CS380: Computer Graphics
2D Imaging and Transformation

Sung-Eui Yoon
(윤성의)

Course URL:
http://sgvr.kaist.ac.kr/~sungeui/CG

KAIST

Announcements

- **Lab class (video) related to OpenGL and PA sometime in this week**
    - **Check KLMS regularly**

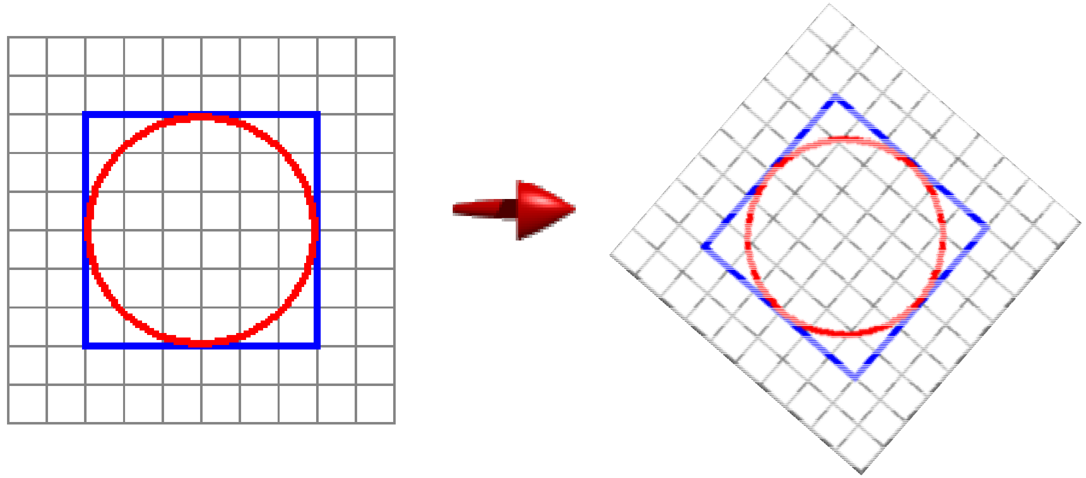
Class Objectives

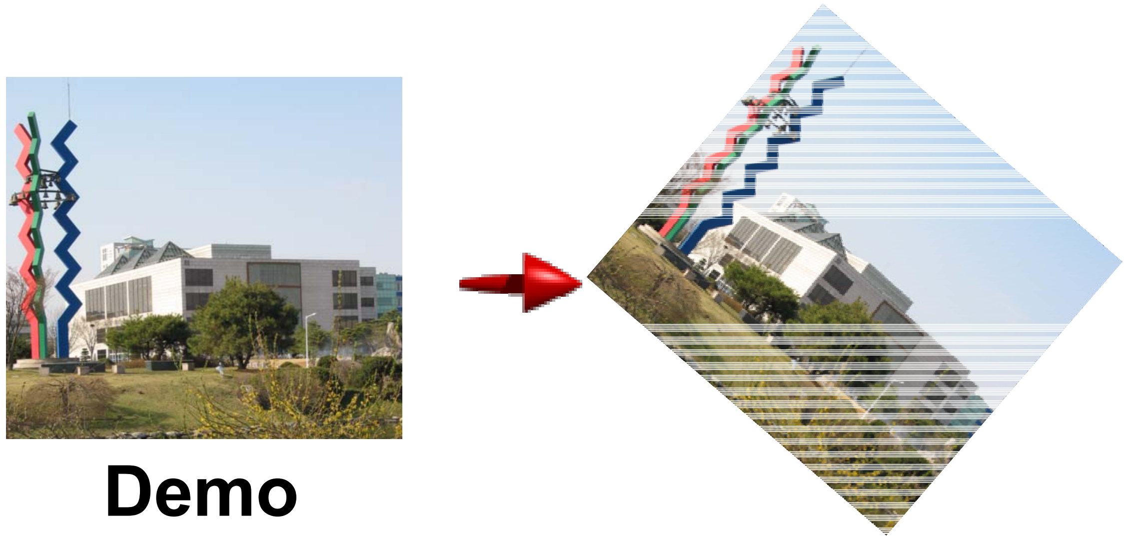
- **Write down simple 2D transformation matrixes**
  - **Understand the homogeneous coordinates and its benefits**
- **Know OpenGL-transformation related API**
  - **Implement idle-based animation method**
- **Covered in 3.2 2D Transformation of my book**

