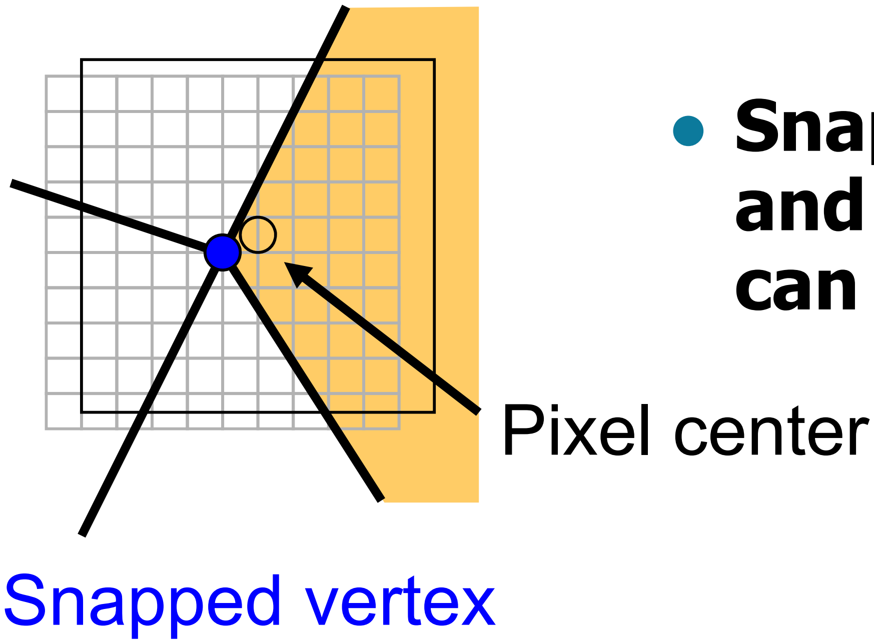


- **Coverage determination becomes**
  if(E(x,y) >0 || (E(x,y)==0 && t))
  pixel is covered

Shared Vertices



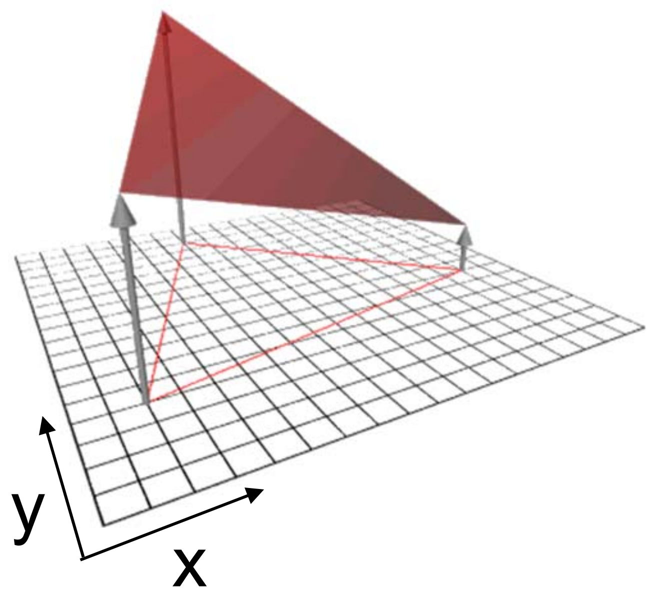
- Use "inclusion direction" as a tie breaker
- Any direction can be used



- Snap vertices to subpixel grid and displace so that no vertex can be at the pixel center

