CS380: Computer Graphics
Triangle Rasterization

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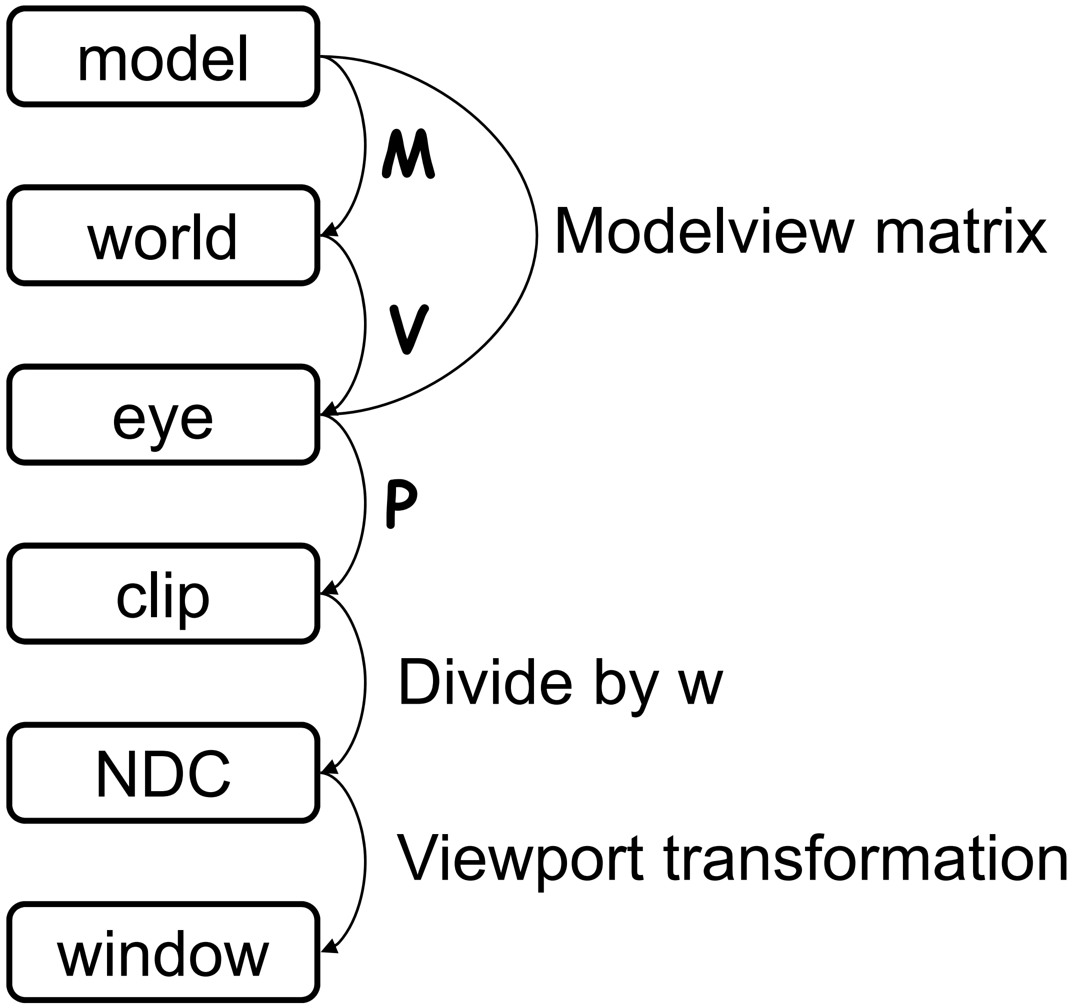
(윤성의)

Course URL:
http://sglab.kaist.ac.kr/~sungeui/CG/

Class Objectives (Ch. 7)

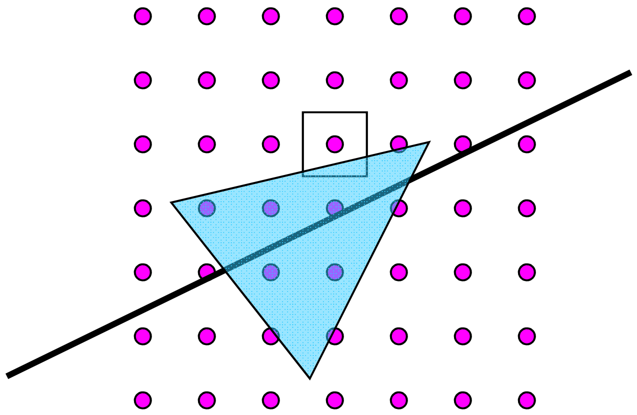
- **Understand triangle rasterization using edge-equations**
- **Understand mechanics for parameter interpolations**
- **Realize benefits of incremental algorithms**

Coordinate Systems



Primitive Rasterization

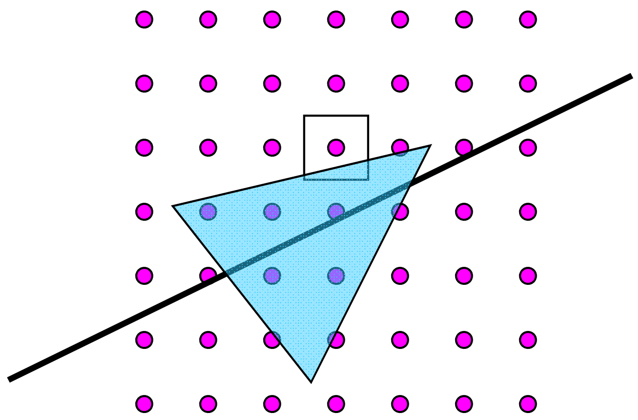
- **Rasterization converts vertex representation to pixel representation**



- **Coverage determination**
  - **Computes which pixels (samples) belong to a primitive**
- **Parameter interpolation**
  - **Computes parameters at covered pixels from parameters associated with primitive vertices**

Coverage Determination

- **Coverage is a 2D sampling problem**
- **Possible coverage criteria:**
  - **Distance of the primitive to sample point (often used with lines)**
  - **Percent coverage of a pixel (used to be popular)**
  - **Sample is inside the primitive (assuming it is closed)**



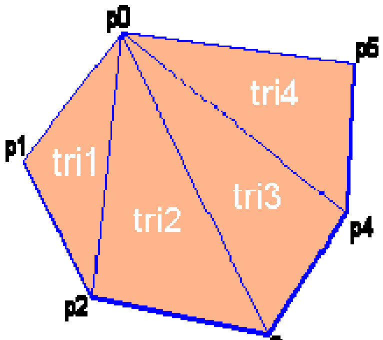
Why Triangles?

- **Triangles are convex**

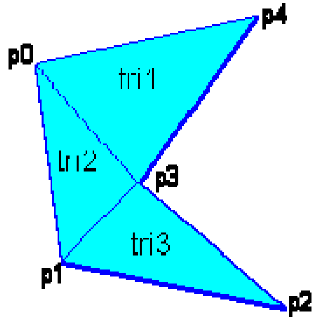
- **Why is convexity important?**
  - **Regardless of a triangle's orientation on the screen a given scan line will contain only a single segment or *span* of that triangle**
  - **Simplify rasterization processes**

Why Triangles?