- **At last time:**
  - **OpenGL structure with event-based programming (e.g., callback functions)**
  - **Went over codes of PA1 (Julia set)**

2D Geometric Transforms

- **Functions to map points from one place to another**
- **Geometric transforms can be applied to**
  - **Drawing primitives (points, lines, conics, triangles)**
  - **Pixel coordinates of an image**





Demo

Translation

- **Translations have the following form:**

$$x' = x + t_x$$
$$y' = y + t_y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

- ***inverse function:*** **undoes the translation:**

$$x = x' - t_x$$
$$y = y' - t_y$$

- ***identity*****: leaves every point unchanged**

$$x' = x + 0$$
$$y' = y + 0$$

2D Rotations

- **Another group - rotation about the origin:**

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = R \begin{bmatrix} x \\ y \end{bmatrix}$$

$$R^{-1} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$R_{\theta=0} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Rotations in Series

- **We want to rotate the object 30 degree and, then, 60 degree**

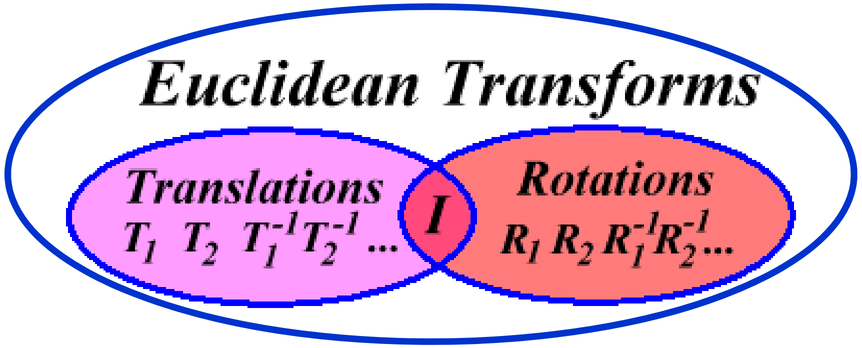
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(60) & -\sin(60) \\ \sin(60) & \cos(60) \end{bmatrix} \begin{bmatrix} \cos(30) & -\sin(30) \\ \sin(30) & \cos(30) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

We can merge multiple rotations into one rotation matrix

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(90) & -\sin(90) \\ \sin(90) & \cos(90) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Euclidean Transforms

- Euclidean group
  - Translations + rotations
  - Rigid body transforms
- Properties:
  - Preserve distances
  - Preserve angles
  - How do you represent these functions?



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

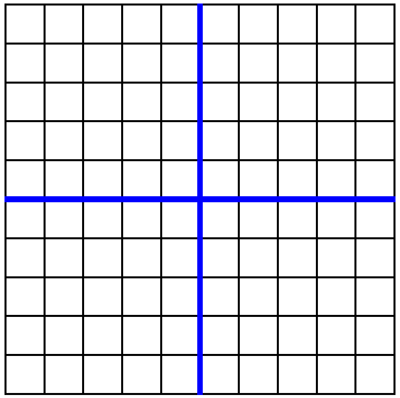
Problems with this Form

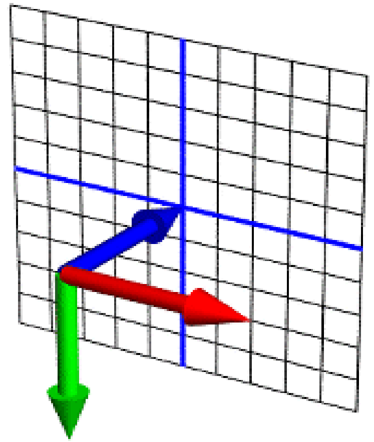
- **Translation and rotation considered separately**
  - **Typically we perform a series of rotations and translations to place objects in world space**
  - **It's inconvenient and inefficient in the previous form**
  - **Inverse transform involves multiple steps**

- **How can we address it?**
  - **How can we represent the translation as a matrix multiplication?**

