CS482: Radiometry and Rendering Equation

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(윤성의)

Course URL:
http://sglab.kaist.ac.kr/~sungeui/ICG/

Class Objectives (Ch. 12 and 13)

- **Know terms of:**
  - **Hemispherical coordinates and integration**
  - **Various radiometric quantities (e.g., radiance)**
  - **Basic material function, BRDF**
  - **Understand the rendering equation**

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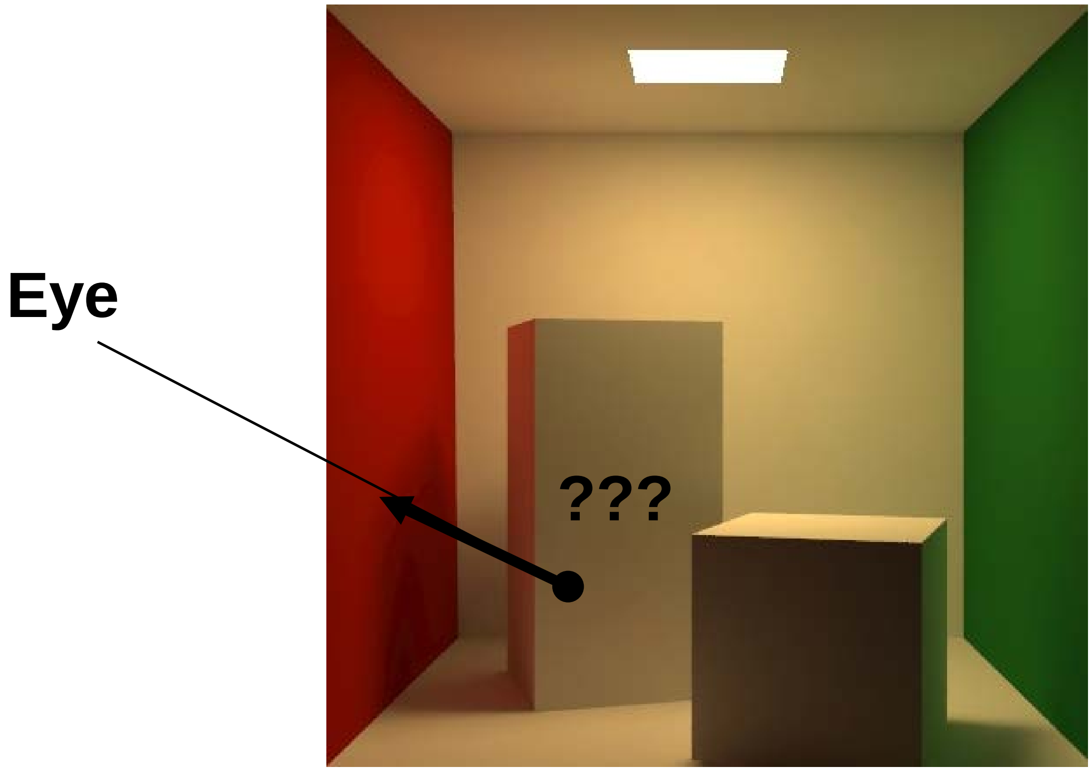
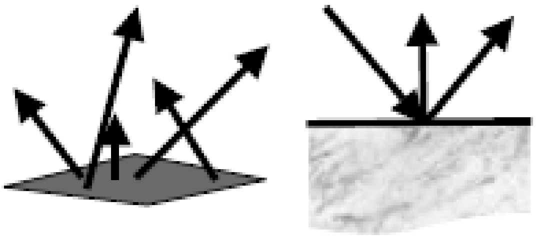
Motivation



Light and Material Interactions

- Physics of light



- Radiometry
- Material properties

From kavita’s slides

- Rendering equation

Models of Light

- Quantum optics
  - Fundamental model of the light
  - Explain the dual wave-particle nature of light
- Wave model
  - Simplified quantum optics
  - Explains diffraction, interference, and polarization

- Geometric optics
  - Most commonly used model in CG
  - Size of objects >> wavelength of light
  - Light is emitted, reflected, and transmitted

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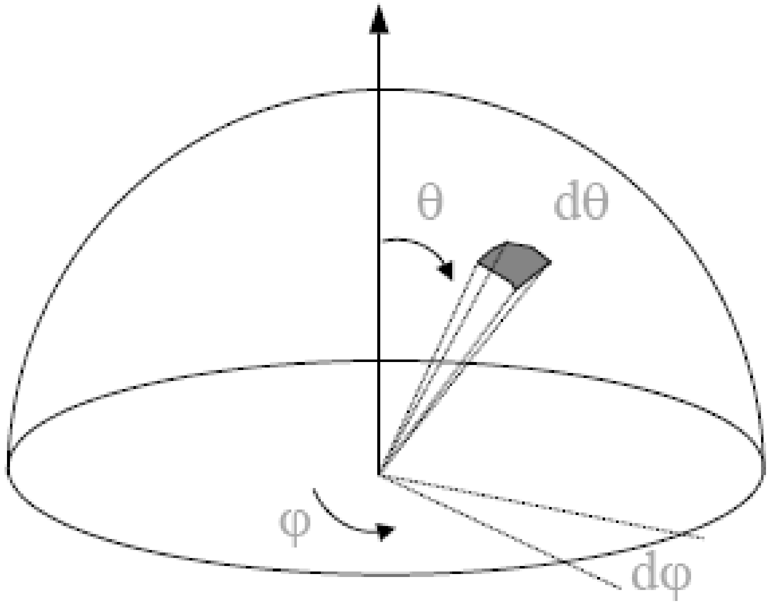
Hemispheres