- **Arbitrary polygons can be decomposed into triangles**



Convex polygon

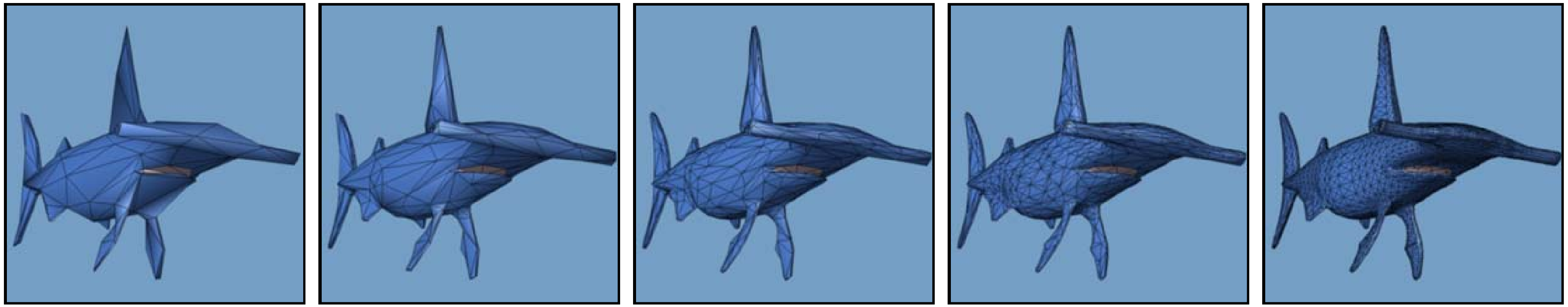


Non-convex

- **Decomposing a convex n-sided polygon is trivial**
  - **Suppose the polygon has ordered vertices $\{v_0, v_1, \ldots v_n\}$**
  - **It can be decomposed into triangles $\{(v_0,v_1,v_2), \{v_0,v_2,v_3), (v_0,v_i,v_{i+1}), \ldots (v_0,v_{n-1},v_n)\}$**
- **Decomposing a non-convex polygon is non-trivial**
  - **Sometimes have to introduce new vertices**

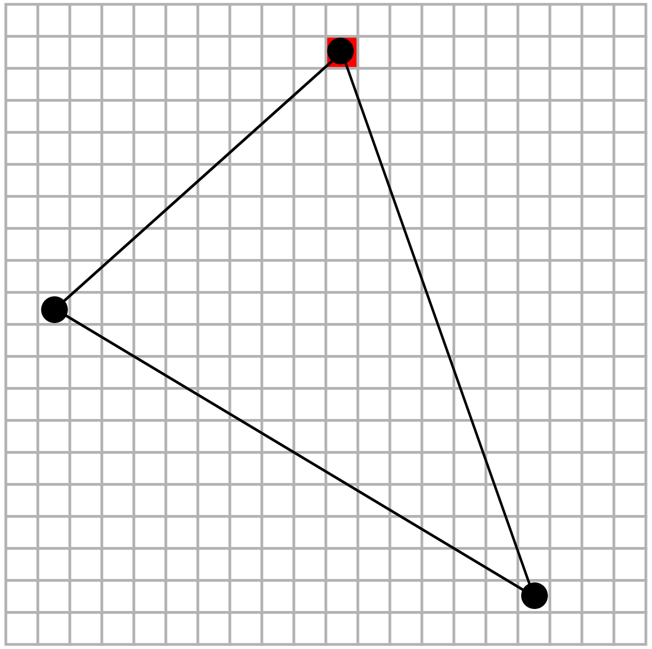
Why Triangles?

- **Triangles can approximate any 2-dimensional shape (or 3D surface)**
  - **Polygons are a locally linear (planar) approximation**
- **Improve the quality of fit by increasing the number edges or faces**



Scanline Triangle Rasterizer

- **Walk along edges and process one scanline at a time; also called edge walk method**
- **Rasterize spans between edges**



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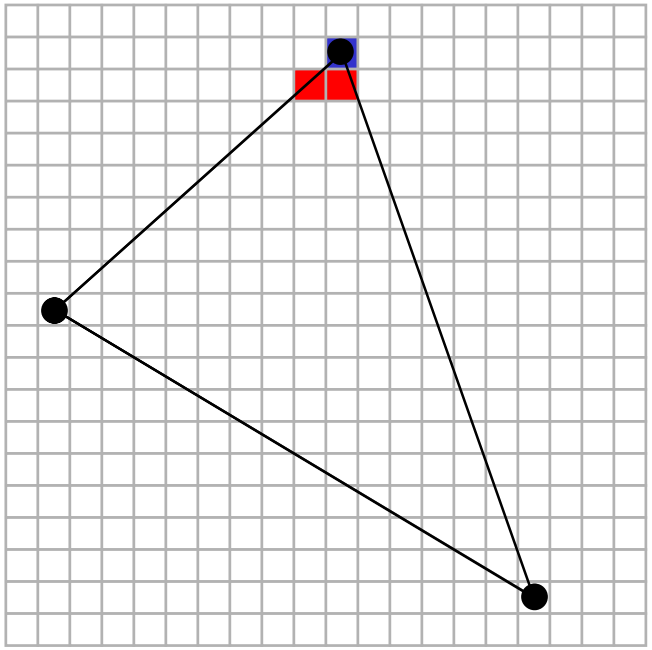

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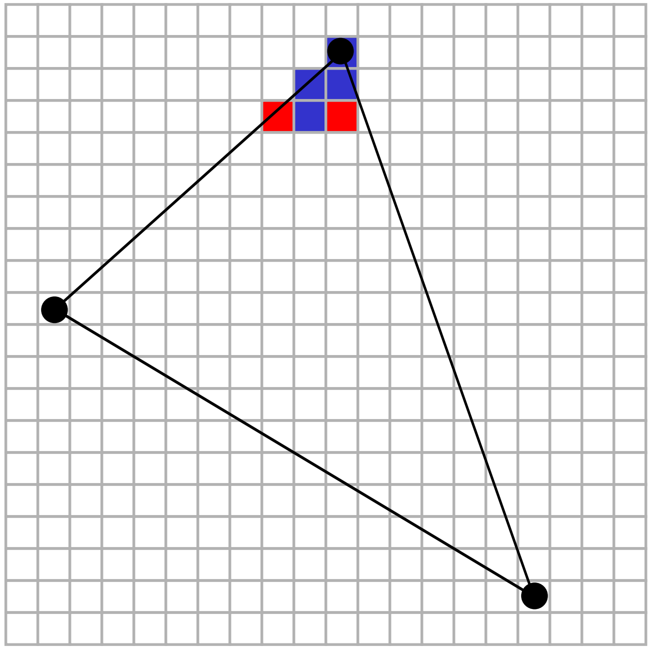

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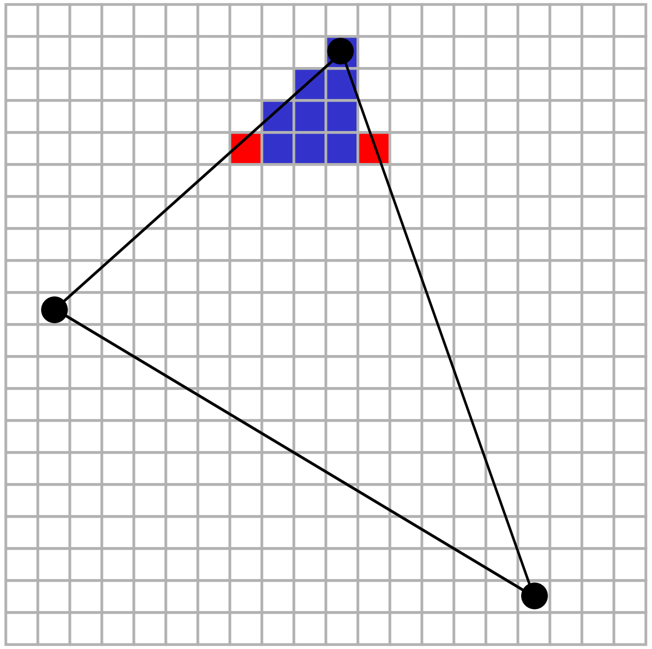

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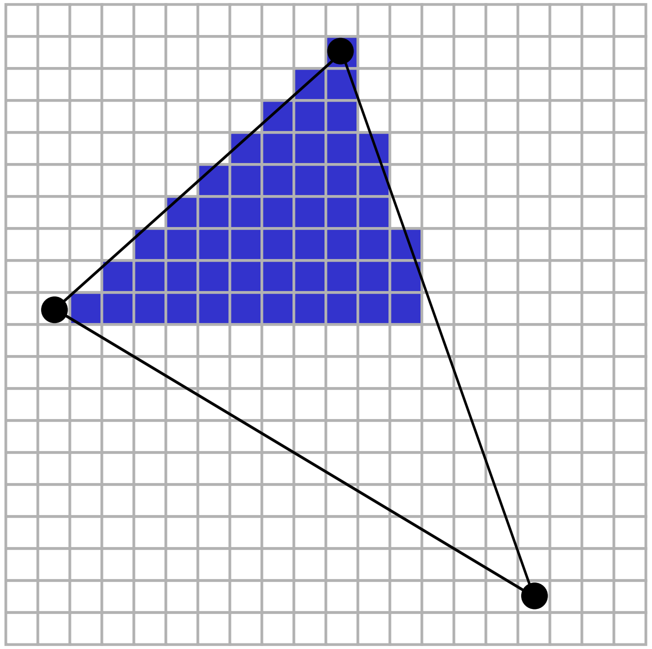

