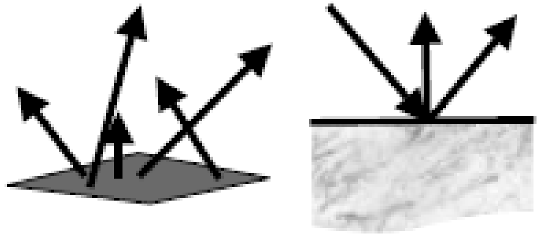
CS580:
Rendering Equation

Sung-Eui Yoon
(윤성의)

Course URL:
http://sglab.kaist.ac.kr/~sungeui/GCG

Light and Material Interactions

- Physics of light



- Radiometry

- Material properties

From kavita's slides

- Rendering equation

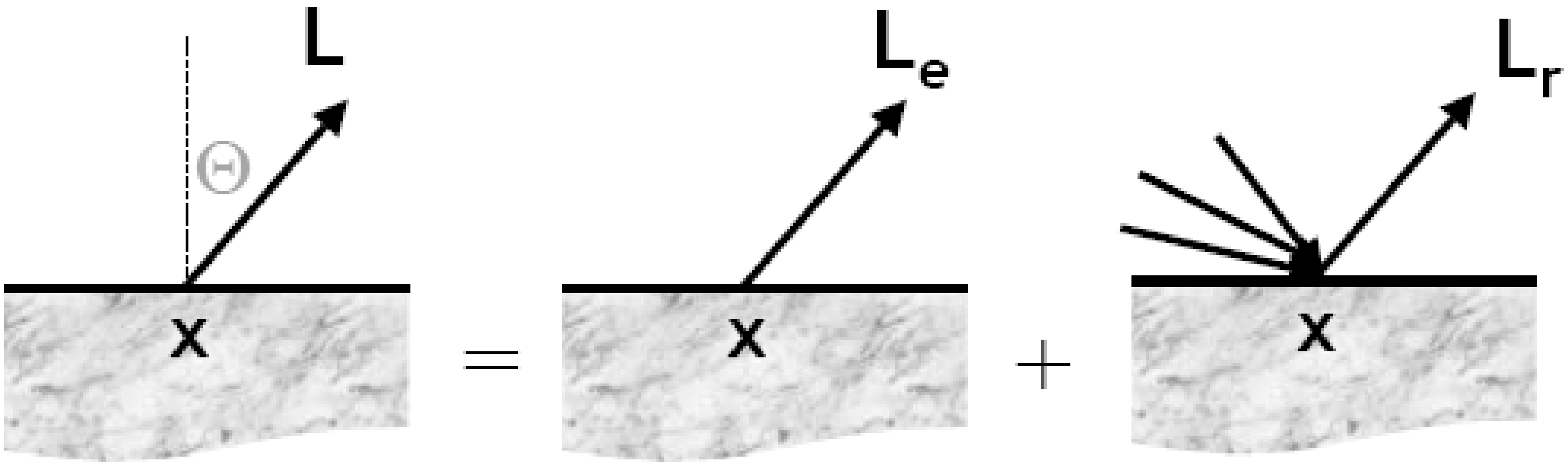
Light Transport

- Goal
  - Describe steady-state radiance distribution in the scene
- Assumptions
  - Geometric optics
  - Achieves steady state instantaneously

Rendering Equation

- Describes energy transport in the scene
- Input
  - Light sources
  - Surface geometry
  - Reflectance characteristics of surfaces
- Output
  - Value of radiances at all surface points in all directions