Homogeneous Coordinates

- **Consider our 2D plane as a subspace within 3D**





(x, y) **(x, y, z)**

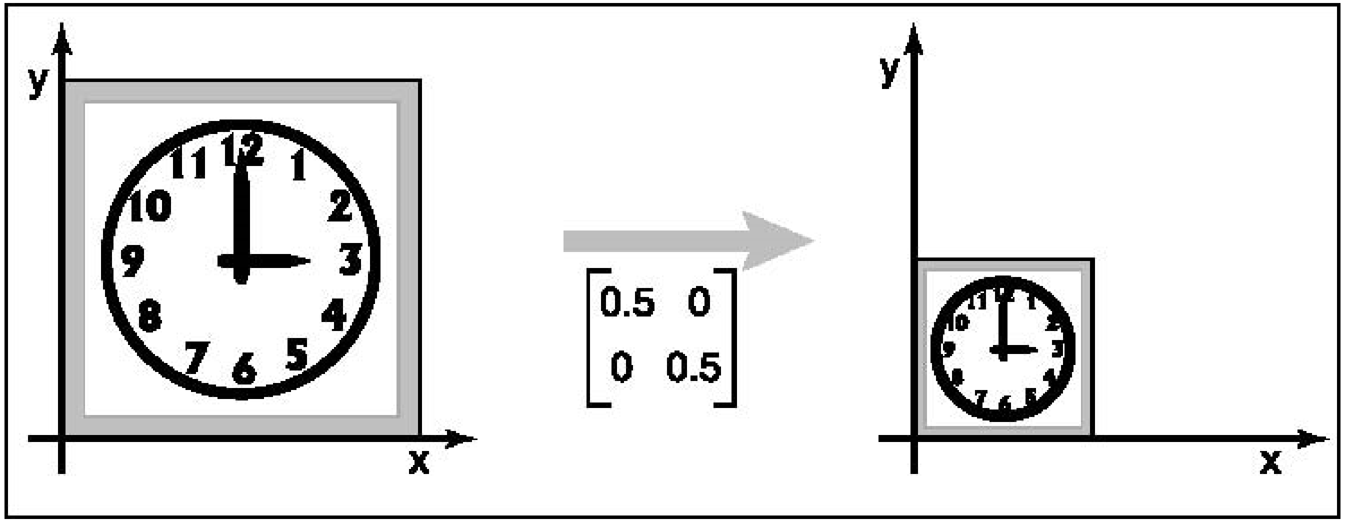
Matrix Multiplications and Homogeneous Coordinates

- **Can use any planar subspace that does not contain the origin**
- **Assume our 2D space lies on the 3D plane *z = 1***
  - **Now we can express all Euclidean transforms in matrix form:**