Interpolating Parameters

- **Specify a parameter, say redness (r) at each vertex of the triangle**
  - **Linear interpolation creates a planar function**



$$r(x,y) = A_r x + B_r y + C_r$$

Solving for Linear Interpolation Equations

- **Given the redness of the three vertices, we can set up the following linear system:**

$$\begin{bmatrix} r_0 & r_1 & r_2 \end{bmatrix} = \begin{bmatrix} A_r & B_r & C_r \end{bmatrix} \begin{bmatrix} x_0 & x_1 & x_2 \\ y_0 & y_1 & y_2 \\ 1 & 1 & 1 \end{bmatrix}$$

with the solution:

$$\begin{bmatrix} A_r & B_r & C_r \end{bmatrix} = \begin{bmatrix} r_0 & r_1 & r_2 \end{bmatrix} \frac{\begin{bmatrix} (y_1 - y_2) & (x_2 - x_1) & (x_1 y_2 - x_2 y_1) \\ (y_0 - y_2) & (x_2 - x_0) & (x_0 y_2 - x_2 y_0) \\ (y_0 - y_1) & (x_1 - x_0) & (x_0 y_1 - x_1 y_0) \end{bmatrix}}{\det \begin{bmatrix} x_0 & x_1 & x_2 \\ y_0 & y_1 & y_2 \\ 1 & 1 & 1 \end{bmatrix}}$$

Triangle Area

$$\text{Area} = \tfrac{1}{2}\det\begin{bmatrix} x_0 & x_1 & x_2 \\ y_0 & y_1 & y_2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\bar{e} = \dot{v}_0 \times \dot{v}_1$$
$$= [x_0 \quad y_0 \quad 1]^t \times [x_1 \quad y_1 \quad 1]^t$$
$$= [(y_0 - y_1) \quad (x_1 - x_0) \quad (x_0 y_1 - x_1 y_0)]$$

A_{01} B_{01} C_{01}

$$= \tfrac{1}{2}((x_1 y_2 - x_2 y_1) - (x_0 y_2 - x_2 y_0) + (x_0 y_1 - x_1 y_0))$$

$$= \tfrac{1}{2}(C_0 + C_1 + C_2)$$ // they are from edge equations; $C_2 = C_{01}$

- **Area = 0 means that the triangle is not visible**
- **Area < 0 means the triangle is back facing:**
  - **Reject triangle if performing back-face culling**
  - **Otherwise, flip edge equations by multiplying by -1**

Interpolation Equation

- **The parameter plane equation is just a linear combination of the edge equations**

$$[A_r \quad B_r \quad C_r] = \frac{1}{2 \cdot area} [r_0 \quad r_1 \quad r_2] \begin{bmatrix} \bar{e}_0 \\ \bar{e}_1 \\ \bar{e}_2 \end{bmatrix}$$

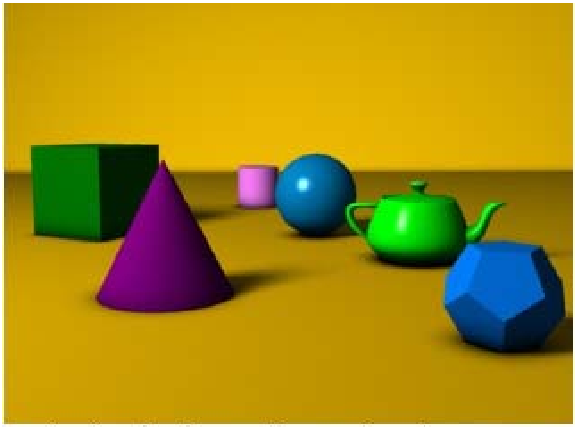
$\overline{e_o}, \overline{e_1}, \overline{e_2}$ are vectors of edge equations

$\overline{e_2} \quad are\ from \quad A_{01}, B_{01}, C_{01}$

Clearer notations are used in the book

Z-Buffering

- When rendering multiple triangles we need to determine which triangles are visible
- Use z-buffer to resolve visibility
  - Stores the depth at each pixel
- Initialize z-buffer to 1 (far value)
  - Post-perspective z values lie between 0 and 1
- Linearly interpolate depth (z_{tri}) across triangles
- If $z_{tri}(x,y)$ < zBuffer[x][y]
  write to pixel at (x,y)
  zBuffer[x][y] = $z_{tri}(x,y)$