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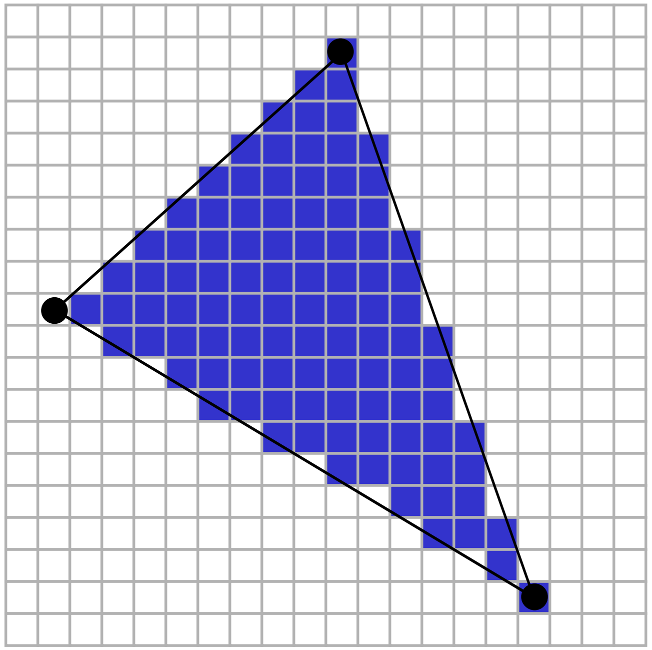
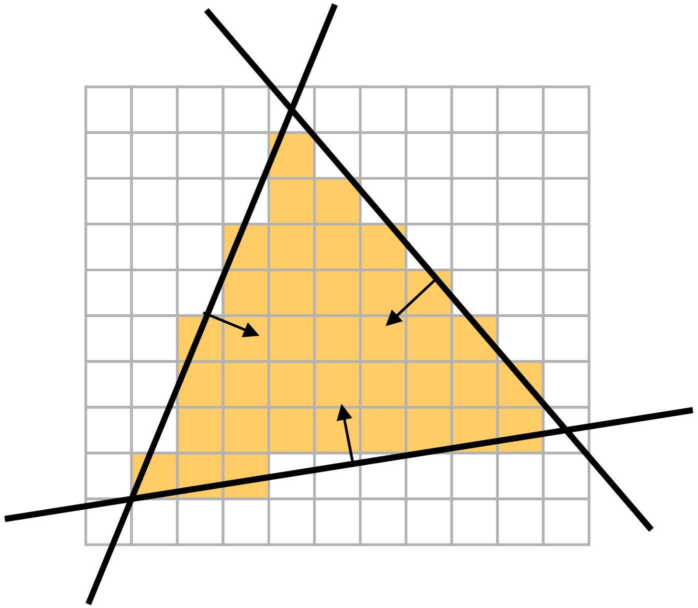


Scanline Rasterization

- **Advantages:**
  - **Can be made quite fast**
  - **Low memory usage for small scenes**
  - **Do not need full 2D z-buffer (can use 1D z-buffer on the scanline)**
- **Disadvantages:**
  - **Does not scale well to large scenes**
  - **Lots of special cases**

Rasterizing with Edge Equations

- Compute edge equations from vertices
- Compute interpolation equations from vertex parameters
- Traverse pixels evaluating the edge equations
- Draw pixels for which all edge equations are positive
- Interpolate parameters at pixels



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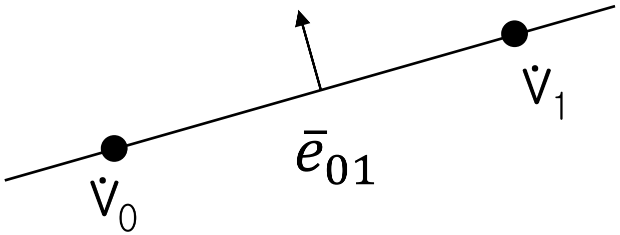
Edge Equation Coefficients

- **The cross product between 2 homogeneous points generates the line between them**



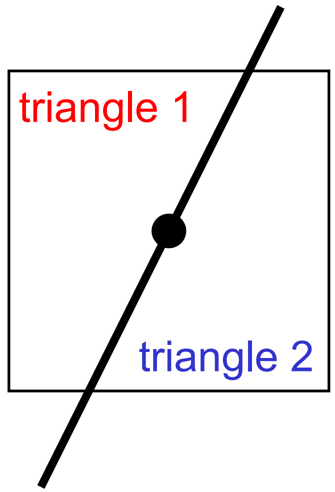
$$
\begin{aligned}
\bar{e} &= \dot{v}_0 \times \dot{v}_1 \\
&= [x_0 \quad y_0 \quad 1]^t \times [x_1 \quad y_1 \quad 1]^t \\
&= [\underbrace{(y_0 - y_1)}_{A_{01}} \quad \underbrace{(x_1 - x_0)}_{B_{01}} \quad \underbrace{(x_0 y_1 - x_1 y_0)}_{C_{01}}]
\end{aligned}
$$

$$E(x,y) = \bar{e}_{01}(x,y) = A_{01}x + B_{01}y + C_{01}$$

- **A pixel at (x,y) is "inside" an edge if E(x,y)>0**

Shared Edges

- **Suppose two triangles share an edge. Which covers the pixel when the edge passes through the sample (E(x,y)=0)?**

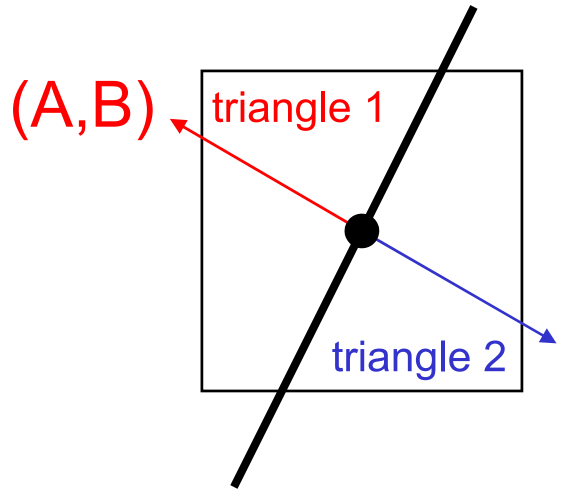


- **Both**
  - **Pixel color becomes dependent on order of triangle rendering**
  - **Creates problems when rendering transparent objects - "double hitting"**
- **Neither**
  - **Missing pixels create holes in otherwise solid surface**
- **We need a consistent tie-breaker!**

Shared Edges

- **A common tie-breaker:**

$$\text{bool } t = \begin{cases} A > 0 & \text{if } A \neq 0 \\ B > 0 & \text{otherwise} \end{cases}$$

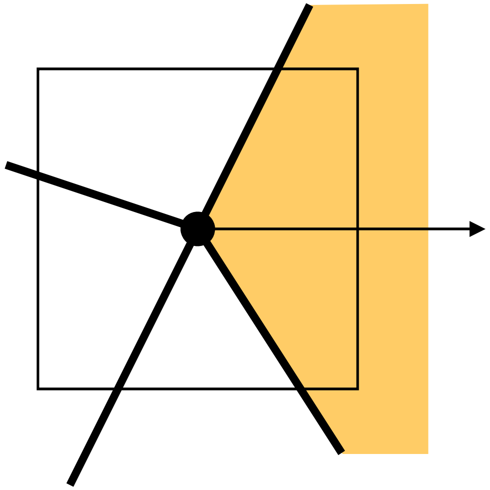


- **Coverage determination becomes**

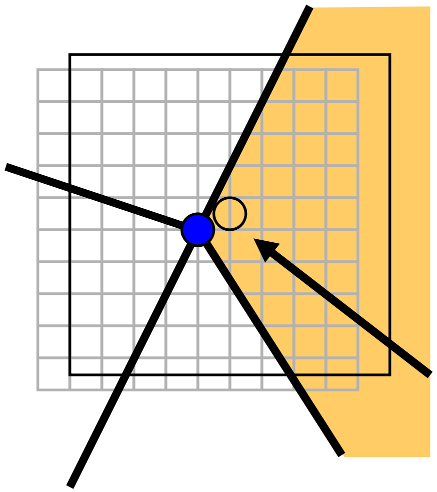
if(E(x,y) >0 || (E(x,y)==0 && t))

pixel is covered

Shared Vertices



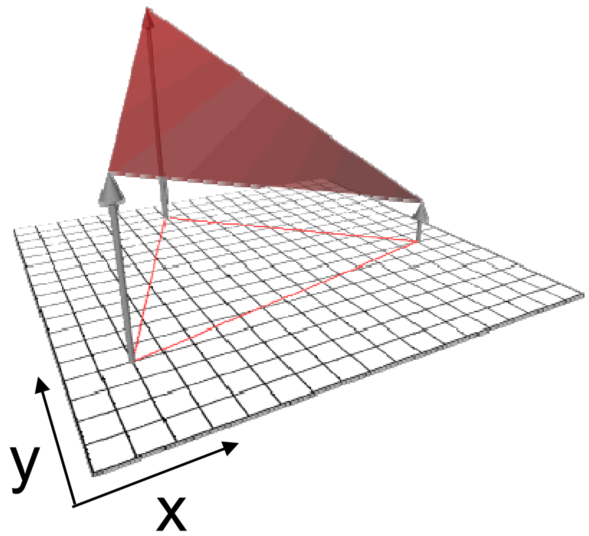
- Use "inclusion direction" as a tie breaker
- Any direction can be used



