
CS580: Rendering Equation

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Course URL:
<http://sglab.kaist.ac.kr/~sungeui/GCG>

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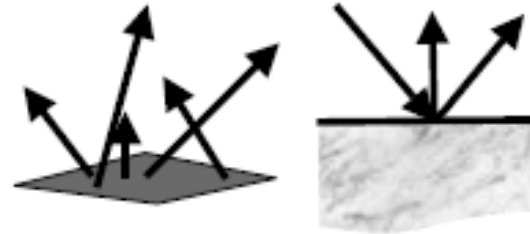


Class Objectives

- Derive the rendering equation
- Transform it to area integration

Light and Material Interactions

- Physics of light
- Radiometry
- Material properties
- **Rendering equation**



From kavita's slides

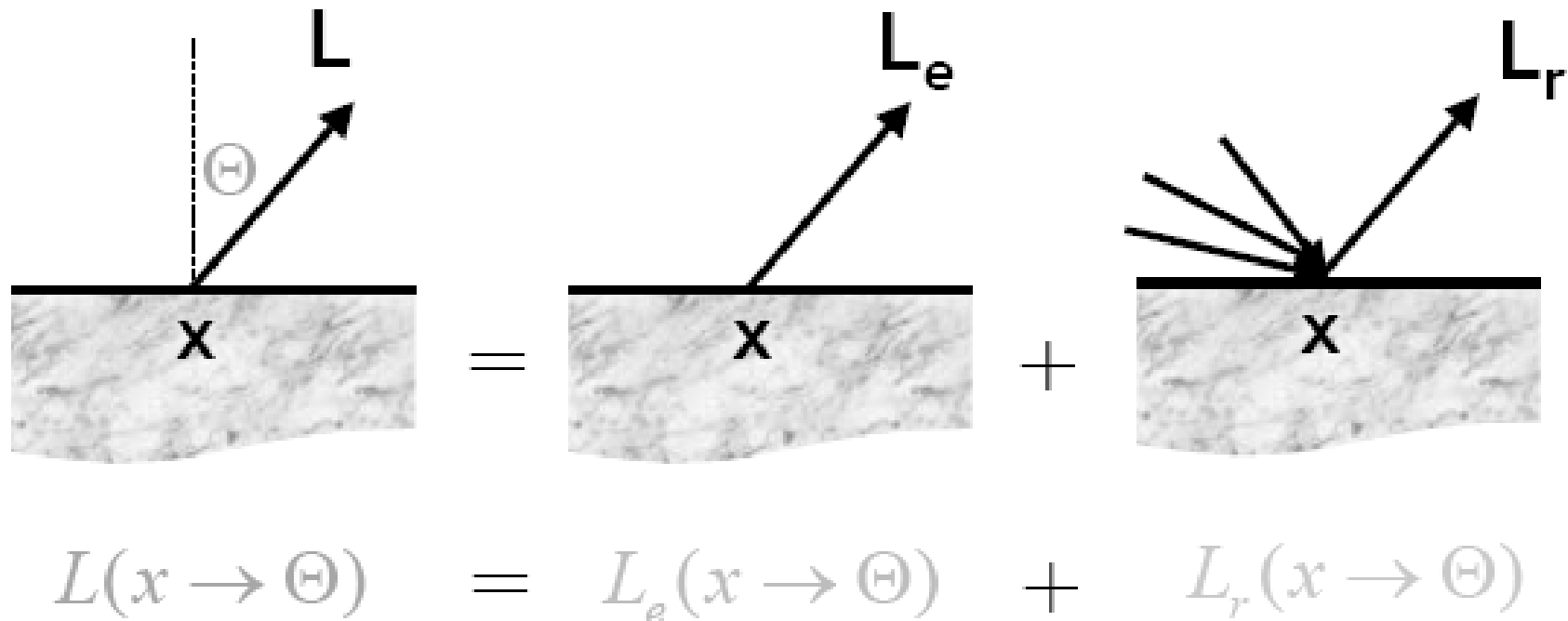
Light Transport

- **Goal**
 - Describe steady-state radiance distribution in the scene
- **Assumptions**
 - Geometric optics
 - Achieves steady state instantaneously

Rendering Equation

- Describes energy transport in the scene
- Input
 - Light sources
 - Surface geometry
 - Reflectance characteristics of surfaces
- Output
 - Value of radiances at all surface points in all directions

Rendering Equation



Rendering Equation

$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{\text{hemisphere}} L(x \leftarrow \Psi) \dots$$

