CS580:
Radiometry

Sung-Eui Yoon
(윤성의)

Course URL:
http://sglab.kaist.ac.kr/~sungeui/GCG

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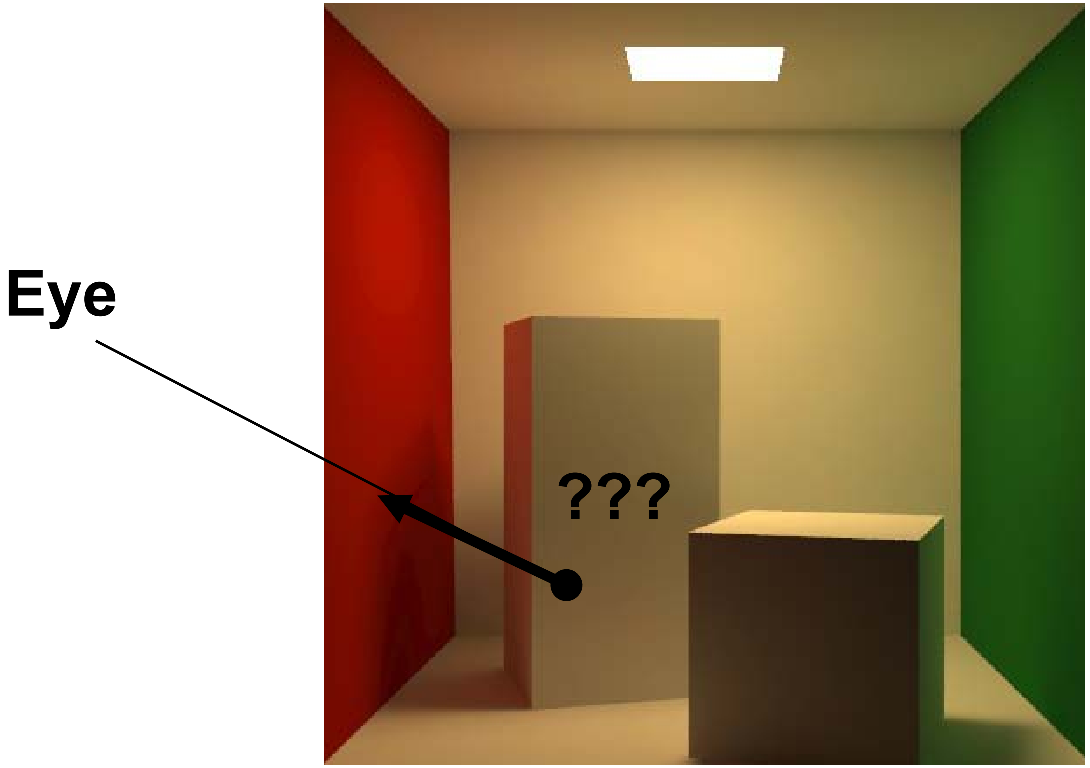
Class Objectives

- **Know terms of:**
  - **Hemispherical coordinates and integration**
  - **Various radiometric quantities (e.g., radiance)**
  - **Basic material function, BRDF**

Announcements

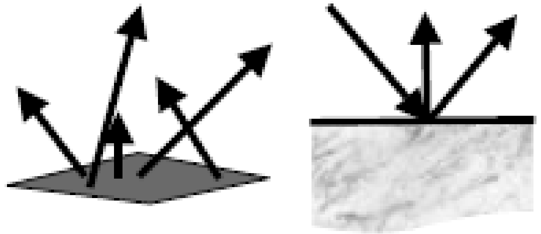
- Final project
  - Make a team of two
  - Think about which paper you want to implement
  - Present a paper related to it
- Scope of papers
  - Papers since the year of 2008

Motivation



Light and Material Interactions

- **Physics of light**



- **Radiometry**

- **Material properties**

From kavita's slides

- **Rendering equation**

Models of Light

- Quantum optics
  - Fundamental model of the light
  - Explain the dual wave-particle nature of light
- Wave model
  - Simplified quantum optics
  - Explains diffraction, interference, and polarization

- Geometric optics
  - Most commonly used model in CG
  - Size of objects >> wavelength of light
  - Light is emitted, reflected, and transmitted

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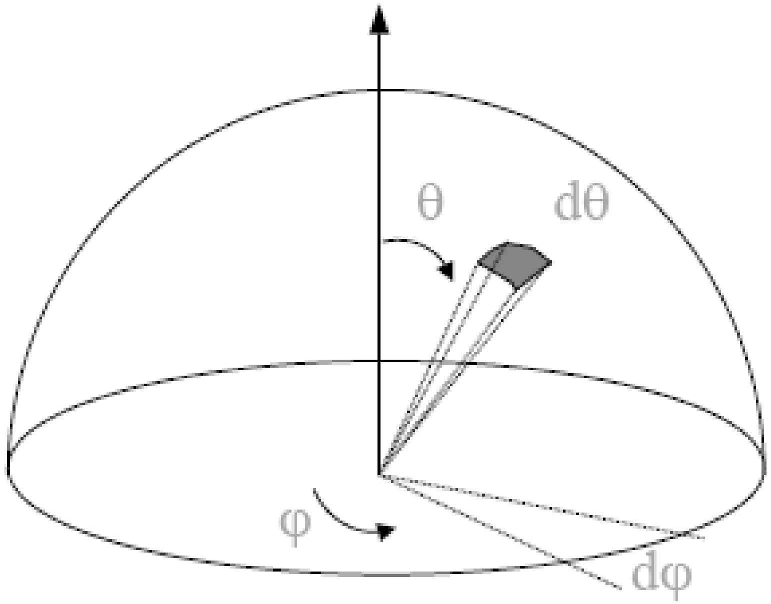
Radiometry

- Measurement of light energy
  - Critical component for photo-realistic rendering
- Light energy flows through space
  - Varies with time, position, and direction
- Radiometric quantities
  - Densities of energy at particular places in time, space, and direction
- Photometry
  - Quantify the perception of light energy