- Snap vertices to subpixel grid and displace so that no vertex can be at the pixel center

Interpolating Parameters

- **Specify a parameter, say redness (r) at each vertex of the triangle**
  - **Linear interpolation creates a planar function**



$$r(x,y) = A_r x + B_r y + C_r$$

Solving for Linear Interpolation Equations

- **Given the redness of the three vertices, we can set up the following linear system:**

$$\begin{bmatrix} r_0 & r_1 & r_2 \end{bmatrix} = \begin{bmatrix} A_r & B_r & C_r \end{bmatrix} \begin{bmatrix} x_0 & x_1 & x_2 \\ y_0 & y_1 & y_2 \\ 1 & 1 & 1 \end{bmatrix}$$

with the solution:

$$\begin{bmatrix} A_r & B_r & C_r \end{bmatrix} = \begin{bmatrix} r_0 & r_1 & r_2 \end{bmatrix} \frac{\begin{bmatrix} (y_1 - y_2) & (x_2 - x_1) & (x_1 y_2 - x_2 y_1) \\ (y_0 - y_2) & (x_2 - x_0) & (x_0 y_2 - x_2 y_0) \\ (y_0 - y_1) & (x_1 - x_0) & (x_0 y_1 - x_1 y_0) \end{bmatrix}}{\det \begin{bmatrix} x_0 & x_1 & x_2 \\ y_0 & y_1 & y_2 \\ 1 & 1 & 1 \end{bmatrix}}$$

Triangle Area

$$Area = \tfrac{1}{2}\det\begin{bmatrix} x_0 & x_1 & x_2 \\ y_0 & y_1 & y_2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \tfrac{1}{2}((x_1y_2 - x_2y_1) - (x_0y_2 - x_2y_0) + (x_0y_1 - x_1y_0))$$

$$= \tfrac{1}{2}(C_0 + C_1 + C_2) \quad \text{// they are from edge equations}$$

- **Area = 0 means that the triangle is not visible**
- **Area < 0 means the triangle is back facing:**
  - **Reject triangle if performing back-face culling**
  - **Otherwise, flip edge equations by multiplying by -1**

Interpolation Equation

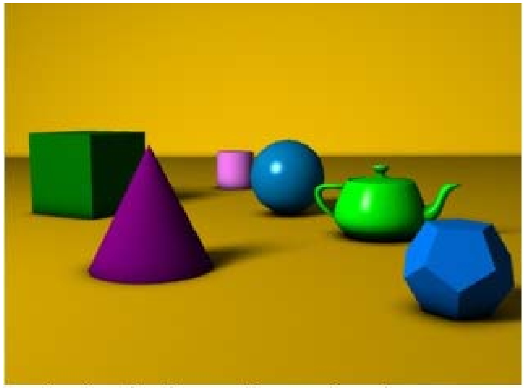
- **The parameter plane equation is just a linear combination of the edge equations**

$$[A_r \quad B_r \quad C_r] = \frac{1}{2 \cdot area}[r_0 \quad r_1 \quad r_2]\begin{bmatrix} \overline{e}_0 \\ \overline{e}_1 \\ \overline{e}_2 \end{bmatrix}$$

$\overline{e_o}, \overline{e_1}, \overline{e_2}$ are vectors of edge equations

Z-Buffering

- **When rendering multiple triangles we need to determine which triangles are visible**
- **Use z-buffer to resolve visibility**
  - **Stores the depth at each pixel**
- **Initialize z-buffer to 1 (far value)**
  - **Post-perspective z values lie between 0 and 1**
- **Linearly interpolate depth (z_{tri}) across triangles**
- **If $z_{tri}(x,y)$ < zBuffer[x][y]**
  write to pixel at (x,y)
  zBuffer[x][y] = $z_{tri}(x,y)$



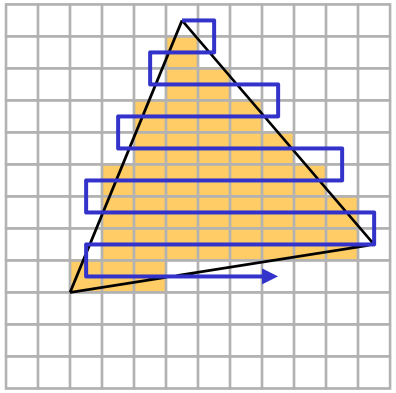
A simple three dimensional scene

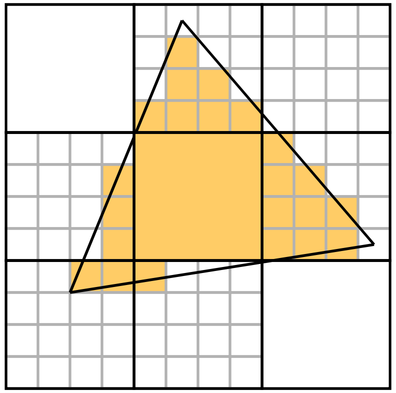
Z-buffer representation

image from wikipedia.com

Traversing Pixels

- Free to traverse pixels
  - Edge and interpolation equations can be computed at any point
- Try to minimize work
  - Restrict traversal to primitive bounding box
  - Hierarchical traversal
    - Knock out tiles of pixels (say 4x4) at a time
    - Test corners of tiles against equations
    - Test individual pixels of tiles not entirely inside or outside





Incremental Algorithms

- **Some computation can be saved by updating the edge and interpolation equations incrementally:**

$$
\begin{aligned}
E(x,y) &= Ax + By + C \\
E(x+\Delta, y) &= A(x+\Delta) + By + C \\
&= E(x,y) + A \cdot \Delta \\
E(x, y+\Delta) &= Ax + B(y+\Delta) + C \\
&= E(x,y) + B \cdot \Delta
\end{aligned}
$$

- **Equations can be updated with a single addition!**

Triangle Setup

- Compute edge equations
  - 3 cross products
- Compute triangle area
  - A few additions
- Cull zero area and back-facing triangles and/or flip edge equations
- Compute interpolation equations
  - Matrix/vector product per parameter

Massive Models

100,000,000 primitives

1,000,000 pixels

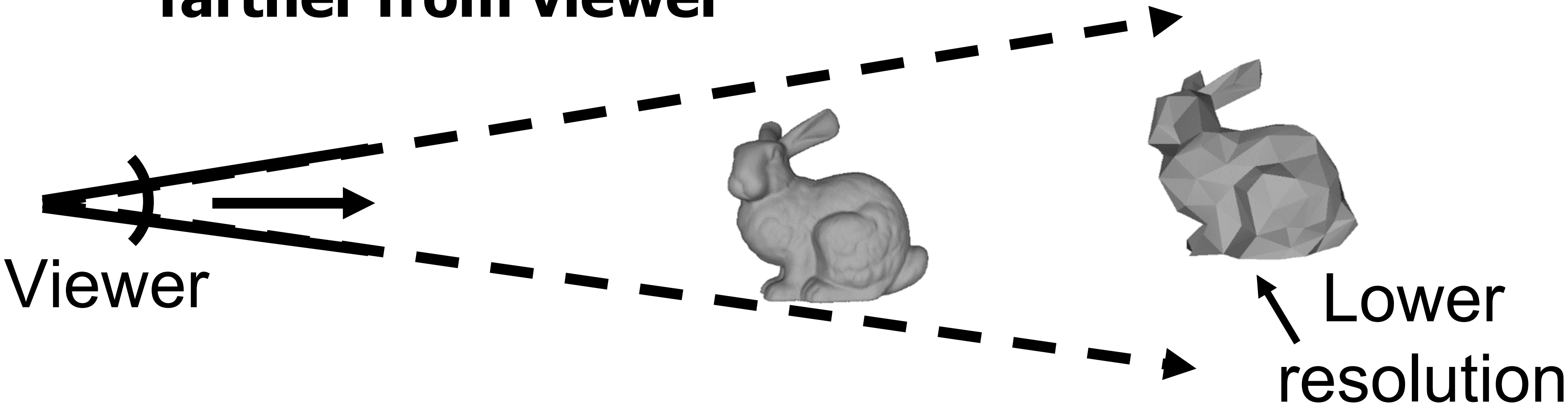
100 visible primitives/pixel

- **Cost to render a single triangle**
  - **Specify 3 vertices**
  - **Compute 3 edge equations**
  - **Evaluate equations one**