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

KAIST

Scaling



- **S is a scaling factor**

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

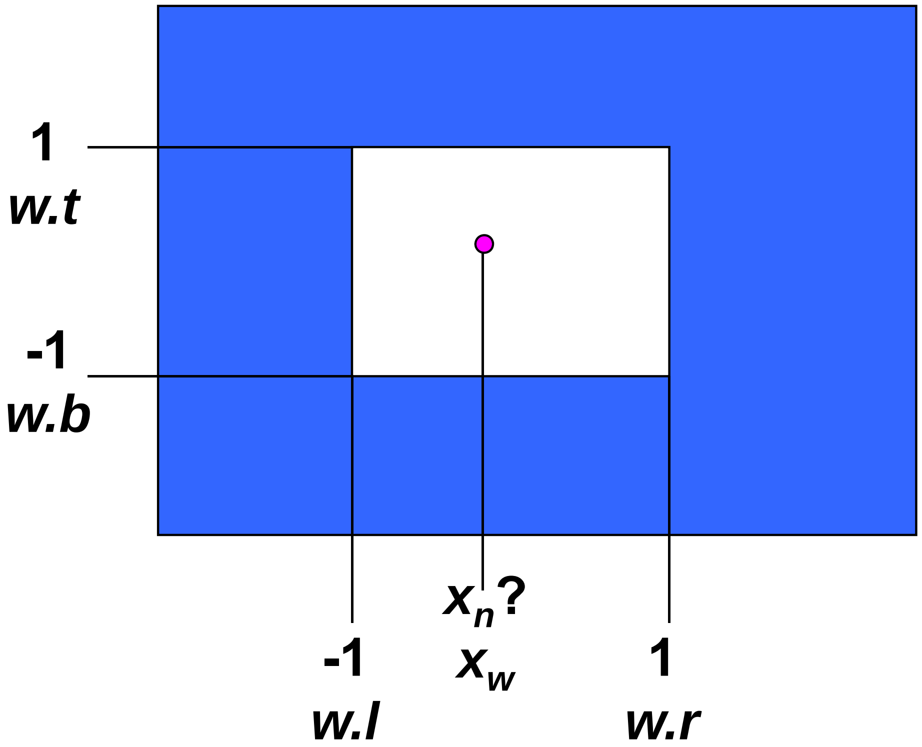
Example: World Space to NDC

$$\frac{x_n - (-1)}{1 - (-1)} = \frac{x_w - (w.l)}{w.r - w.l}$$

$$x_n = 2\frac{x_w - (w.l)}{w.r - w.l} - 1$$

$$x_n = Ax_w + B$$

$$A = \frac{2}{w.r - w.l}, \quad B = -\frac{w.r + w.l}{w.r - w.l}$$



Example: World Space to NDC

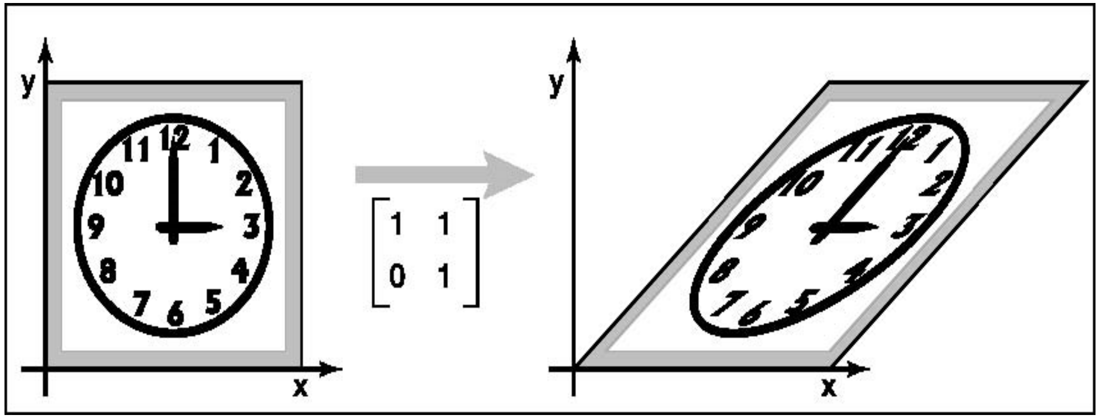
- **Now, it can be accomplished via a matrix multiplication**
  - **Also, conceptually simple**

$$\begin{bmatrix} x_n \\ y_n \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{w.r - w.l} & 0 & -\frac{w.r + w.l}{w.r - w.l} \\ 0 & \frac{2}{w.t - w.b} & -\frac{w.t + w.b}{w.t - w.b} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ 1 \end{bmatrix}$$