Hemispheres

- Hemisphere
  - Two-dimensional surfaces
- Direction
  - Point on (unit) sphere



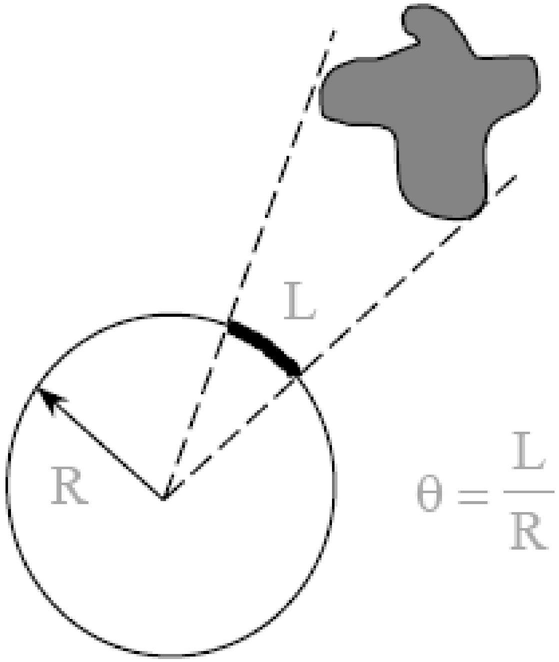
$$\theta \in [0, \frac{\pi}{2}]$$

$$\varphi \in [0, 2\pi]$$

From kavita's slides

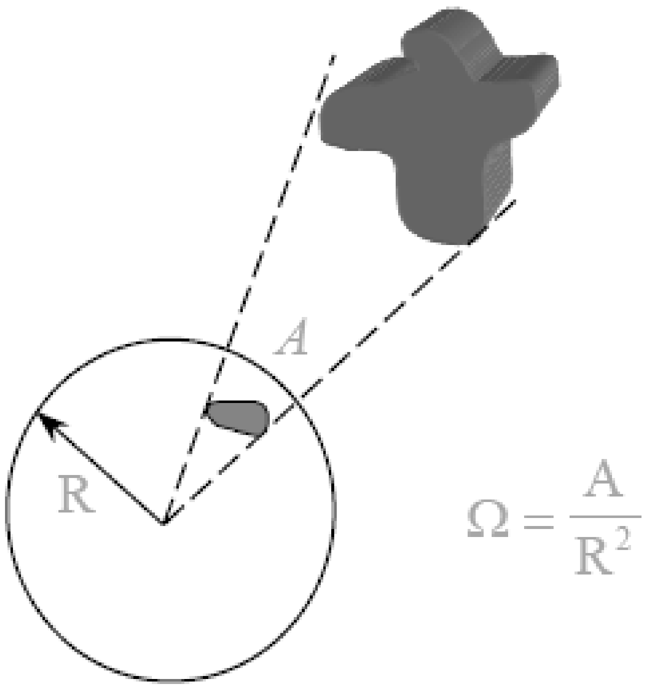
Solid Angles

2D



Full circle
= 2pi radians

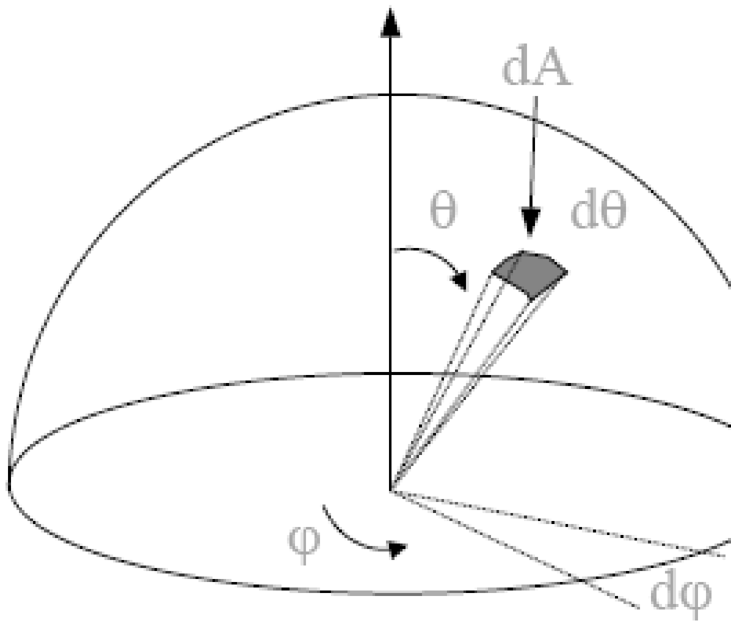
3D



Full sphere
= 4pi steradians

Hemispherical Coordinates

- **Direction, Θ**
  - **Point on (unit) sphere**



$$dA = (r \sin\theta d\varphi)(r d\theta)$$

From kavita's slides

Hemispherical Coordinates

- **Differential solid angle**

$$d\omega = \frac{dA}{r^2} = \sin\theta d\theta d\varphi$$

Hemispherical Integration

- **Area of hemispehre:**

$$\int_{\Omega_x} d\omega = \int_0^{2\pi} d\varphi \int_0^{\pi/2} \sin\theta d\theta$$

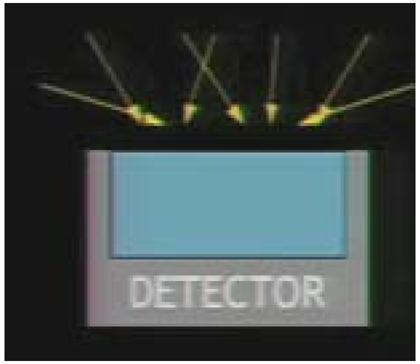
$$= \int_0^{2\pi} d\varphi [-\cos\theta]_0^{\pi/2}$$

$$= \int_0^{2\pi} d\varphi$$

$$= 2\pi$$

Energy

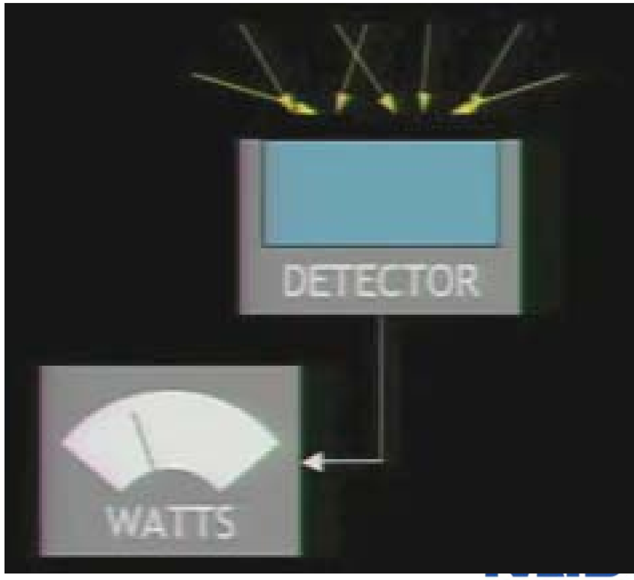
- **Symbol: Q**
  - **# of photons in this context**
  - **Unit: Joules**



From Steve Marschner's talk

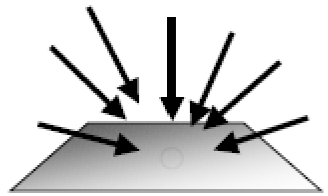
Power (or Flux)

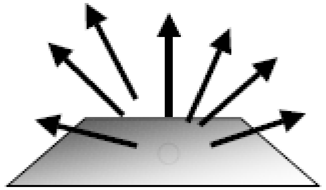
- Symbol, P or Φ
  - Total amount of energy through a surface per unit time, dQ/dt
  - Radiant flux in this context
  - Unit: Watts (=Joules / sec.)
  - Other quantities are derivatives of P
- Example
  - A light source emits 50 watts of radiant power
  - 20 watts of radiant power is incident on a table