St. Mathew models consisting of about 400M triangles (Michelangelo Project)

Multi-Resolution or Levels-of-Detail (LOD) Techniques

- **Basic idea**
  - **Render with fewer triangles when model is farther from viewer**



- **Methods**
  - **Polygonal simplification**

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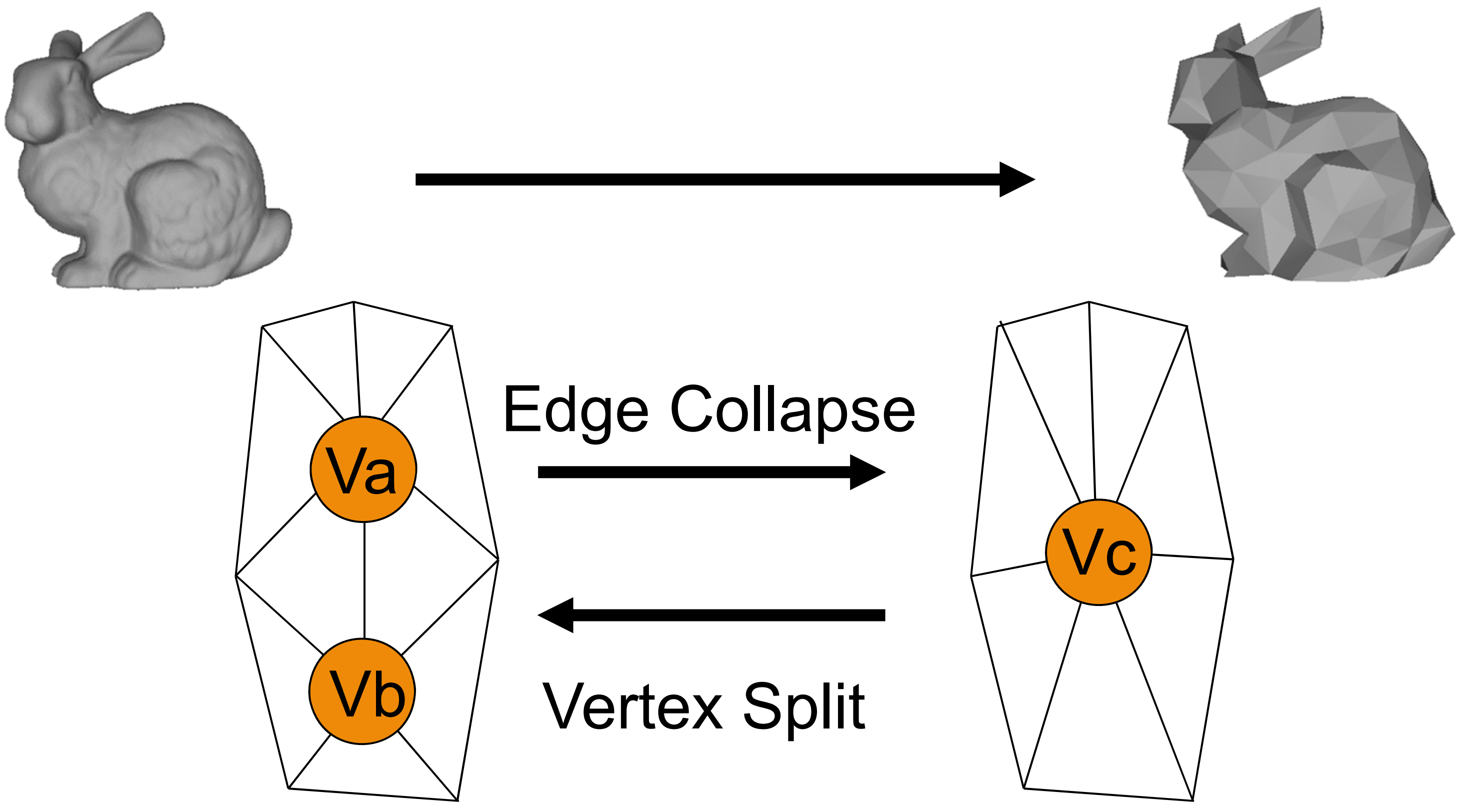
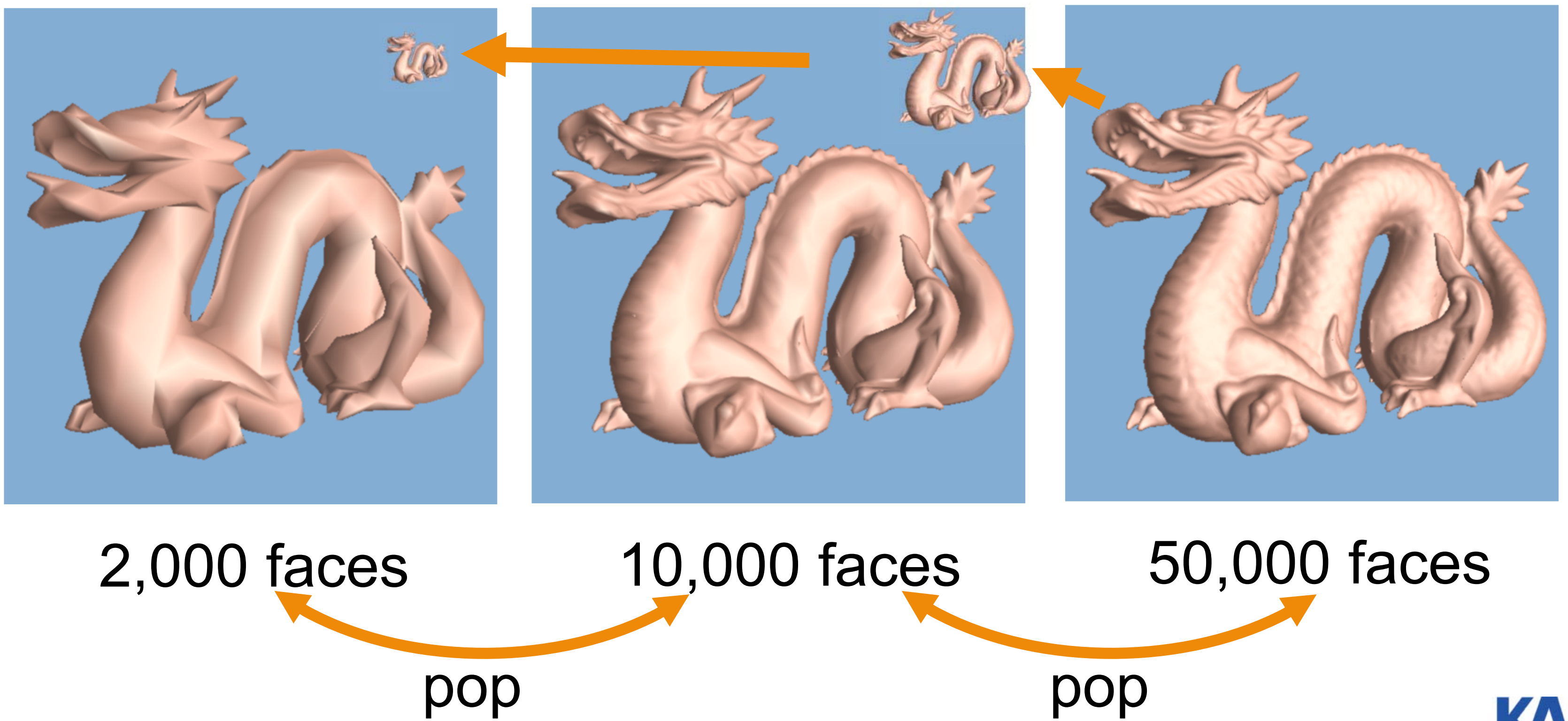
Polygonal Simplification