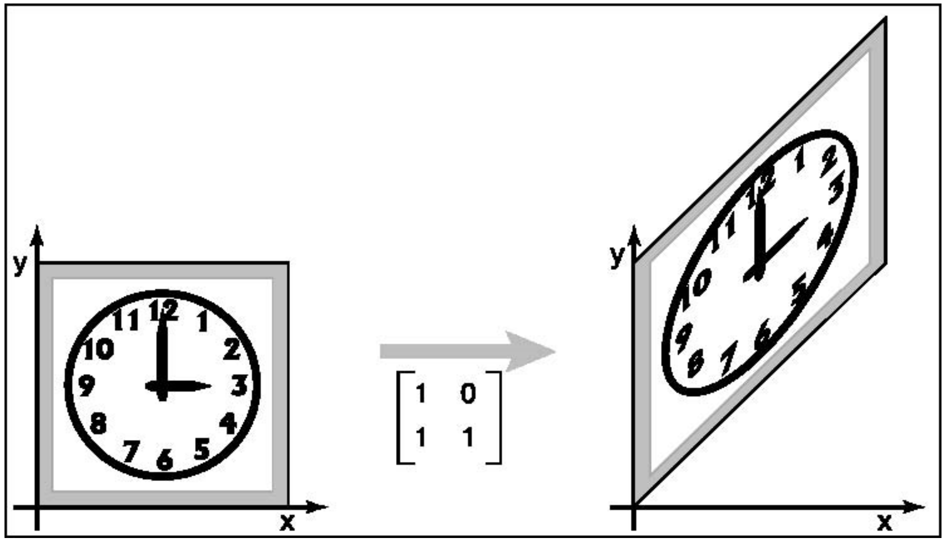
KAIST

Shearing

- Push things sideways
- Shear along x-axis

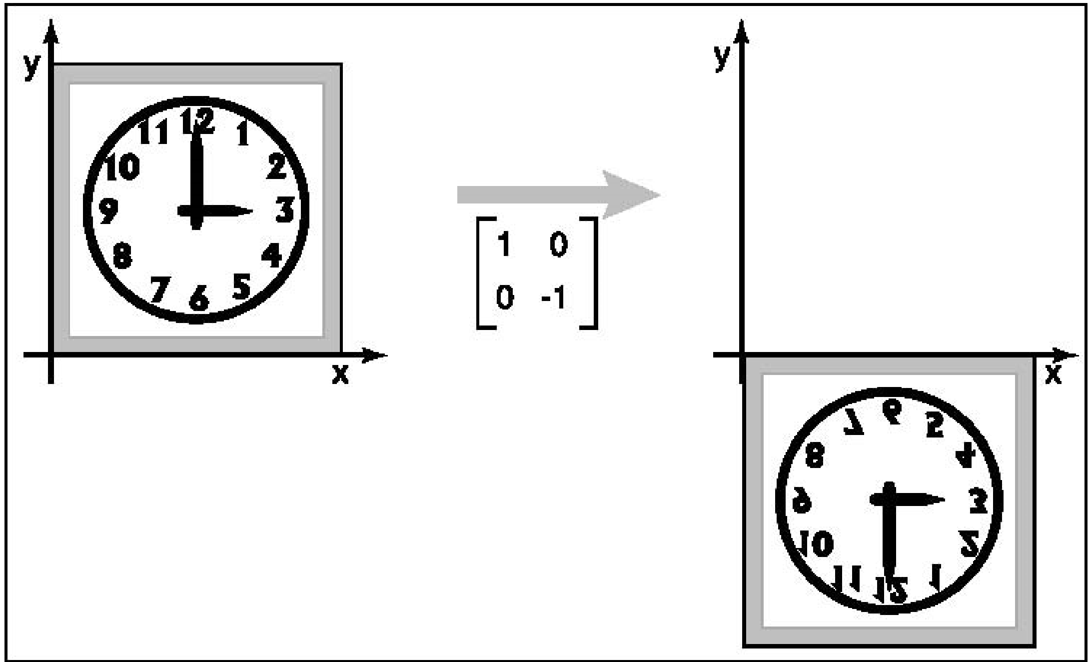


- Shear along y-axis

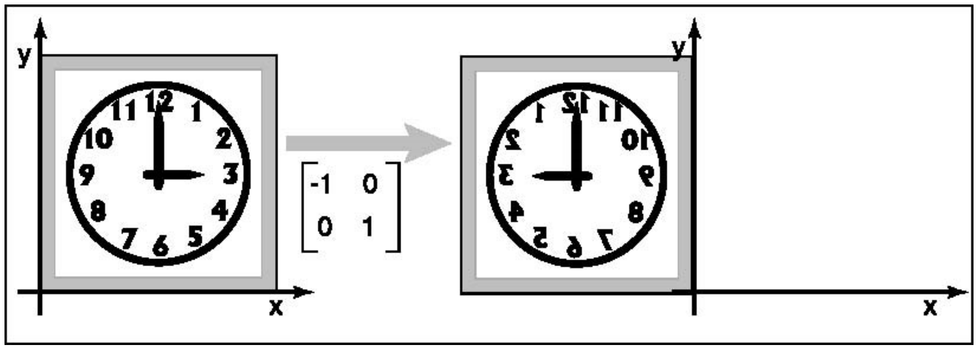


Reflection

- **Reflection about x-axis**

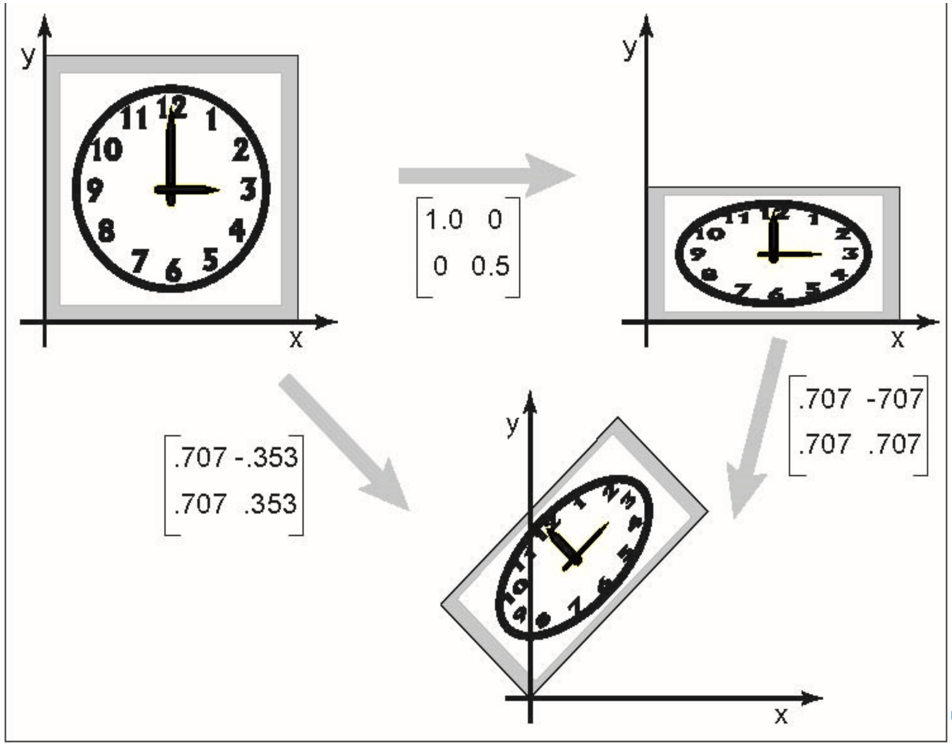


- **Reflection about y-axis**



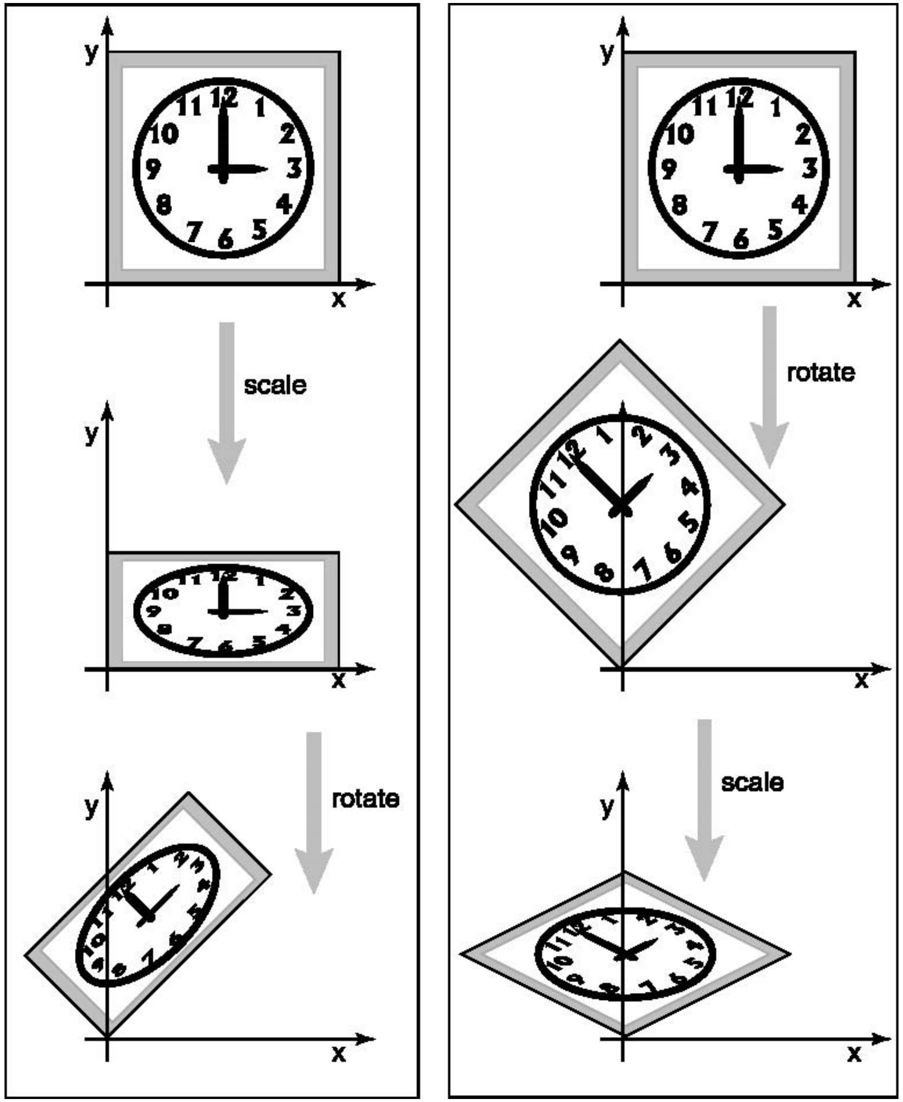
Composition of 2D Transformation