Rendering Equation

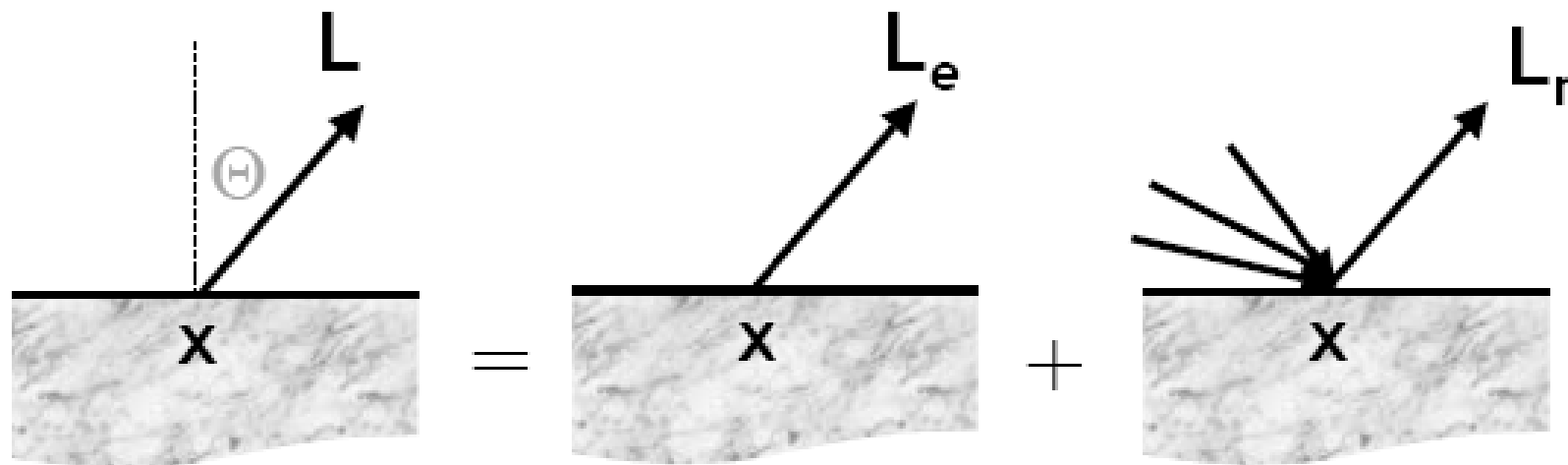
$$f_r(x, \Psi \leftrightarrow \Theta) = \frac{dL(x \rightarrow \Theta)}{dE(x \leftarrow \Psi)}$$

$$dL(x \rightarrow \Theta) = f_r(x, \Psi \leftrightarrow \Theta) dE(x \leftarrow \Psi)$$

$$dL(x \rightarrow \Theta) = f_r(x, \Psi \leftrightarrow \Theta) L(x \leftarrow \Psi) \cos(N_x, \Psi) d\omega_\Psi$$

$$L_r(x \rightarrow \Theta) = \int_{\text{hemisphere}} f_r(x, \Psi \leftrightarrow \Theta) L(x \leftarrow \Psi) \cos(N_x, \Psi) d\omega_\Psi$$

Rendering Equation

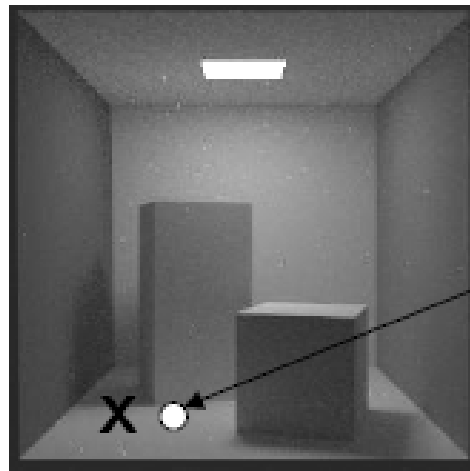


$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{\text{hemisphere}} L(x \leftarrow \Psi) f_r(x, \Psi \leftrightarrow \Theta) \cos(\mathbf{N}_x, \Psi) d\omega_\Psi$$