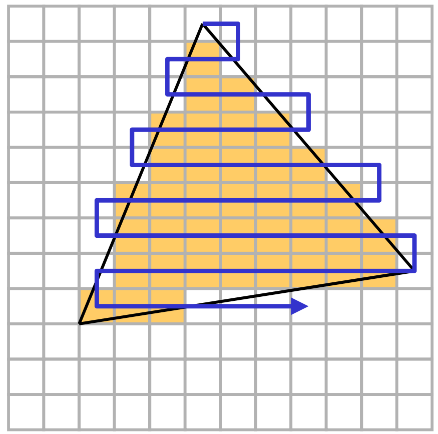
A simple three dimensional scene

Z-buffer representation

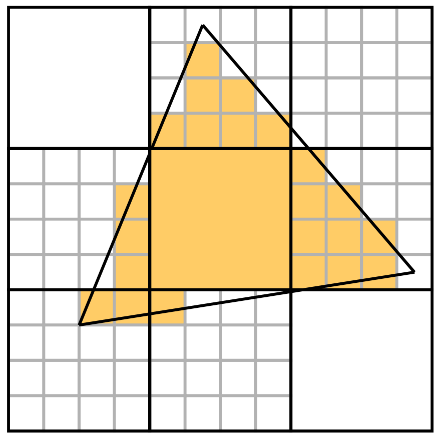
image from wikipedia.com

Traversing Pixels

- Free to traverse pixels
  - Edge and interpolation equations can be computed at any point



- Try to minimize work
  - Restrict traversal to primitive bounding box
  - Hierarchical traversal
    - Knock out tiles of pixels (say 4x4) at a time



Incremental Algorithms

- **Some computation can be saved by updating the edge and interpolation equations incrementally:**

$$E(x, y) = Ax + By + C$$
$$E(x + \Delta, y) = A(x + \Delta) + By + C$$
$$= E(x, y) + A \cdot \Delta$$
$$E(x, y + \Delta) = Ax + B(y + \Delta) + C$$
$$= E(x, y) + B \cdot \Delta$$

- **Equations can be updated with a single addition!**

Triangle Setup

- Compute edge equations
  - 3 cross products
- Compute triangle area
  - A few additions
- Cull zero area and back-facing triangles and/or flip edge equations
- Compute interpolation equations
  - Matrix/vector product per parameter