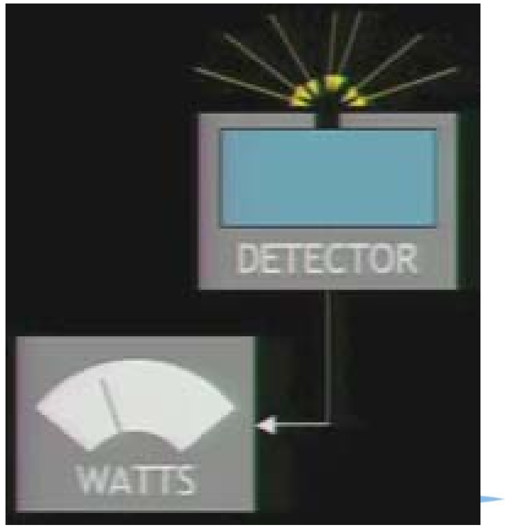
Irradiance

- **Incident radiant power per unit area (dP/dA)**
  - **Area density of power**





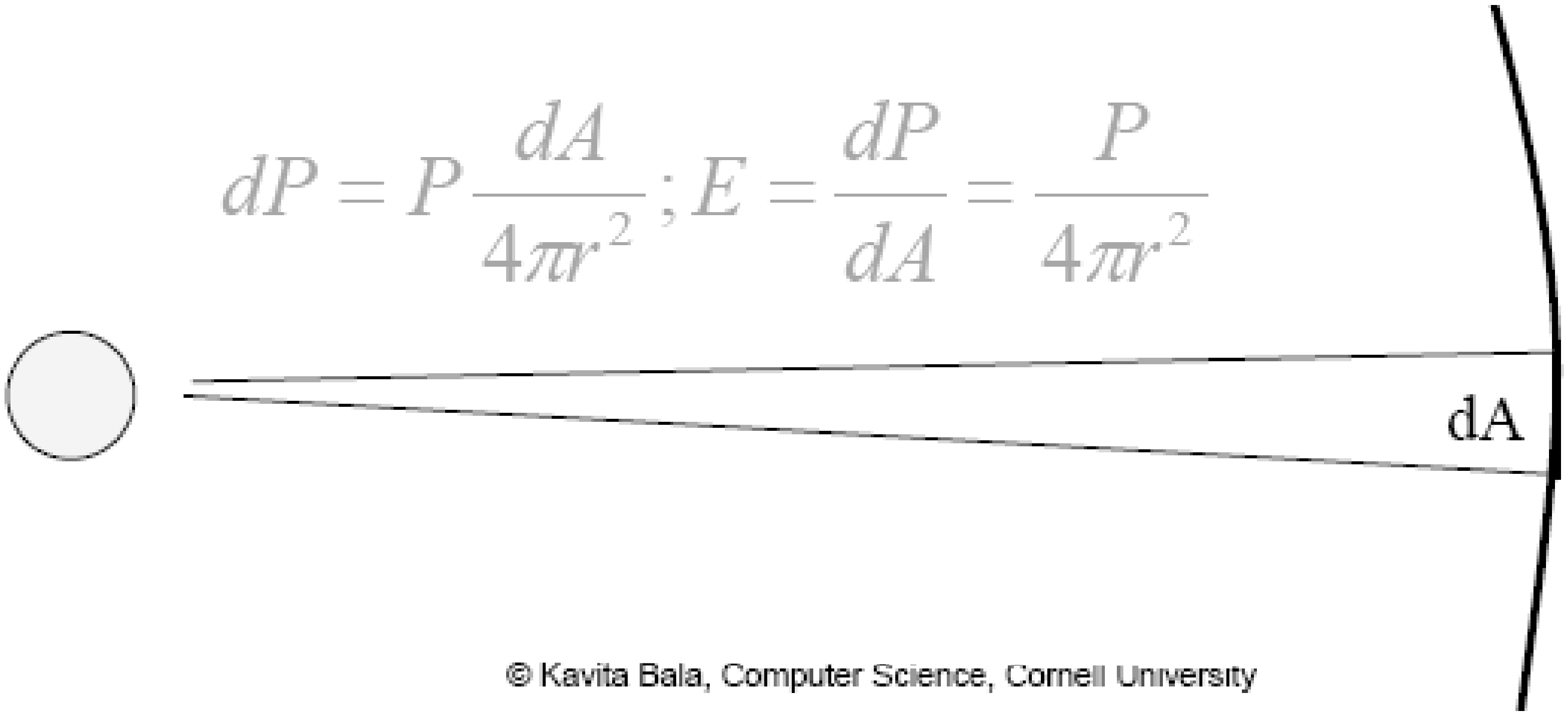
- **Symbol: E, unit: W/ m²**
  - **Area power density existing a surface is called radiance exitance (M) or radiosity (B)**
- **For example**
  - **A light source emitting 100 W of area 0.1 m²**
  - **Its radiant exitance is 1000 W/ m²**



Irradiance Example

- **Uniform point source illuminates a small surface dA from a distance r**
  - **Power P is uniformly spread over the area of the sphere**