- **Method for reducing the polygon count of mesh**



Static LODs

- Pre-compute discrete simplified meshes
  - Switch between them at runtime
  - Has very low LOD selection overhead

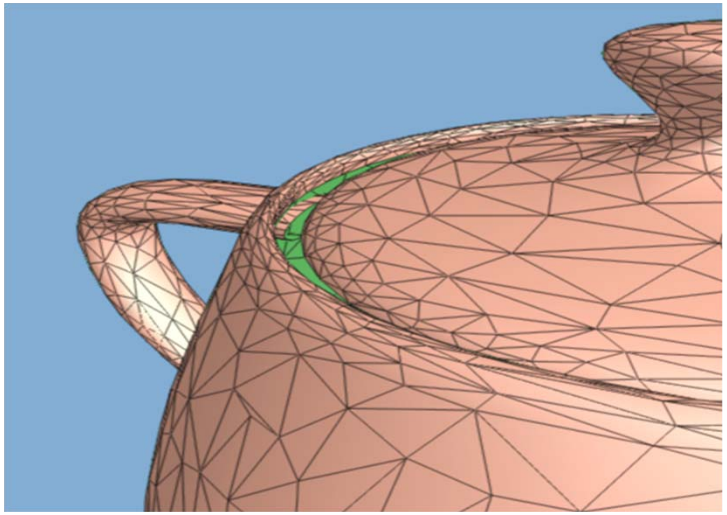


Excerpted from Hoppe's slides

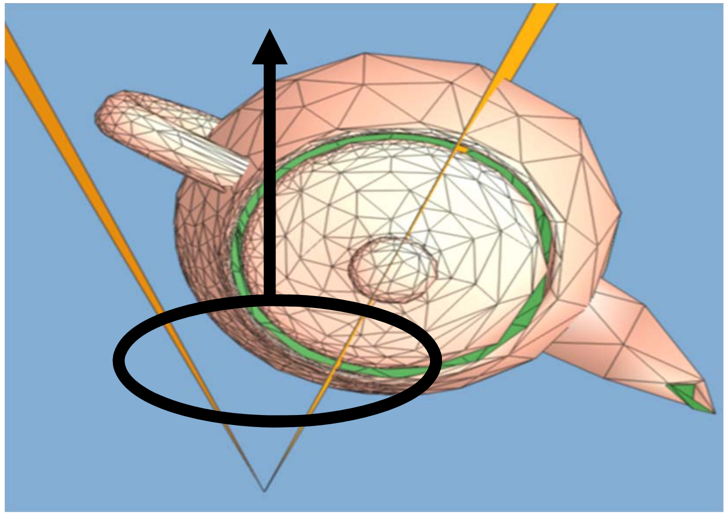
Dynamic Simplification

- **Provides smooth and varying LODs over the mesh [Hoppe 97]**

1st person's view



3rd person's view



Play video

View-Dependent Rendering [Yoon et al., SIG 05]

30 Pixels of error

Pentium 4

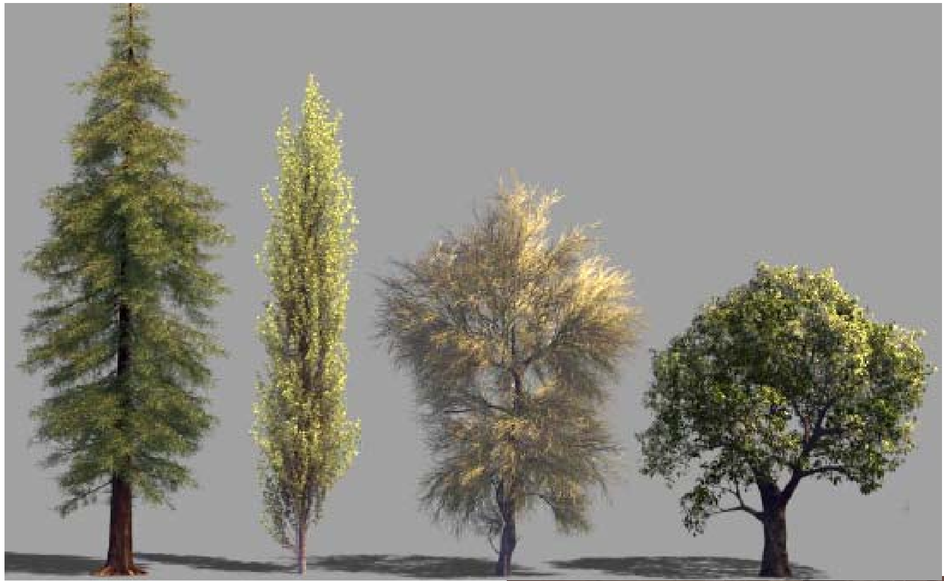
GeForce Go 6800 Ultra

1GB RAM

What if there are so many objects?

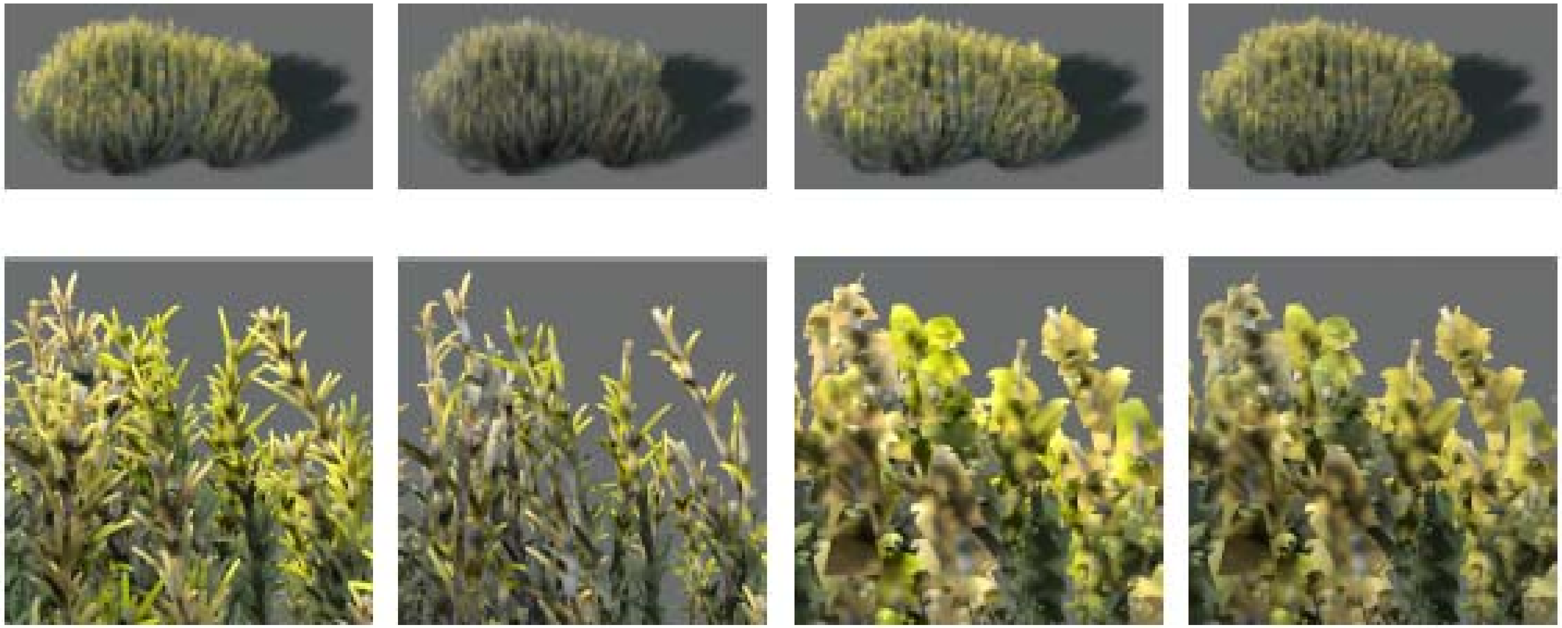
From “cars”, a Pixar movie

What if there are so many objects?