Massive Models

100,000,000 primitives

<u>1,000,000 pixels</u>

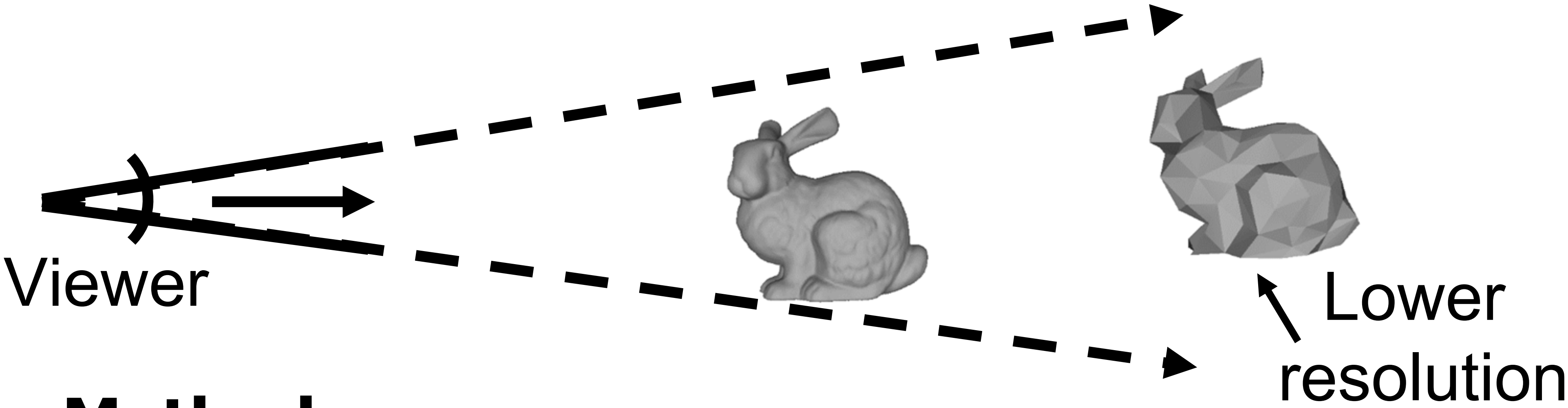
100 visible primitives/pixel

- **Cost to render a single triangle**
  - **Specify 3 vertices**
  - **Compute 3 edge equations**
  - **Evaluate equations one**

St. Mathew models consisting of about 400M triangles (Michelangelo Project)

Multi-Resolution or Levels-of-Detail (LOD) Techniques

- **Basic idea**
  - **Render with fewer triangles when model is farther from viewer**