$$dP = P\frac{dA}{4\pi r^2};\ E = \frac{dP}{dA} = \frac{P}{4\pi r^2}$$



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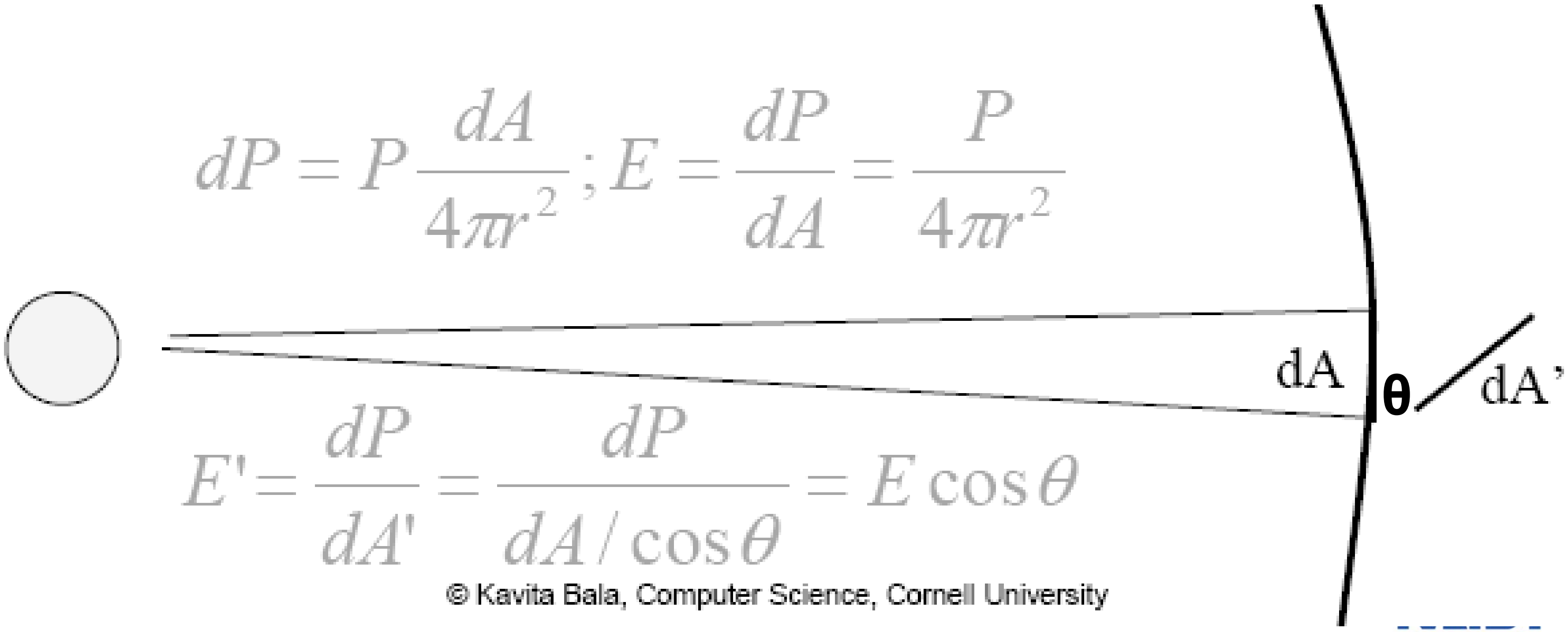
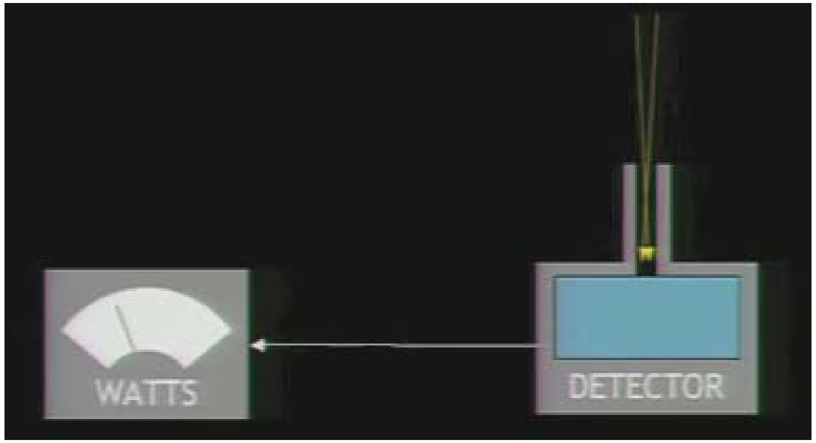


$$E' = \frac{dP}{dA'} = \frac{dP}{dA/\cos\theta} = E\cos\theta$$

Radiance

- **Radiant power at x in direction θ**
  - $L(x \rightarrow \Theta)$ **: 5D function**
    - **Per unit area**
    - **Per unit solid angle**

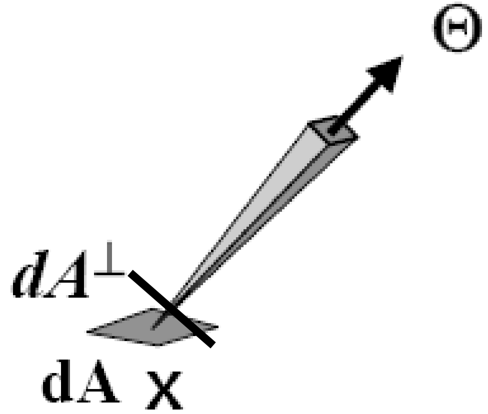


- **Important quantity for rendering**

Radiance

- **Radiant power at x in direction θ**
  - $L(x \rightarrow \Theta)$ **: 5D function**
    - **Per unit area**
    - **Per unit solid angle**

$$L(x \rightarrow \Theta) = \frac{d^2 P}{dA^{\perp} d\omega_{\Theta}}$$

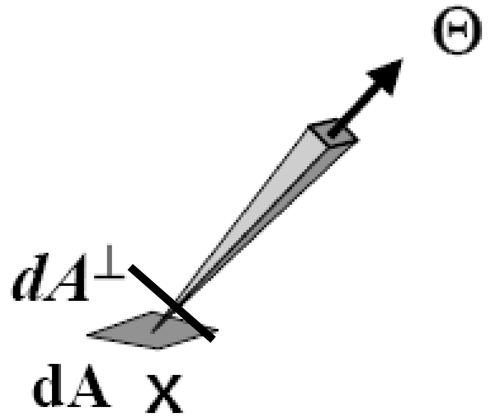


  - **Units: Watt / (m^2 sr)**
  - **Irradiance per unit solid angle**
  - **2nd derivative of P**
  - **Most commonly used term**

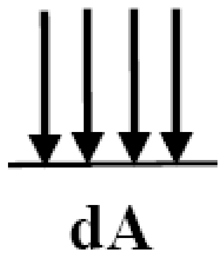
Radiance: Projected Area

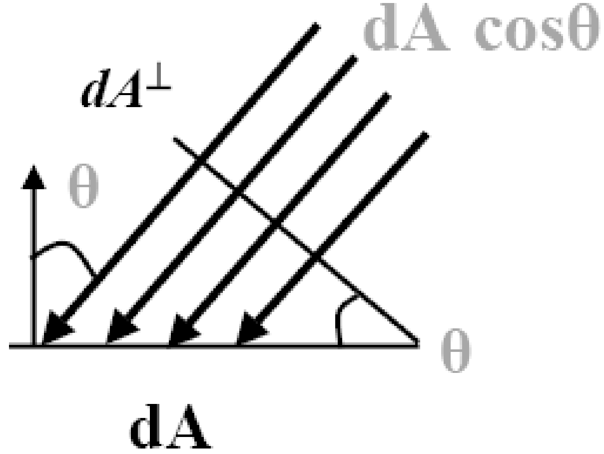
$$L(x \to \Theta) = \frac{d^2 P}{dA^{\perp} d\omega_{\Theta}}$$

$$= \frac{d^2 P}{d\omega_{\Theta} dA \cos\theta}$$



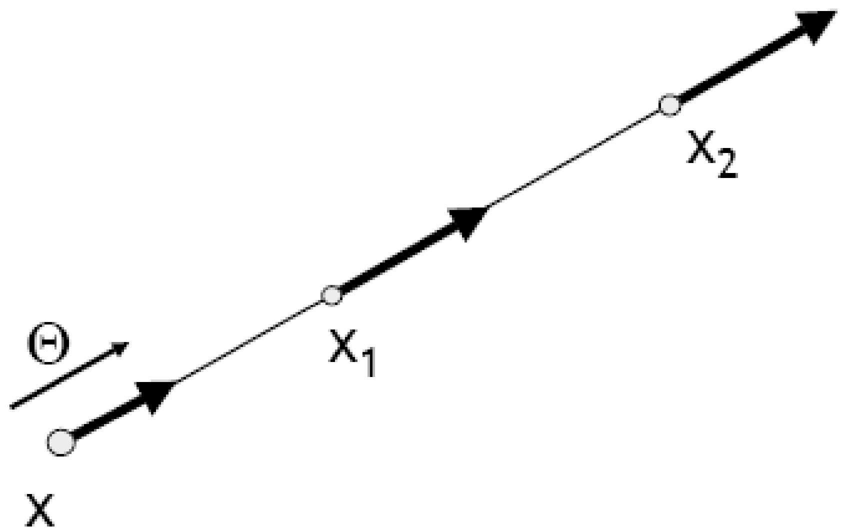
- **Why per unit projected surface area**