Rendering Equation



$$L(x \to \Theta) = L_e(x \to \Theta) + L_r(x \to \Theta)$$

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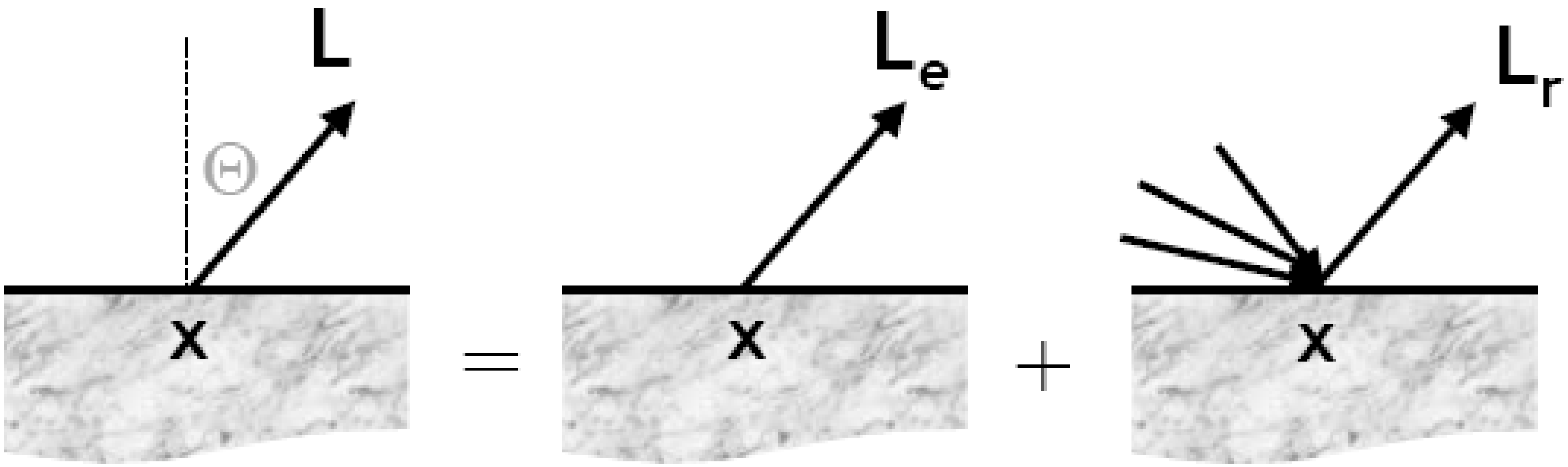
Rendering Equation



$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{hemisphere} L(x \leftarrow \Psi) \cdots$$

Rendering Equation

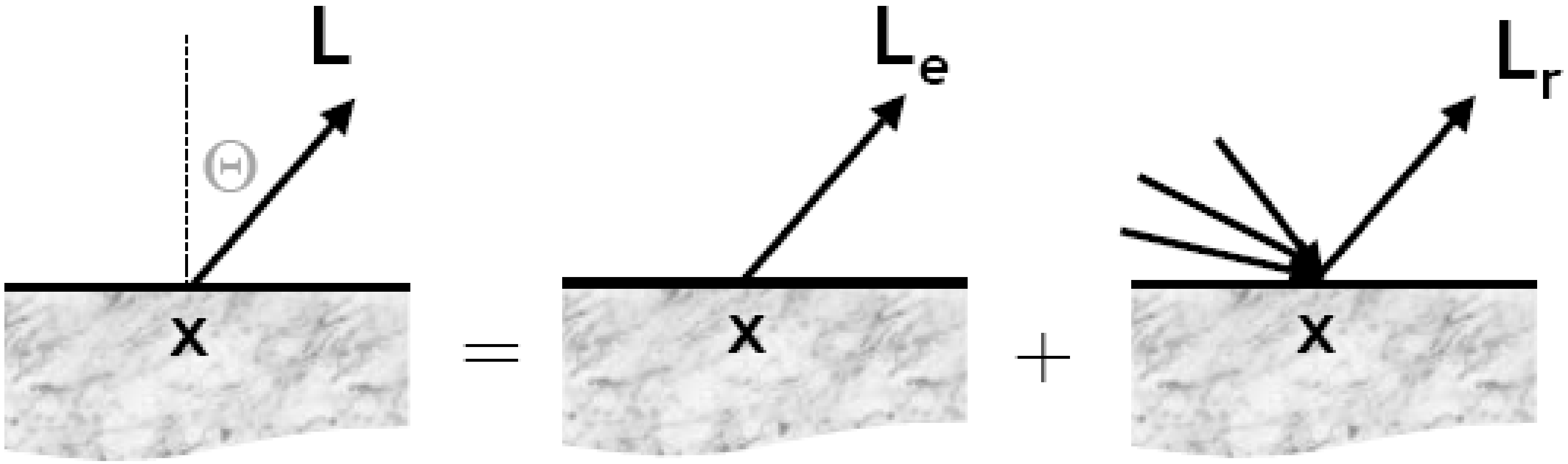
$$f_r(x, \Psi \leftrightarrow \Theta) = \frac{dL(x \rightarrow \Theta)}{dE(x \leftarrow \Psi)}$$