From a Pixar movie

Stochastic Simplification of Aggregate Detail
Cook et al., ACM SIGGRAPH 2007

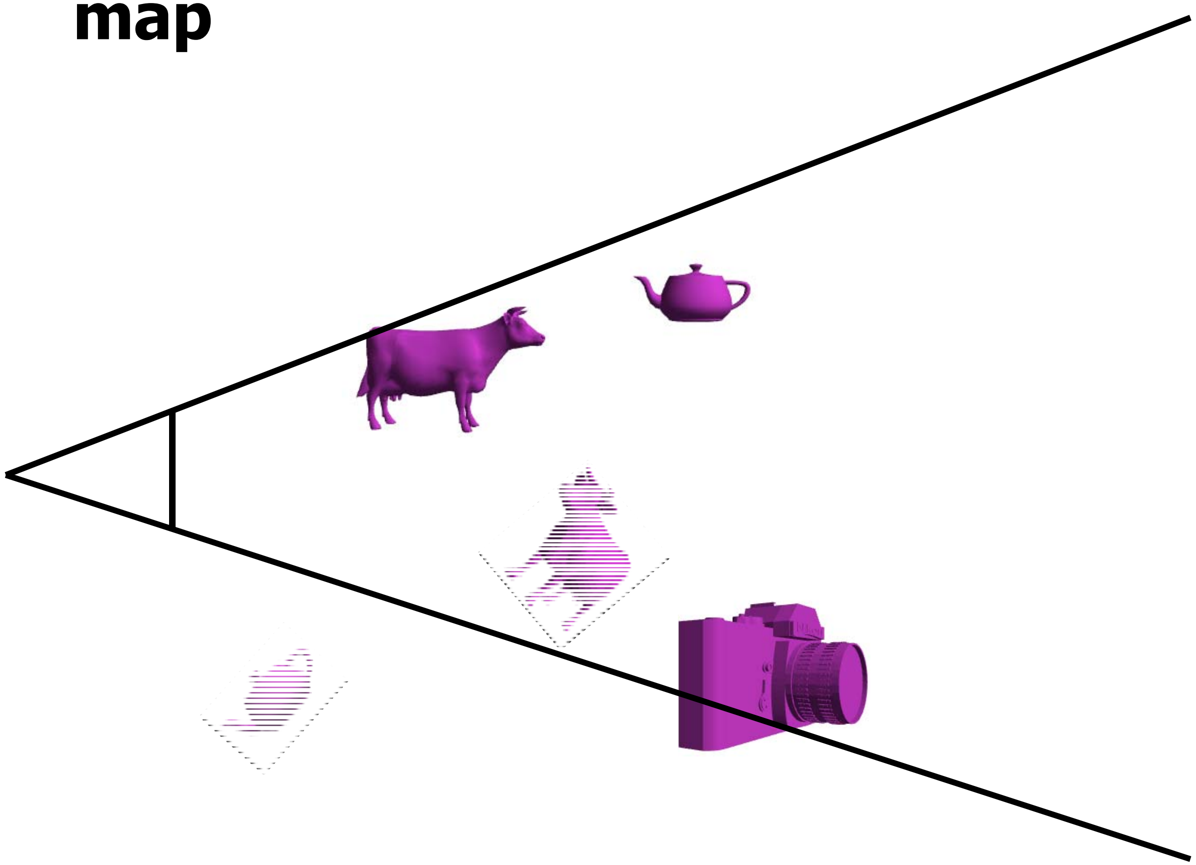


(a) (b) (c) (d)

Figure 2: *Distant views of the plant from Figure 1 with close-ups below: (a) unsimplified, (b) with 90% of its leaves excluded, (c) with area correction, (d) with area and contrast correction.*

Occlusion Culling with Occlusion Queries

- **Render objects visible in previous frame**
  - **Known as occlusion representation or occlusion map**



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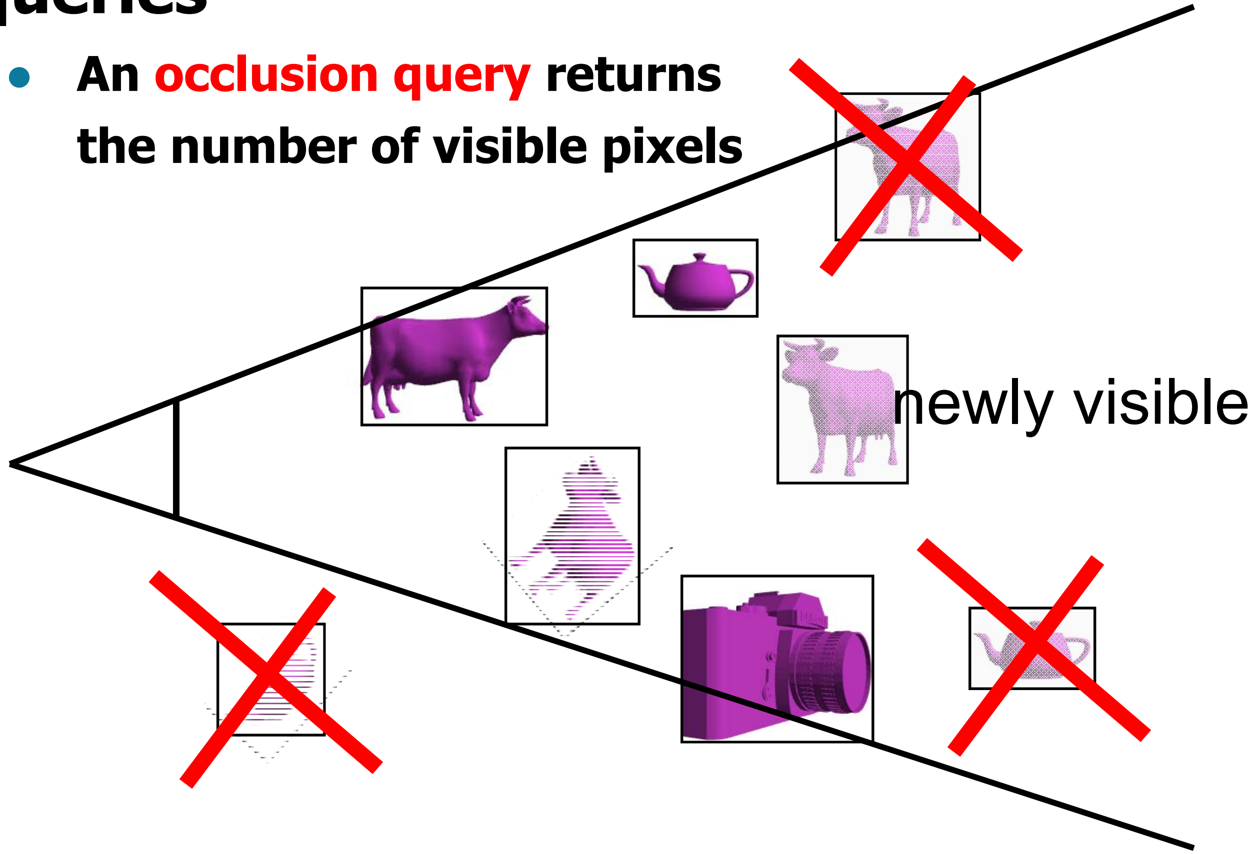
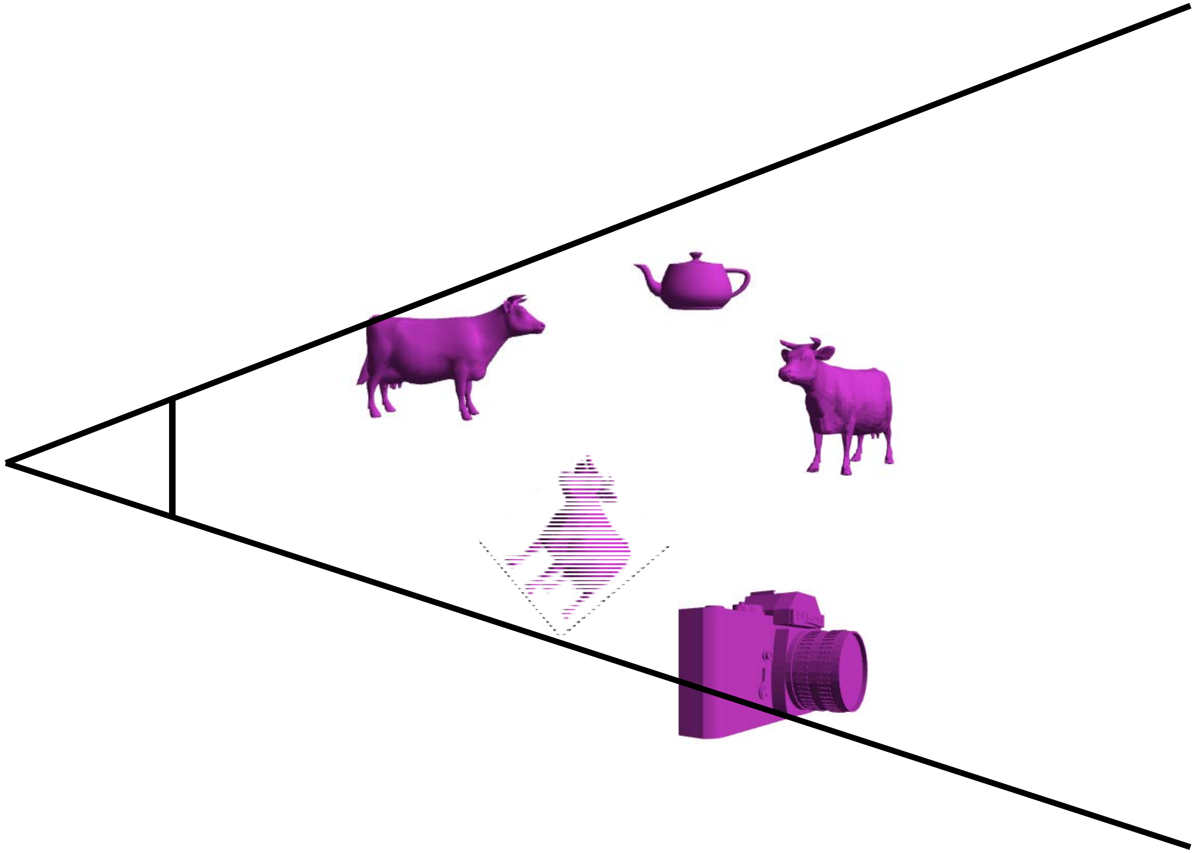
Occlusion Culling with Occlusion Queries