Properties of Radiance

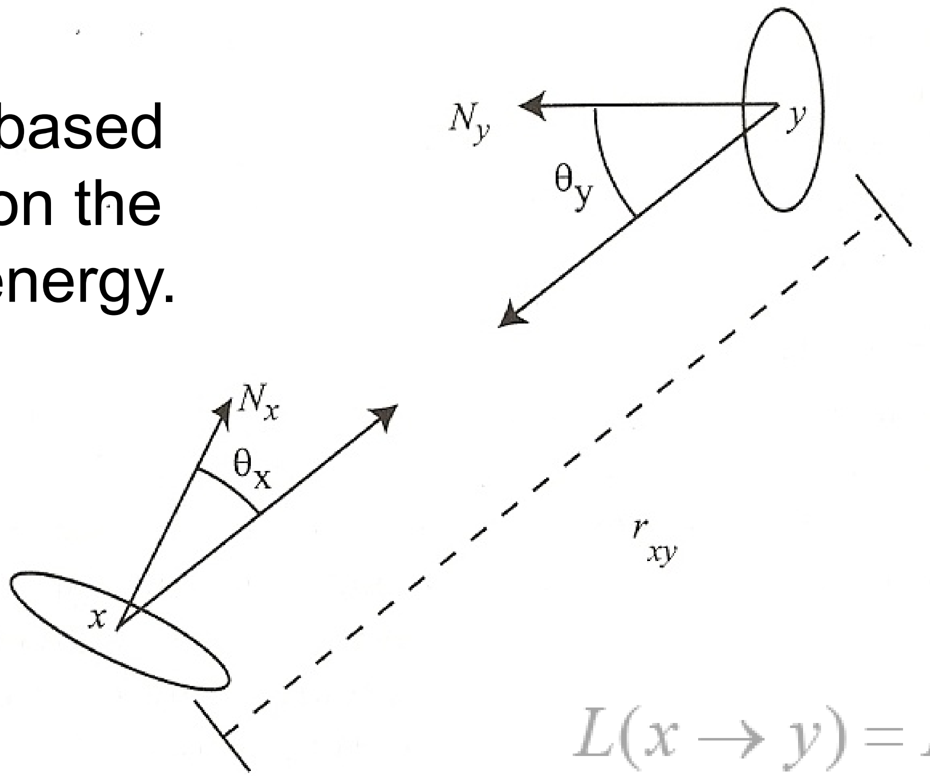
- **Invariant along a straight line (in vacuum)**



From kavita's slides

Invariance of Radiance

We can prove it based on the assumption the conservation of energy.

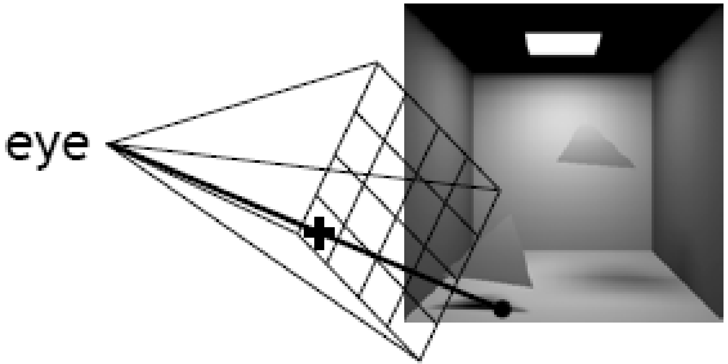


$$L(x \rightarrow y) = L(y \leftarrow x)$$

Figure 2.3. Invariance of radiance.

Sensitivity to Radiance

- **Responses of sensors (camera, human eye) is proportional to radiance**



From kavita's slides

- **Pixel values in image proportional to radiance received from that direction**

Relationships

- **Radiance is the fundamental quantity**

$$L(x \to \Theta) = \frac{d^2 P}{dA^{\perp} d\omega_{\Theta}}$$

- **Power:**

$$P = \int_{Area} \int_{\substack{Solid\\Angle}} L(x \to \Theta) \cdot \cos\theta \cdot d\omega_{\Theta} \cdot dA$$

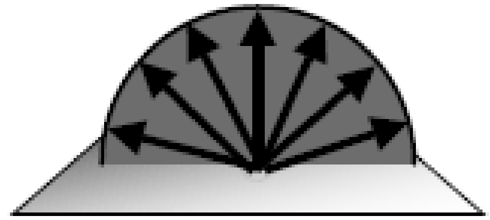
- **Radiosity:**

$$B = \int_{\substack{Solid\\Angle}} L(x \to \Theta) \cdot \cos\theta \cdot d\omega_{\Theta}$$

Example: Diffuse emitter

- Diffuse emitter: light source with equal radiance everywhere

$$L(x \to \Theta) = \frac{d^2 P}{dA^{\perp} d\omega_{\Theta}}$$



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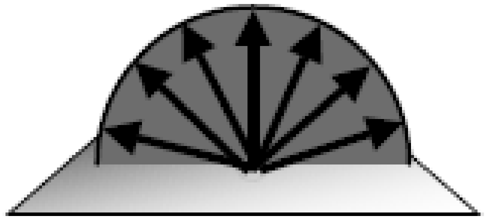

$$L(x \rightarrow \Theta) = \frac{d^2 P}{dA^{\perp} d\omega_{\Theta}}$$

$$P = \int_{Area} \int_{Solid\ Angle} L(x \rightarrow \Theta) \cdot \cos\theta \cdot d\omega_{\Theta} \cdot dA$$