$$dL(x \rightarrow \Theta) = f_r(x, \Psi \leftrightarrow \Theta) dE(x \leftarrow \Psi)$$

$$dL(x \rightarrow \Theta) = f_r(x, \Psi \leftrightarrow \Theta) L(x \leftarrow \Psi) \cos(N_x, \Psi) d\omega_\Psi$$

$$L_r(x \rightarrow \Theta) = \int_{hemisphere} f_r(x, \Psi \leftrightarrow \Theta) L(x \leftarrow \Psi) \cos(N_x, \Psi) d\omega_\Psi$$

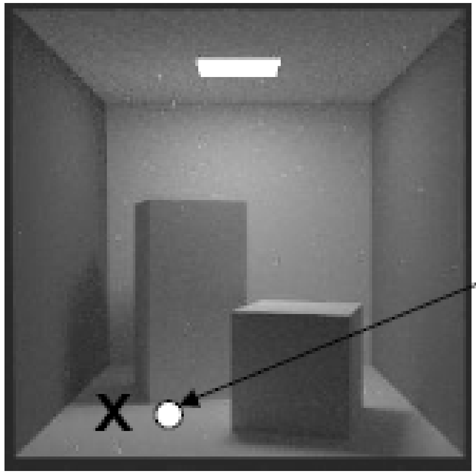
Rendering Equation



$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{hemisphere} L(x \leftarrow \Psi)\, f_r(x, \Psi \leftrightarrow \Theta) \cos(\mathbf{N}_x, \Psi)\, d\omega_\Psi$$