- Applicable for each wavelength



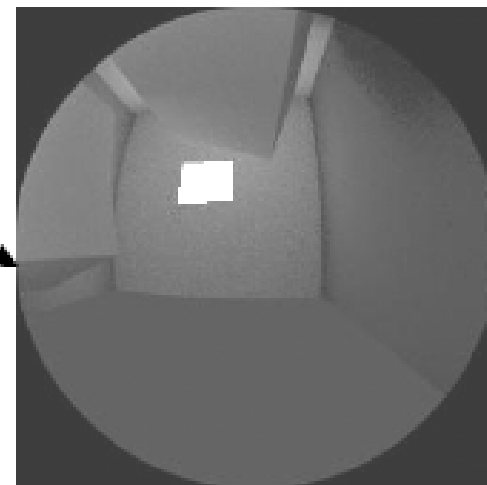
Rendering Equation



$$\underline{L(x \rightarrow \Theta)} = L_e(x \rightarrow \Theta) +$$

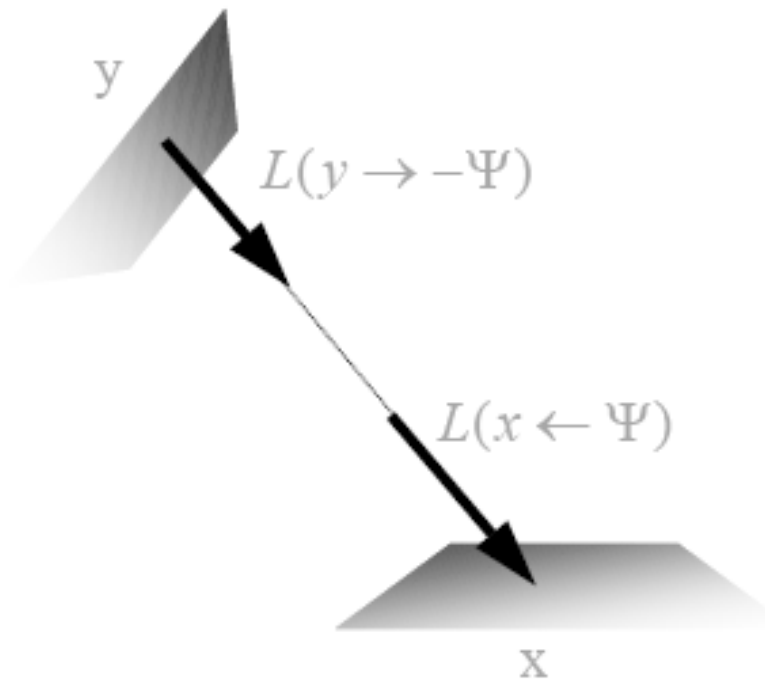
$$\int_{\text{hemisphere}} L(x \leftarrow \Psi) f_r(x, \Psi \leftrightarrow \Theta) \cos(\mathbf{N}_x, \Psi) d\omega_\Psi$$

incoming radiance



Rendering Equation: Area Formulation

$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos \theta_x \cdot d\omega_\Psi$$



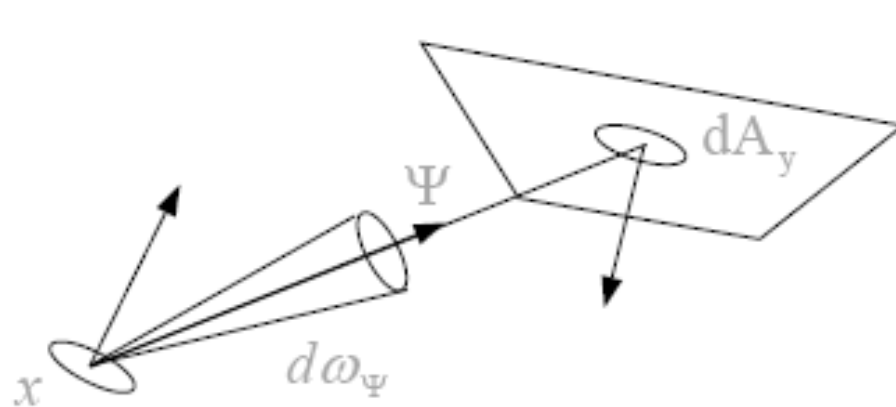
Ray-casting function: what is the nearest visible surface point seen from x in direction Ψ ?

$$y = vp(x, \Psi)$$

$$L(x \leftarrow \Psi) = L(vp(x, \Psi) \rightarrow -\Psi)$$

Rendering Equation

$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos \theta_x \cdot d\omega_\Psi$$



$$y = vp(x, \Psi)$$

$$L(x \leftarrow \Psi) = L(vp(x, \Psi) \rightarrow -\Psi)$$

$$d\omega_\Psi = \frac{dA_y \cos \theta_y}{r_{xy}^2}$$

Rendering Equation: Visible Surfaces

$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos \theta_x \cdot d\omega_\Psi$$

Coordinate transform \downarrow

$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{\substack{y \text{ on} \\ \text{all surfaces}}} f_r(\Psi \leftrightarrow \Theta) \cdot L(y \rightarrow -\Psi) \cos \theta_x \cdot \frac{\cos \theta_y}{r_{xy}^2} \cdot dA_y$$