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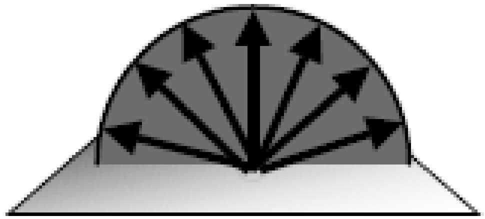


$$L(x \to \Theta) = \frac{d^2 P}{dA^{\perp} d\omega_{\Theta}}$$

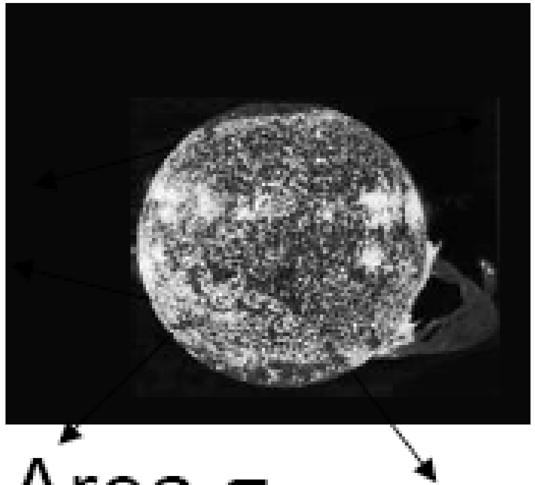
$$P = \int_{Area} \int_{\substack{Solid\\Angle}} L(x \to \Theta) \cdot \cos\theta \cdot d\omega_{\Theta} \cdot dA$$

$$= L \int_{Area} dA \int_{\substack{Solid\\Angle}} \cos\theta \cdot d\omega_{\Theta}$$

$$= L \cdot Area \cdot \pi$$

Sun Example: radiance

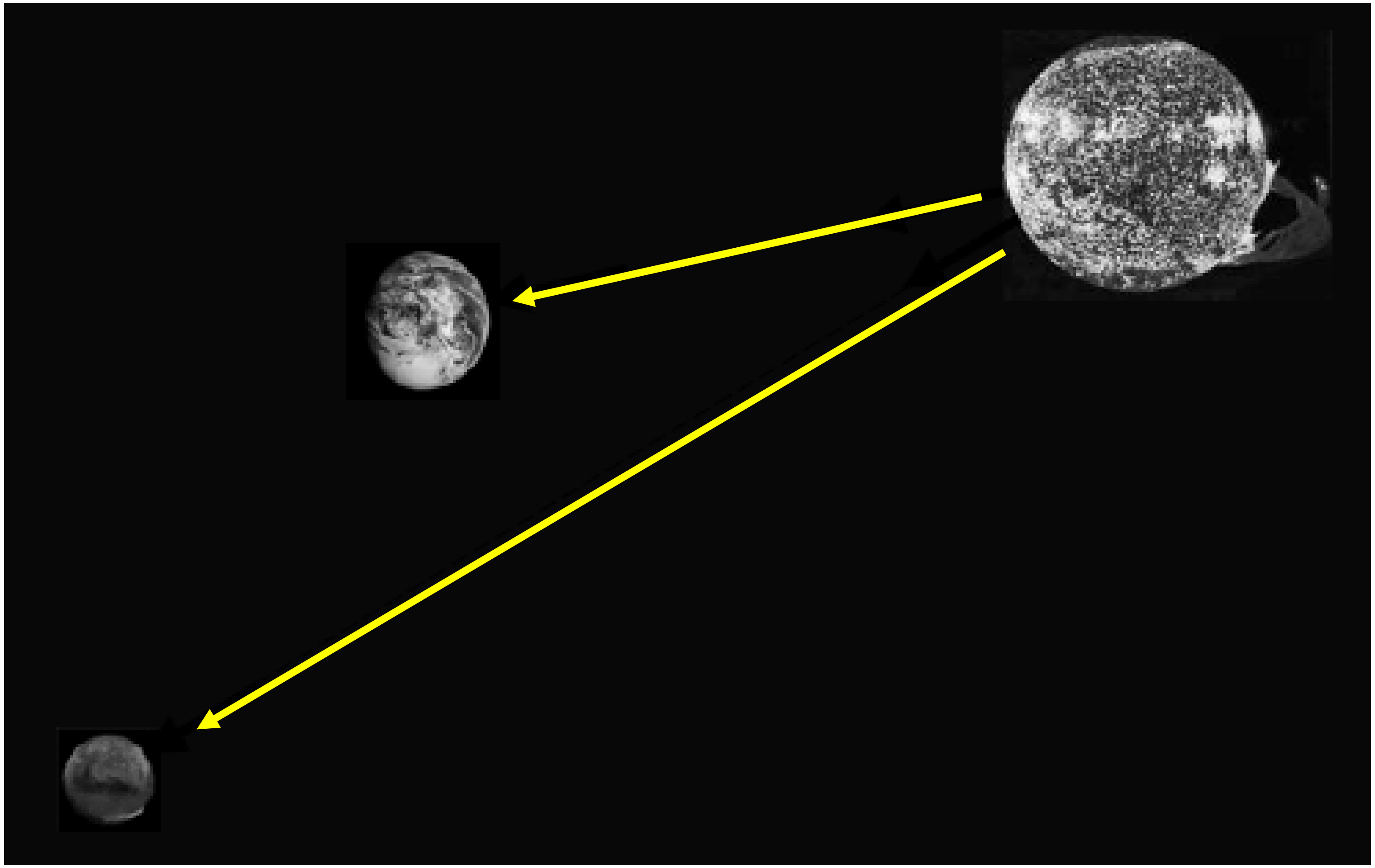
- Power: 3.91×10^{26} W
- Surface Area: 6.07×10^{18} m^2



- Power = Radiance.Surface Area.π
- Radiance = Power/(Surface Area.π)

- Radiance = 2.05×10^{7} W/ m^2.sr

Sun Example

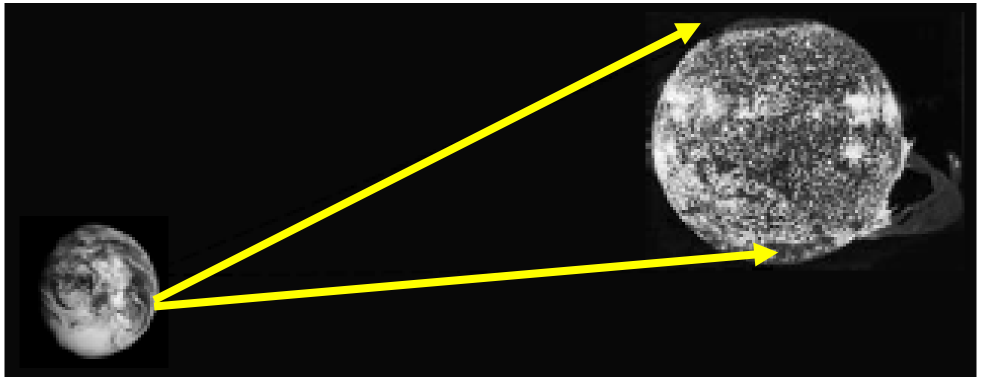


Same radiance on Earth and Mars?