- **Methods**
  - **Polygonal simplification**

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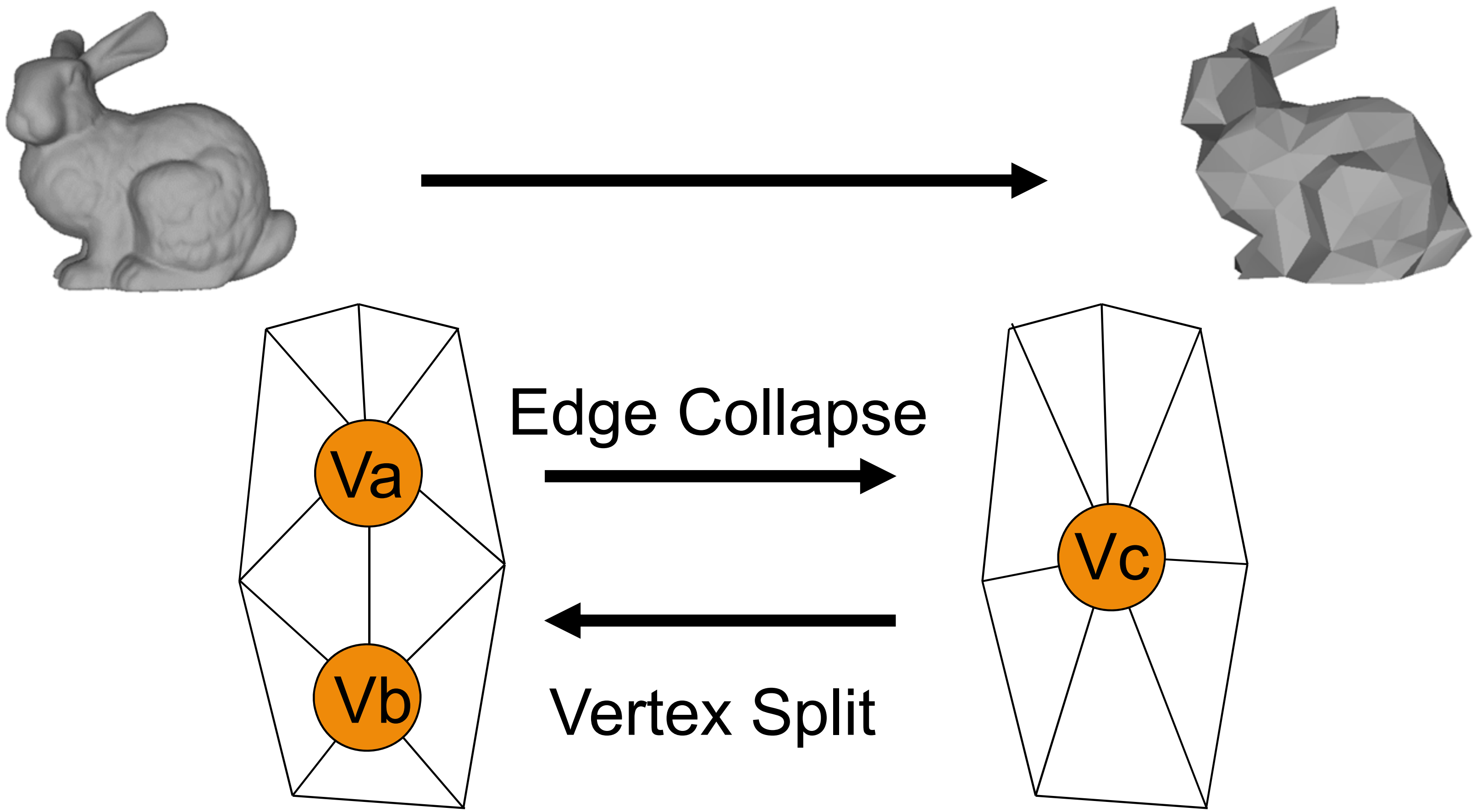
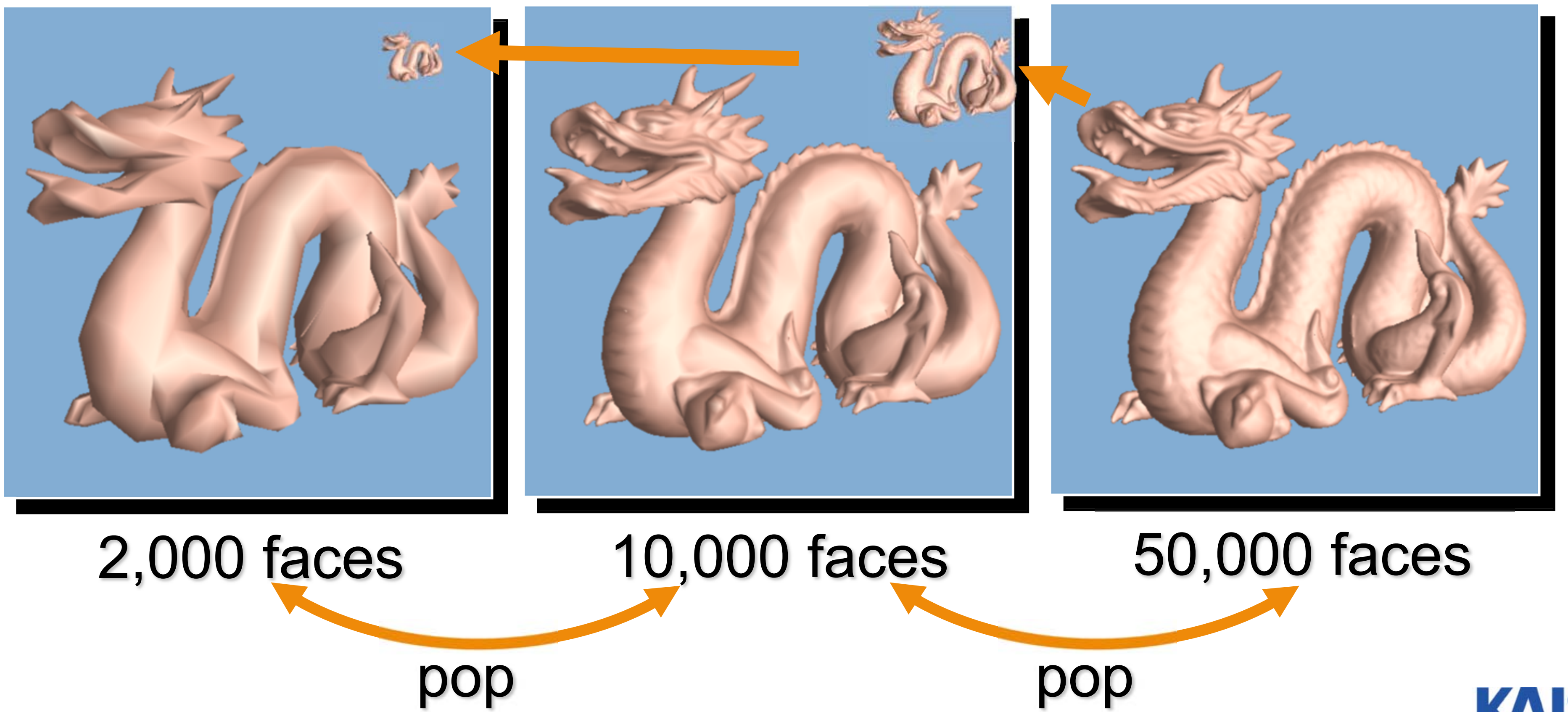
Polygonal Simplification