- Quite common to apply more than one transformations to an object
  - E.g., $v_2 = Sv_1$, $v_3 = Rv_2$, where S and R are scaling and Rotation matrix
- Then, we can use the following representation:
  - $v_3 = R(Sv_1)$ or
  - $v_3 = (RS)v_1$
  - why? (associative)



Transformation Order

- **Order of transforms is very important**
  - **Why?**



Affine Transformations

- **Transformed points (x', y') have the following form:**

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- **Combinations of translations, rotations, scaling, reflection, shears**
- **Properties**
  - **Parallel lines are preserved**
  - **Finite points map to finite points**

KAIST

Rigid-Body Transforms in OpenGL

glTranslate (tx, ty, tz);

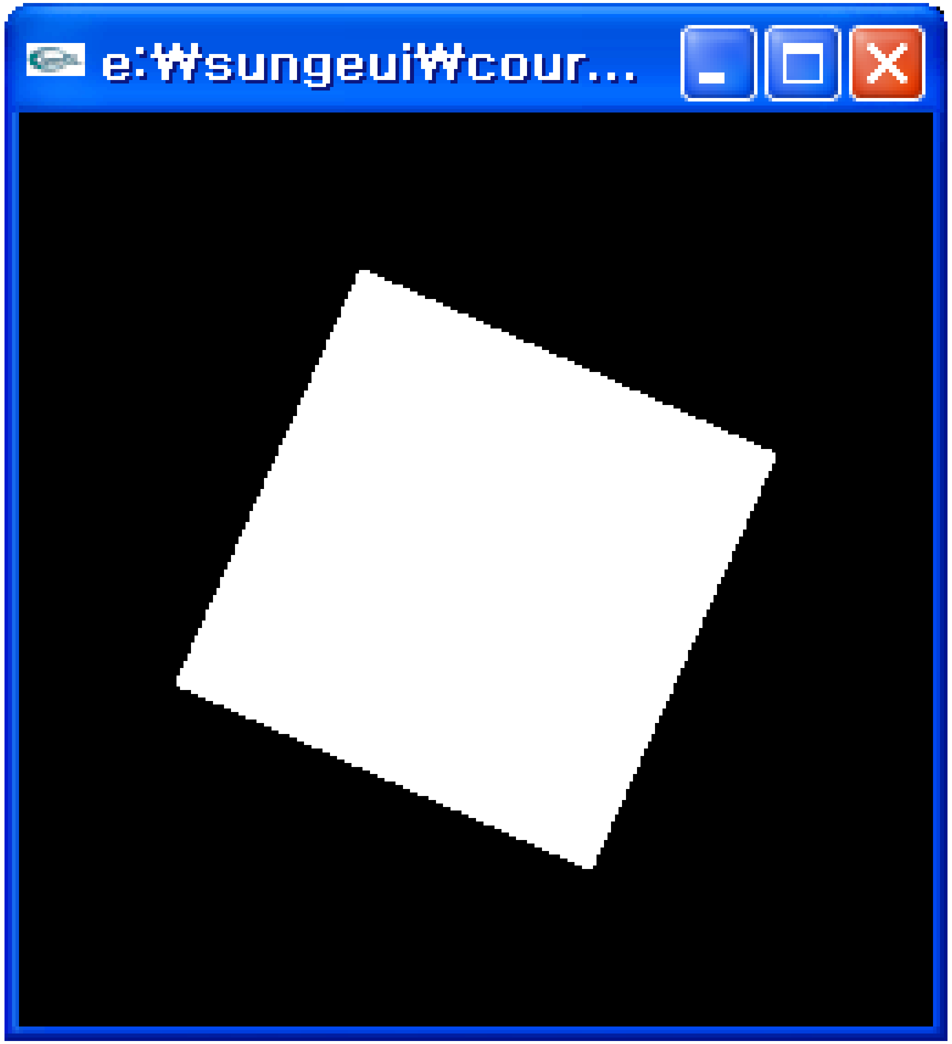
glRotate (angleInDegrees, axisX, axisY, axisZ);

glScale(sx, sy, sz);

OpenGL uses matrix format internally.