Sun Example: Power on Earth

- Power reaching earth on a 1m² square:

$$P = L \int_{Area} dA \int_{Solid\ Angle} \cos\theta \cdot d\omega_{\Theta}$$

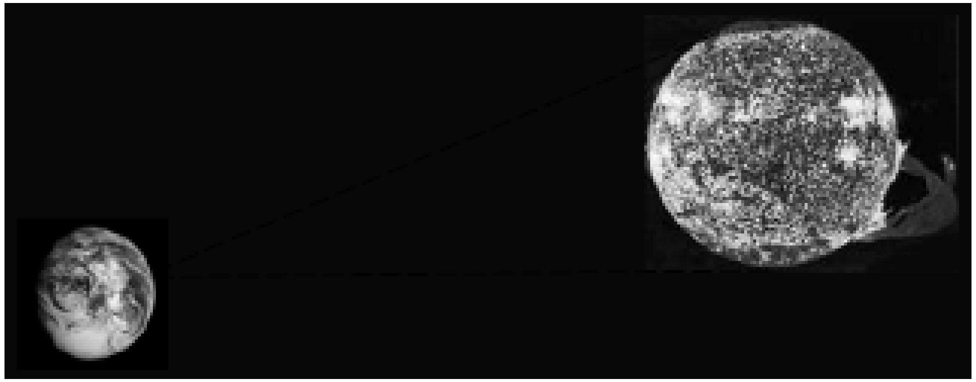


- Assume $\cos\theta = 1$ (sun in zenith)

$$P = L \int_{Area} dA \int_{Solid\ Angle} d\omega_{\Theta}$$

Sun Example: Power on Earth

Power = Radiance.Area.Solid Angle



Solid Angle = Projected $\text{Area}_{\text{Sun}}/(\text{distance}_{\text{earth\_sun}})^2$
= $6.7\ 10^{-5}$ sr

P = $(2.05 \times 10^7\ \text{W}/\ \text{m}^2.\text{sr}) \times (1\ \text{m}^2) \times (6.7\ 10^{-5}\ \text{sr})$
= 1373.5 Watt

Sun Example: Power on Mars

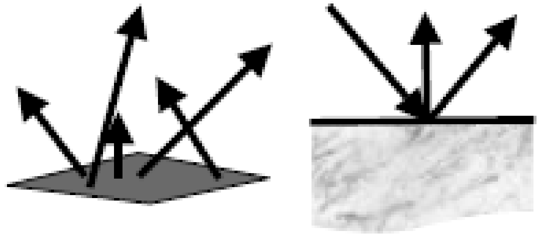
Power = Radiance.Area.Solid Angle

Solid Angle = Projected $\text{Area}_{Sun}/(\text{distance}_{mars\_sun})^2$
= $2.92\ 10^{-5}$ sr

P = $(2.05 \times 10^7\ W/\ m^2.sr) \times (1\ m^2) \times (2.92\ 10^{-5}\ sr)$
= 598.6 Watt