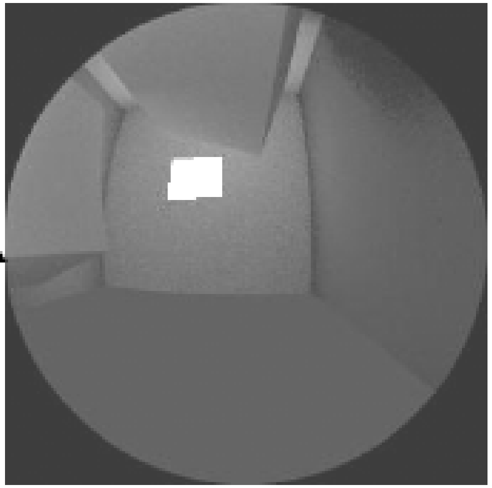
- Applicable for each wavelength

Rendering Equation



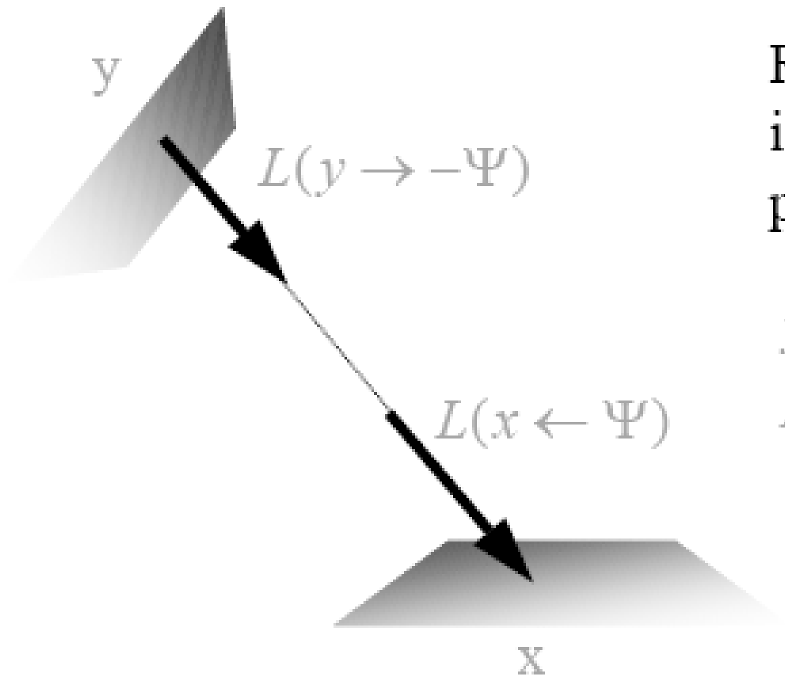
$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{hemisphere} L(x \leftarrow \Psi) f_r(x, \Psi \leftrightarrow \Theta) \cos(\mathbf{N}_x, \Psi) d\omega_\Psi$$

incoming radiance



Rendering Equation: Area Formulation

$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos\theta_x \cdot d\omega_\Psi$$



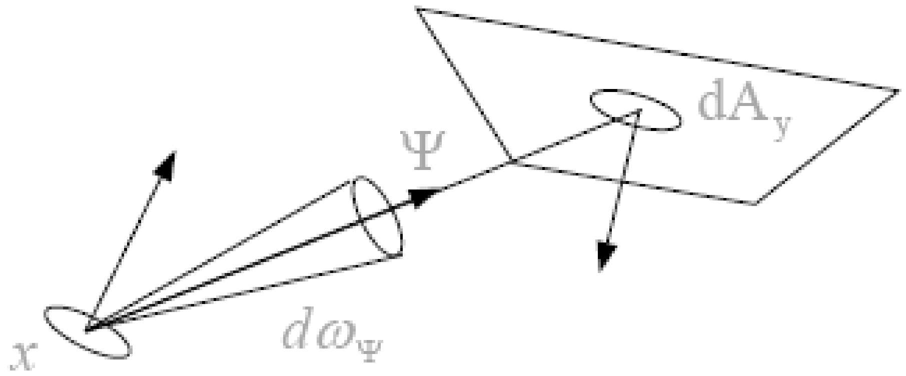
Ray-casting function: what is the nearest visible surface point seen from x in direction Ψ?

$$y = vp(x, \Psi)$$

$$L(x \leftarrow \Psi) = L(vp(x, \Psi) \to -\Psi)$$

Rendering Equation

$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos\theta_x \cdot d\omega_\Psi$$



$$y = vp(x, \Psi)$$

$$L(x \leftarrow \Psi) = L(vp(x, \Psi) \to -\Psi)$$

$$d\omega_\Psi = \frac{dA_y \cos\theta_y}{r_{xy}^2}$$

Rendering Equation: Visible Surfaces

$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos\theta_x \cdot d\omega_\Psi$$

Coordinate transform

$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{\substack{y \text{ on} \\ \text{all surfaces}}} f_r(\Psi \leftrightarrow \Theta) \cdot L(y \to -\Psi) \ \cos\theta_x \cdot \frac{\cos\theta_y}{r_{xy}^2} \cdot dA_y$$