- **Turn off color and depth writes**
- **Render object bounding boxes with occlusion queries**
  - **An** **occlusion query** **returns the number of visible pixels**



Occlusion Culling with Occlusion Queries

- Re-enable color writes
- Render newly visible objects



Class Objectives were:

- Understand triangle rasterization using edge-equations
- Understand mechanics for parameter interpolations
- Realize benefits of incremental algorithms

Next Time

- Illumination and shading
- Texture mapping

Homework

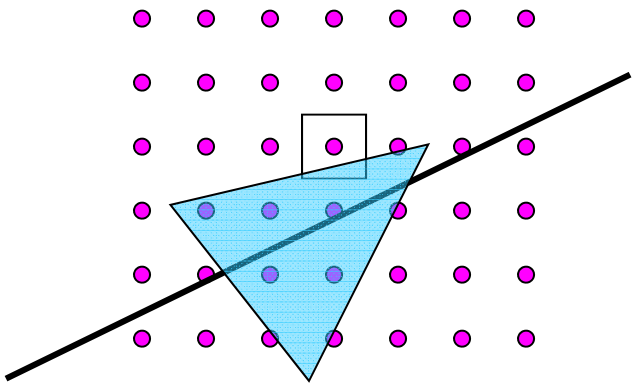
- **Go over the next lecture slides before the class**
- **Watch 2 SIGGRAPH videos and submit your summaries before every Tue. class**
  - **Just one paragraph for each summary**

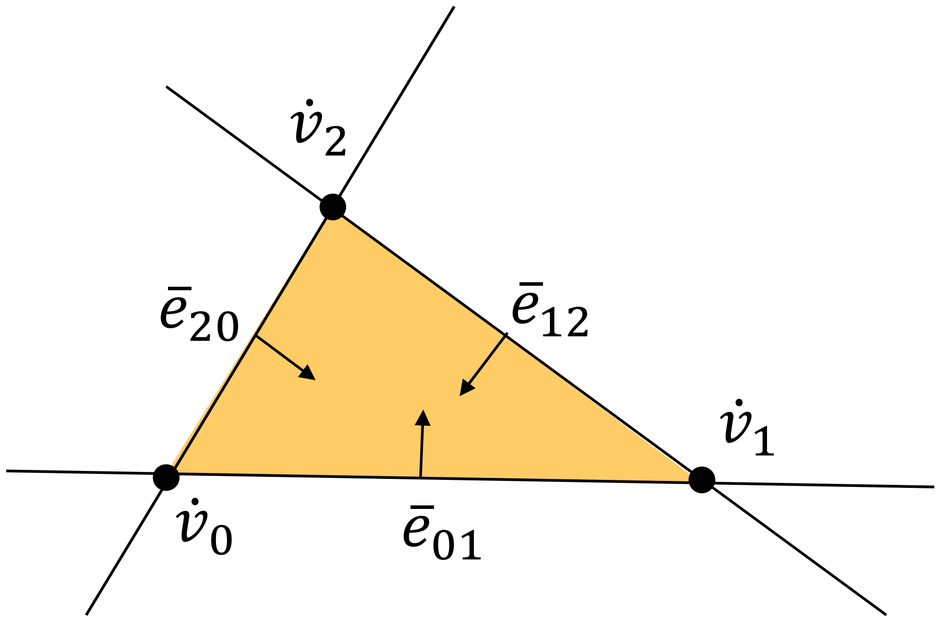
Any Questions?

- **Come up with one question on what we have discussed in the class and submit at the end of the class**
  - **1 for already answered questions**
  - **2 for typical questions**
  - **3 for questions with thoughts or that surprised me**

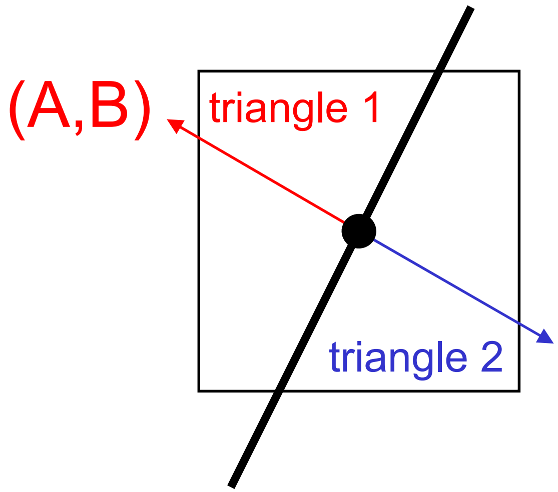
- **Submit at least four times during the whole semester**

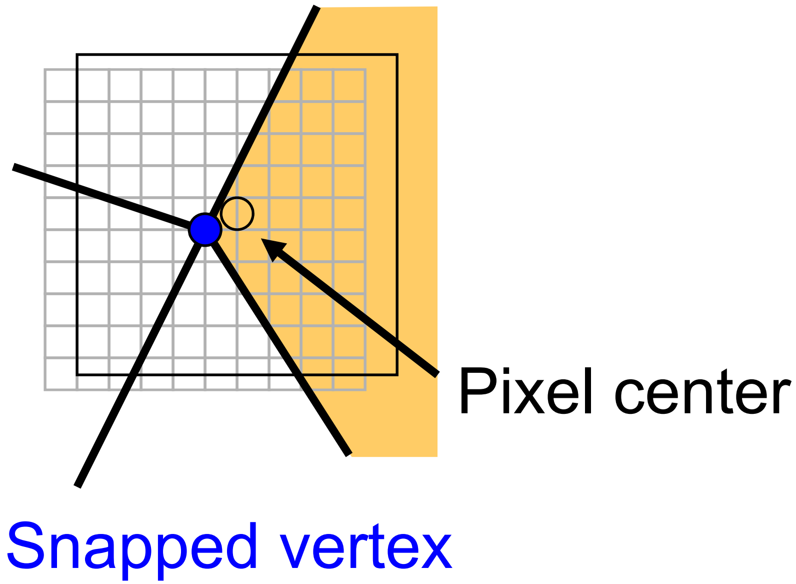
Figs





$\dot{v}_2$
$\bar{e}_{20}$
$\bar{e}_{12}$
$\dot{v}_1$
$\dot{v}_0$
$\bar{e}_{01}$


(A,B)
triangle 1
triangle 2


Pixel center
Snapped vertex