- glm (Ver. 4.3) stands for OpenGL Mathematics

OpenGL Example – Rectangle Animation (double.c)



Demo

Main Display Function

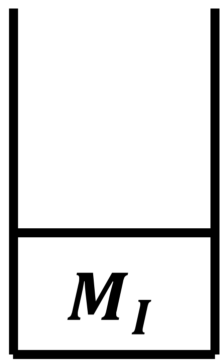
M_I : initial matrix

```
void display(void)
{
  glClear(GL_COLOR_BUFFER_BIT);

  glPushMatrix();
  glRotatef(spin, 0.0, 0.0, 1.0);
  glColor3f(1.0, 1.0, 1.0);
  glRectf(-25.0, -25.0, 25.0, 25.0);
  glPopMatrix();

  glutSwapBuffers();
}
```

glRotatef(spin, 0.0, 0.0, 1.0); — M_R

glRectf(-25.0, -25.0, 25.0, 25.0); — v

glPopMatrix(); — M_I



Frame Buffer

- **Contains an image for the final visualization**
- **Color buffer, depth buffer, etc.**

- **Buffer initialization**
  - **glClear(GL_COLOR_BUFFER_BIT);**
  - **glClearColor (..);**
- **Buffer creation**
  - **glutInitDisplayMode (GLUT_DOUBLE | GLUT_RGB);**
- **Buffer swap**
  - **glutSwapBuffers();**

Matrix Stacks

- **OpenGL maintains matrix stacks**
  - **Provides pop and push operations**
  - **Convenient for transformation operations**

- **glMatrixMode() sets the current stack**
  - **GL_MODELVIEW, GL_PROJECTION, or GL_TEXTURE**
- **glPushMatrix() and glPopMatrix() are used to manipulate the stacks**

OpenGL Matrix Operations

glTranslate(tx, ty, tz)

glRotate(angleInDegrees, axisX, axisY, axisZ)

glMultMatrix(*arrayOf16InColumnMajorOrder)

} Concatenate with the current matrix

glLoadMatrix (*arrayOf16InColumnMajorOrder)

glLoadIdentity()

} Overwrite the current matrix

Matrix Specification in OpenGL

- **Column-major ordering**

$$M = \begin{bmatrix} m_1 & m_5 & m_9 & m_{13} \\ m_2 & m_6 & m_{10} & m_{14} \\ m_3 & m_7 & m_{11} & m_{15} \\ m_4 & m_8 & m_{12} & m_{16} \end{bmatrix}$$

  - **Reverse to the typical C-convention (e.g., m [i][j] : row i & column j)**
  - **Better to declare m [16]**

- **Also, glLoadTransportMatrix*() & glMultTransposeMatrix*() are available**

Animation

- **It consists of “redraw” and “swap”**
- **It’s desirable to provide more than 30 frames per second (fps) for interactive applications**
- **We will look at an animation example based on idle-callback function**

KAIST

Idle-based Animation

```
void mouse(int button, int state, int x, int y)
{
  switch (button) {
    case GLUT_LEFT_BUTTON:
      if (state == GLUT_DOWN)
        glutIdleFunc (spinDisplay);
      break;
    case GLUT_RIGHT_BUTTON:
      if (state == GLUT_DOWN)
        glutIdleFunc (NULL);
      break;
  }
}
```

```
void spinDisplay(void)
{
  spin = spin + 2.0;
  if (spin > 360.0)
    spin = spin - 360.0;
  glutPostRedisplay();
}
```


KAIST

Class Objectives were:

- Write down simple 2D transformation matrixes
- Understand the homogeneous coordinates and its benefits
- Know OpenGL-transformation related API
- Implement idle-based animation method

Homework

- **Go over the next lecture slides before the class**
- **Watch 2 SIGGRAPH videos and submit your summaries before every Tue. class**
  - **Submit online**
  - **Just one paragraph for each summary**

Example:

Title: XXX XXXX XXXX
Abstract: this video is about accelerating the performance of ray tracing. To achieve its goal, they design a new technique for reordering rays, since by doing so, they can improve the ray coherence and thus improve the overall performance.

Any Questions?

- **Come up with one question on what we have discussed in the class and submit at the end of the class**
  - **1 for already answered or typical questions**
  - **2 for questions with thoughts or that surprised me**

- **Submit 2 times during the whole semester**

KAIST

Next Time

- **3D transformations**