Sample Midterm Exam

True or False Questions (/3pts)
1 point for each question

1. Perspective projection belongs to the class of affine projection.
   True ( )   False ( )
2. Two vectors of (1, 1) and (0, 1) are an example of basis vector that can span 2D space.
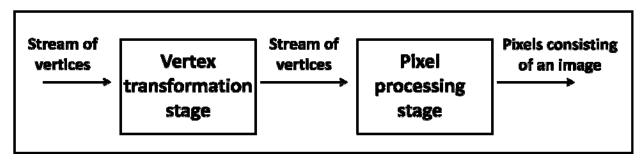   True ( )   False ( )
3. By performing the back-face culling, we can get more than 2 times performance improvement during rasterizing a model consisting of triangles.
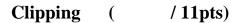   True ( )   False ( )

Rendering Pipeline (/ 7pts)

We studied the rendering pipeline in the class. Although it has many stages, we can simplify those stages into the following two major steps: 1) vertex transformation (e.g., modeling and viewing transformation) step and 2) pixel processing step (e.g., rasterization)

Since these two stages can be performed independently, we can perform these two stages in a parallel manner in graphics hardware. For example, after performing vertex transformation to v1, v2, v3, we perform rasterization step for a triangle consisting of those three vertices. During rasterizing the triangle, we can perform vertex transformation to other vertices, say, v4.

1. Suppose that we increase the image resolution from 500 by 500 to 1K by 1K. Also, suppose that even though we increase the image resolution as described above, we do not observe any performance degradation in terms of total rendering time consisting of times spent on two stages. Could you explain why? (3pts)

2. In this case, if we perform back-face culling in the beginning of the pixel processing stage, will it improve the overall performance? Why do you think so? (2pts)

3. How about performing view-frustum culling right before the vertex transformation stage? Can it improve the performance? Why do you think so? (2pts)
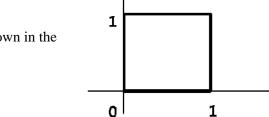
Clipping   (/ 11pts)

Suppose that we use the following 4bit binary outcode assignment:

                    Above Below Right Left
              X   X   X   X

We will use such assignment method in a unit square clipping window shown in the right.

1. Determine the outcodes of the following points: (3pts)
        a. (0.3, 1.2):   _____
        b. (-0.1, -0.1):   _____
        c. (0.2, 0.3):   _____

2. Can a line segment with vertex outcodes (1010, 0110) be trivially rejected?   (2pts) _____

3. Can a line segment with vertex outcodes (1001, 0100) be trivially rejected?   (2pts) _____

4. What is the maximum number of times that a line can be clipped using the Cohen-Sutherland algorithm that we learned at the class?   (2pts) _____
And, draw such a line for the clipping region shown above. (2pts)

Mathematical Foundations   (/6pts)

1. What is the 4 x 4 matrix in homogeneous coordinate form corresponding to a 3D translation by (a, b, c)?   (2pts)

2. a) Show that the following matrix A is orthogonal matrix. Show it with the properties of basis vectors computed from rows (or columns) of the matrix.
(2pts)

$$A = \begin{bmatrix} 0.71 & 0.0 & 0.71 \\ 0.0 & 1.0 & 0.0 \\ -0.71 & 0.0 & 0.71 \end{bmatrix}$$

    b) Compute the inverse matrix of A (2pts)

$$A^{-1} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

Culling   (/10pts)

The following equation shows the implicit plane equation that we learned in the class:
$f(x, y, z) = n_x x + n_y y + n_z z - d$, where (n_x, n_y, n_z) is the normal of the plane and is normalized.

1. If we plugged a position of a point (x, y, z) into the plane equation, we learn that it will give the (signed) distance between the plane and the point.
Prove that f(x, y, z) gives the distance between the point (x, y, z) and the plane. (4pts)

2. We want to cull objects consisting of vertices if those objects are outside a certain plane represented by the implicit plane equation shown above. Suppose that testing a vertex against the implicit equations takes 5ms and rendering any polygon takes 20ms. Will the culling operation improve the performance of rendering a polygon consisting of 3 vertices? _____ Will the culling operation improve the performance of rendering a polygon consisting of 10 vertices? _____ (2pts)

3. What may be a reasonably modification on the culling method explained above in order to make the culling method improve the performance of rendering irrespective of the number of vertices of the polygon? (2pts) What is the disadvantage of your method? (2pts)

Rotations (/8pts)

In class, we studied how to rotate a 3D point $\dot{x}$ with an amount of θ along a rotation axis $\hat{r}$.
1. Compute three natural basis vectors that are used to derive the transformation of the rotation? (3pts)

2. Derive an equation that gives us the rotated point based on the computed basis vectors (3pts); you need to show steps of the derivation.

3. We prefer to represent the computed equation above into a matrix format. Why? (2pts)