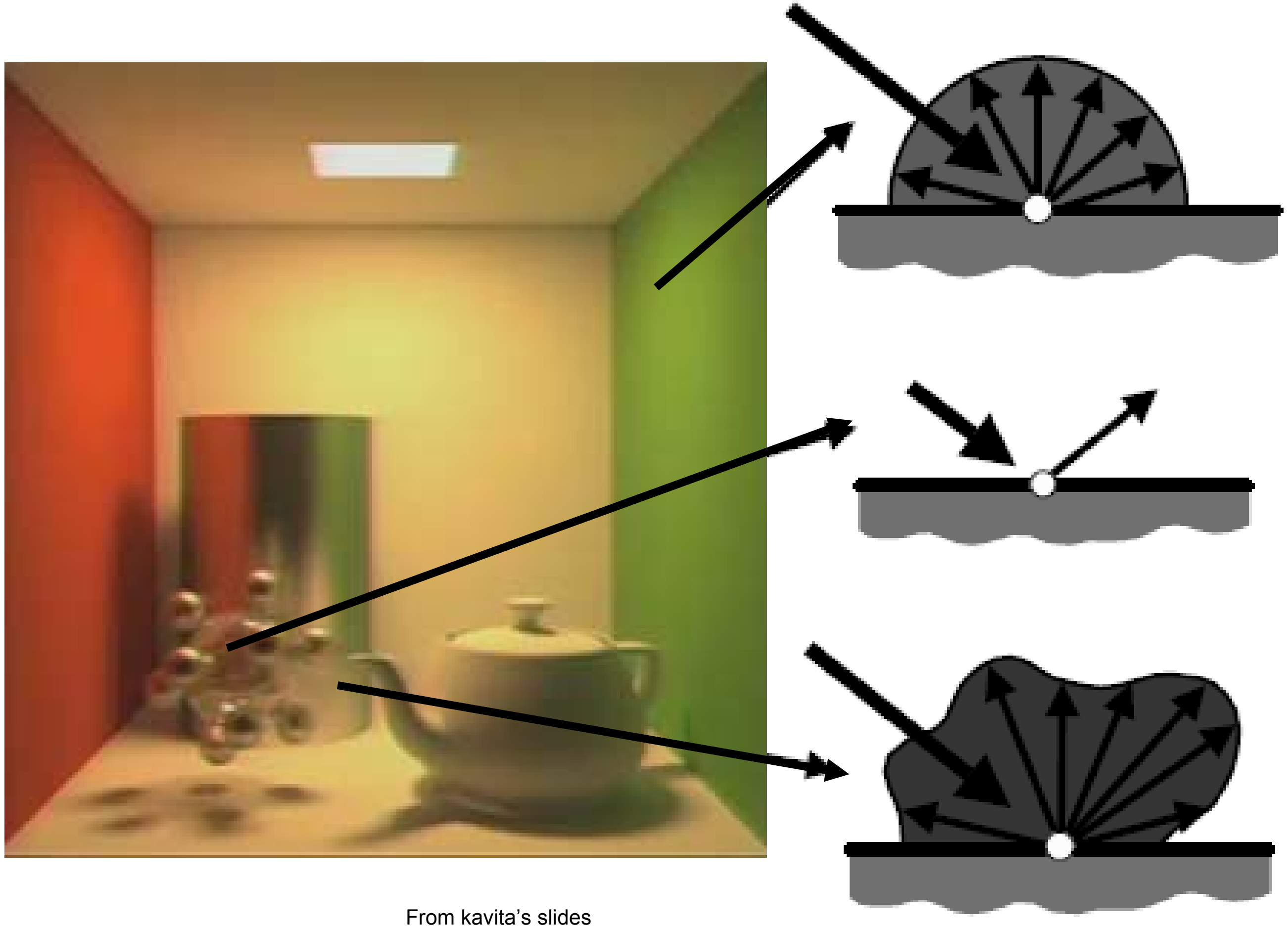
Light and Material Interactions

- Physics of light
- Radiometry
- **Material properties**
- Rendering equation



From kavita's slides

Materials



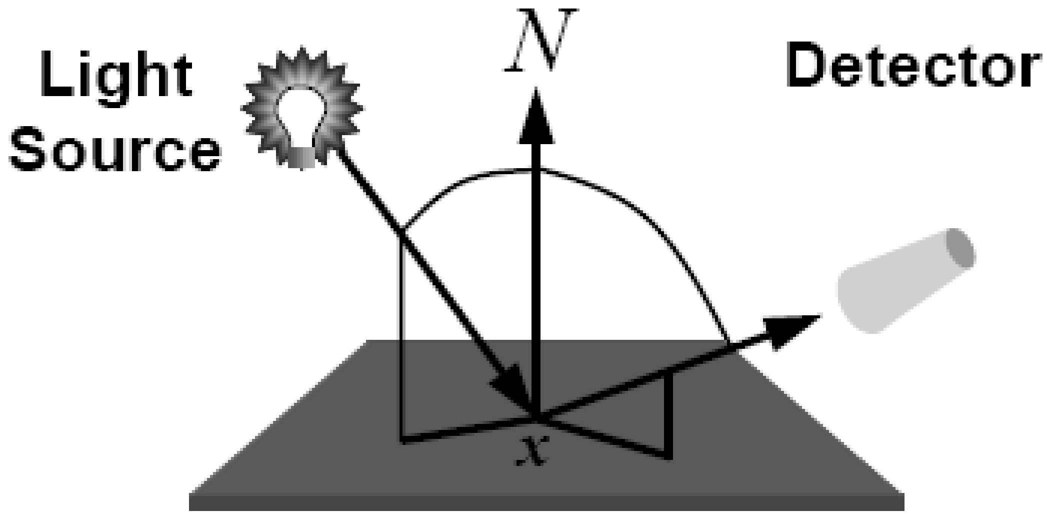
Ideal diffuse (Lambertian)

Ideal specular

Glossy

From kavita's slides

Bidirectional Reflectance Distribution Function (BRDF)

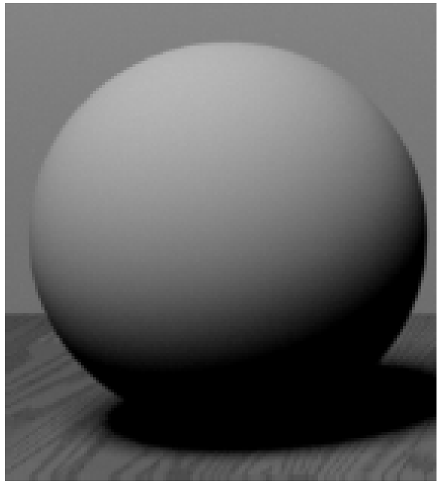


$$f_r(x, \Psi \rightarrow \Theta) = \frac{dL(x \rightarrow \Theta)}{dE(x \leftarrow \Psi)} = \frac{dL(x \rightarrow \Theta)}{L(x \leftarrow \Psi)\cos(N_x, \Psi)\, d\omega_\Psi}$$

BRDF special case: ideal diffuse

Pure Lambertian

$$f_r(x, \Psi \rightarrow \Theta) = \frac{\rho_d}{\pi}$$



$$\rho_d = \frac{Energy_{out}}{Energy_{in}} \qquad 0 \leq \rho_d \leq 1$$

Properties of the BRDF

- Reciprocity:

$$f_r(x, \Psi \rightarrow \Theta) = f_r(x, \Theta \rightarrow \Psi)$$

- Therefore, notation: $f_r(x, \Psi \leftrightarrow \Theta)$

- Important for bidirectional tracing

Properties of the BRDF

- Bounds:

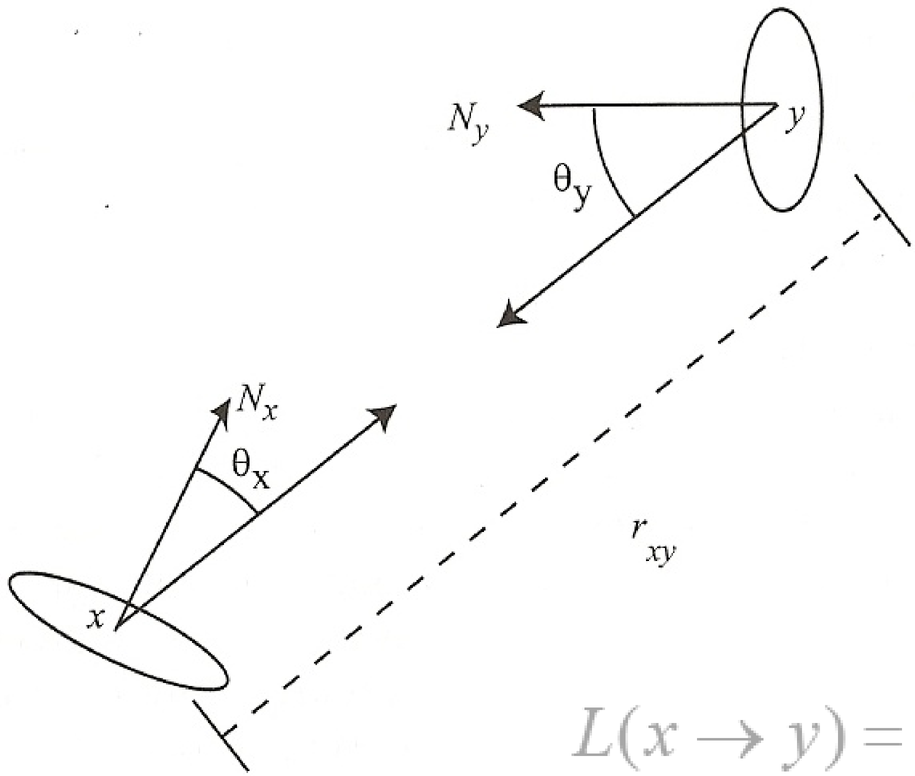
$$0 \leq f_r(x, \Psi \leftrightarrow \Theta) \leq \infty$$

- Energy conservation:

$$\forall \Psi \quad \int_{\Theta} f_r(x, \Psi \leftrightarrow \Theta) \cos(N_x, \Theta) d\omega_{\Theta} \leq 1$$

Homework 1

- **Prove the invaraince**



$$L(x \to y) = L(y \leftarrow x)$$

Figure 2.3. Invariance of radiance.

Speaking of Radiometry

- "I need to sum all those radiances to compute the irradiance based on hemispherical integration."
- "Do you have a BRDF model for that copper model? If you give me that model, I can support its visual appearance."

Next Time

- **Rendering equation**

Any Questions?

- **Come up with one question on what we have discussed in the class and submit at the end of the class**
  - **1 for already answered questions**
  - **2 for typical questions**
  - **3 for questions with thoughts**
  - **4 for questions that surprised me**