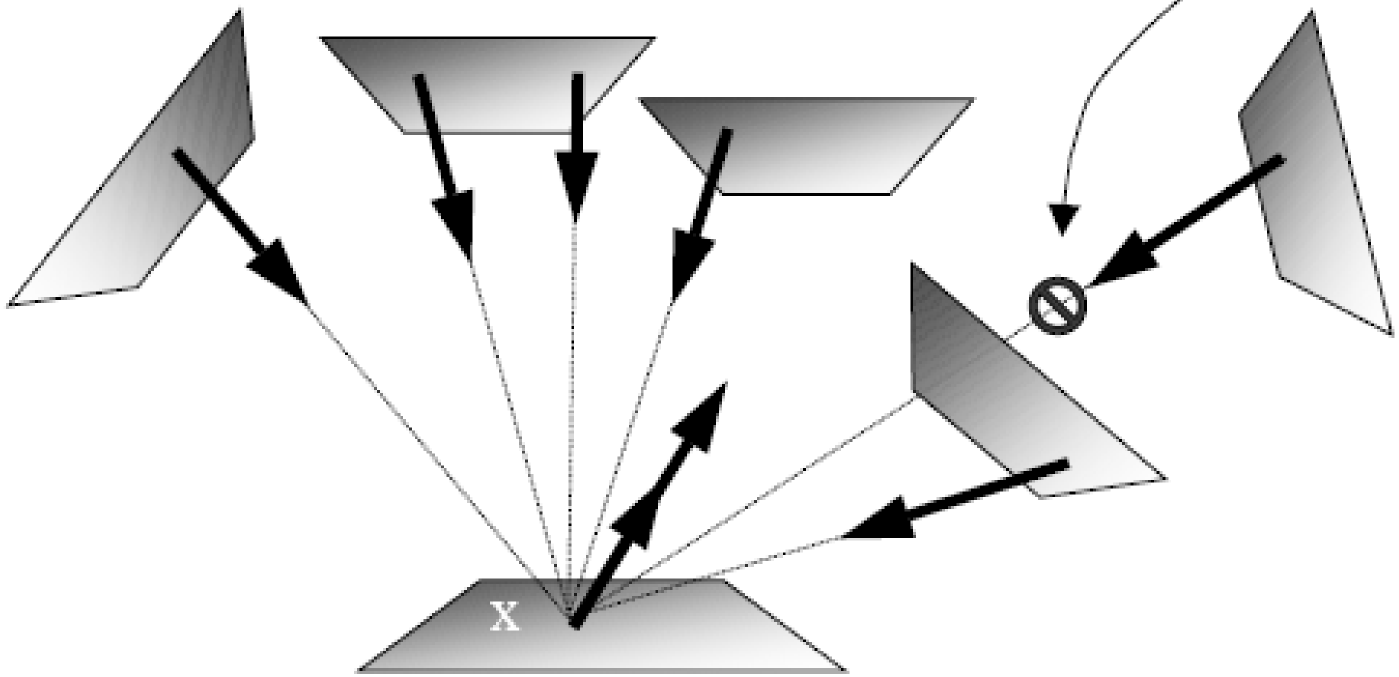
$$y = vp(x, \Psi)$$

Integration domain = visible surface points y

- Integration domain extended to ALL surface points by including visibility function

Rendering Equation: All Surfaces

$$L(x \to \Theta) = L_e(\ldots) + \int_A f_r(\ldots) \cdot L(y \to -\Psi) \cdot \frac{\cos\theta_x \cdot \cos\theta_y}{r_{xy}^2} \cdot \boxed{V(x,y)}\, dA_y$$



Two Forms of the Rendering Equation

- **Hemisphere integration**

$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos\theta_x \cdot d\omega_\Psi$$

- **Area integration**

$$L(x \to \Theta) = L_e(x \to \Theta) + \int_A f_r(\Psi \leftrightarrow \Theta) \cdot L(y \to -\Psi) \cdot \frac{\cos\theta_x \cdot \cos\theta_y}{r_{xy}^2} \cdot V(x,y) \cdot dA_y$$

Speaking of Rendering Equation

- "The rendering equation is derived similarly to the radiosity equation, but I noticed that its main difference is at considering incoming radiance and BRDF."
- "I can also derive the rendering equation based on areas of triangles, not on incoming solid angles."

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Next Time

- Monte Carlo ray tracing

Any Questions?

- Come up with one question on what we have discussed in the class and submit at the end of the class
  - 1 for already answered questions
  - 2 for typical questions
  - 3 for questions with thoughts
  - 4 for questions that surprised me