- **Method for reducing the polygon count of mesh**



Static LODs

- Pre-compute discrete simplified meshes
  - Switch between them at runtime
  - Has very low LOD selection overhead



Excerpted from Hoppe's slides

What if there are so many objects?

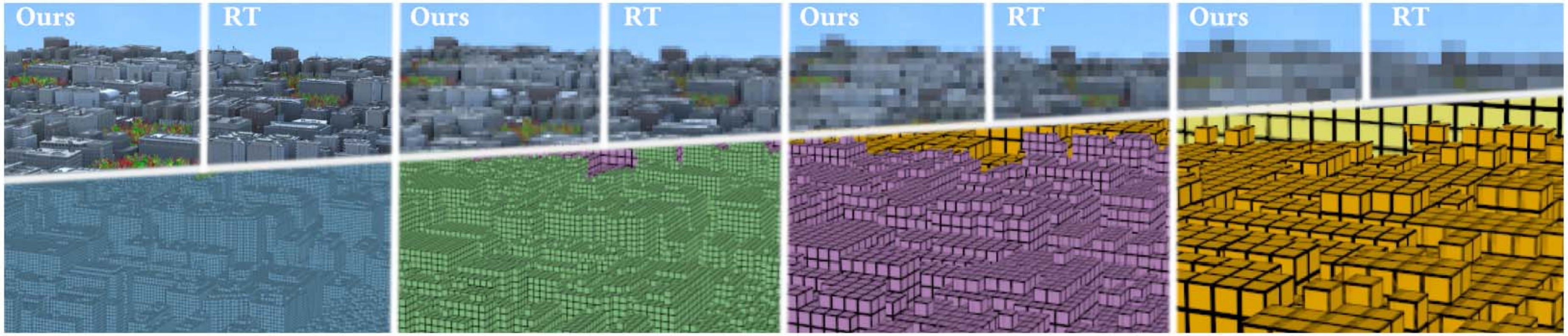
From “cars”, a Pixar movie

Some solution: Stochastic Simplification of Aggregate Detail

Cook et al., ACM SIGGRAPH 2007

Deep Appearance Prefiltering, SIGGRAPH 23

- **Prefiltering complex appearance of the scene using deep learning approaches**



City	Scale 8	Scale 7	Scale 6	Scale 5
Per-pixel memory decrease:	28.1×	44.6×	155×	528×

Class Objectives were:

- **Understand triangle rasterization using edge-equations**
- **Understand mechanics for parameter interpolations**
- **Realize benefits of incremental algorithms**

Homework

- Go over the next lecture slides before the class
- Watch 2 SIGGRAPH videos and submit your summaries before every Mon. class
  - Just one paragraph for each summary
- Submit questions two times during the whole semester

Next Time

- **Illumination and shading**
- **Texture mapping**