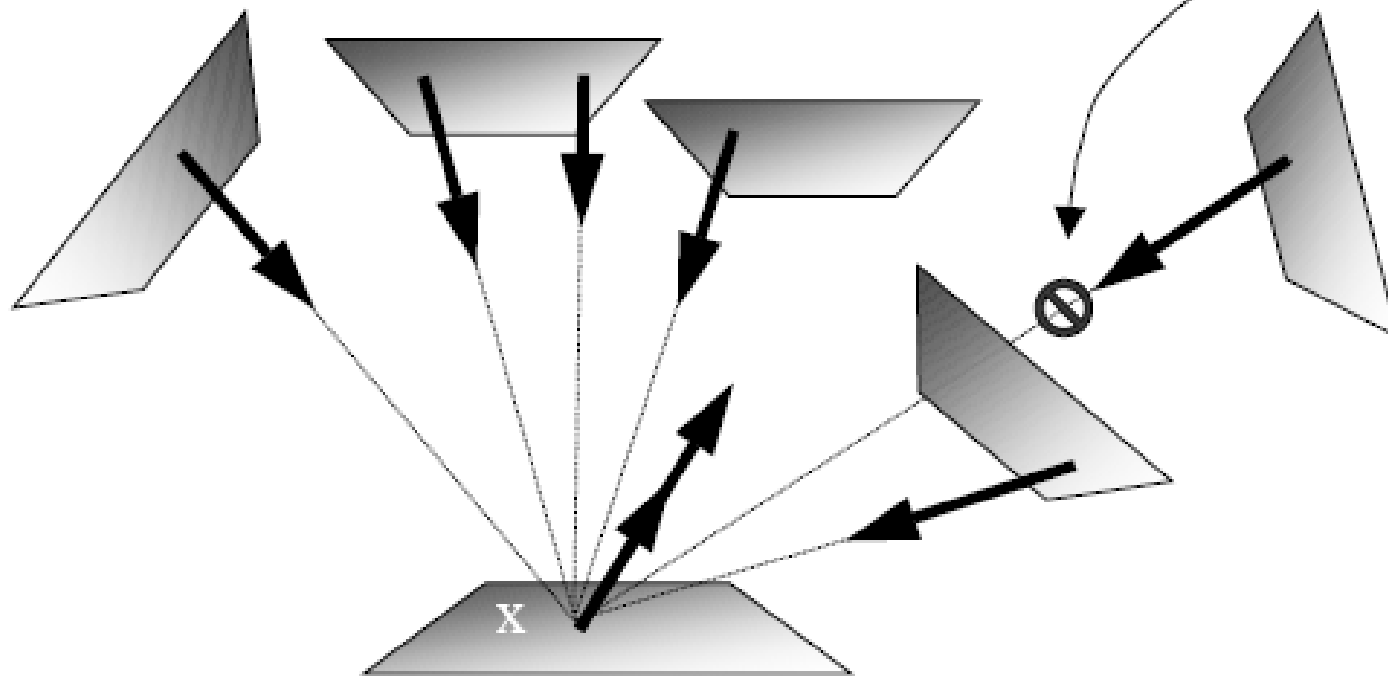
$y = vp(x, \Psi)$

Integration domain = visible surface points y

- Integration domain extended to ALL surface points by including visibility function

Rendering Equation: All Surfaces

$$L(x \rightarrow \Theta) = L_e(\dots) + \int_A f_r(\dots) \cdot L(y \rightarrow -\Psi) \cdot \frac{\cos \theta_x \cdot \cos \theta_y}{r_{xy}^2} \cdot V(x, y) dA_y$$



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Two Forms of the Rendering Equation

- Hemisphere integration

$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos \theta_x \cdot d\omega_\Psi$$

- Area integration

$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_A f_r(\Psi \leftrightarrow \Theta) \cdot L(y \rightarrow -\Psi) \cdot \frac{\cos \theta_x \cdot \cos \theta_y}{r_{xy}^2} \cdot V(x, y) \cdot dA_y$$

Class Objectives were:

- Derive the rendering equation
- Transform it to area integration

Homework

- **Go over the next lecture slides before the class**
- **Watch 2 SIGGRAPH videos and submit your summaries every Tue. class**
 - **Just one paragraph for each summary**

Example:

Title: XXX XXXX XXXX

Abstract: this video is about accelerating the performance of ray tracing. To achieve its goal, they design a new technique for reordering rays, since by doing so, they can improve the ray coherence and thus improve the overall performance.

Any Questions?

- **Come up with one question on what we have discussed in the class and submit at the end of the class**
 - 1 for already answered questions
 - 2 for typical questions
 - 3 for questions with thoughts
- **Submit questions at least four times before the mid-term exam**
 - Multiple questions for the class is counted as only a single time

Next Time

- Monte Carlo ray tracing