- **Hemisphere**
  - **Two-dimensional surfaces**
- **Direction**
  - **Point on (unit) sphere**



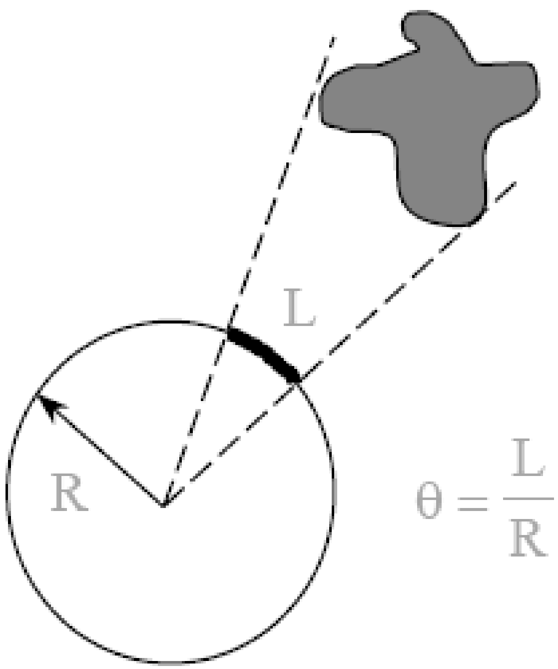
$$\theta \in [0, \frac{\pi}{2}]$$

$$\varphi \in [0, 2\pi]$$

From kavita's slides

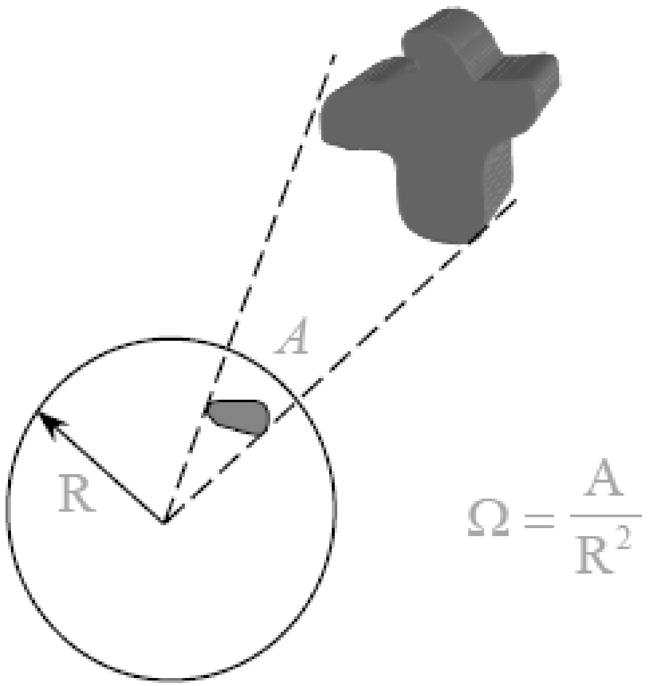
Solid Angles

2D



Full circle
= 2pi radians

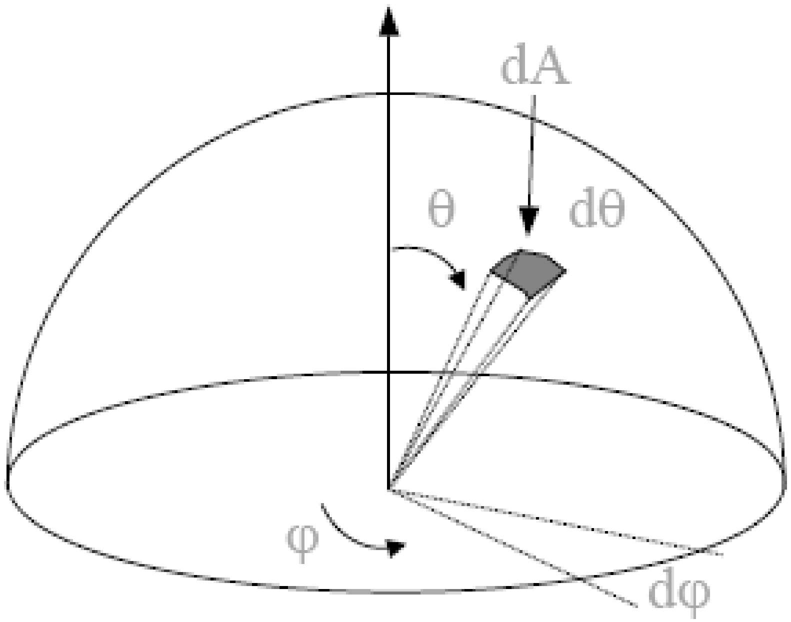
3D



Full sphere
= 4pi steradians

Hemispherical Coordinates

- **Direction, Θ**
  - **Point on (unit) sphere**



$$dA = (r \sin\theta d\varphi)(r d\theta)$$

From kavita's slides

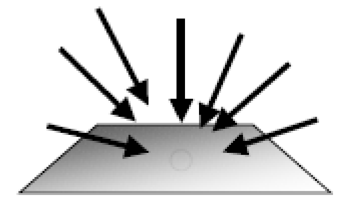
Hemispherical Coordinates

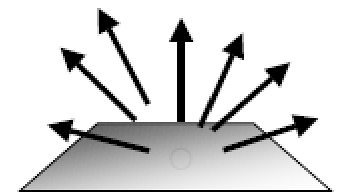
- **Differential solid angle**

$$d\omega = \frac{dA}{r^2} = \sin\theta d\theta d\varphi$$

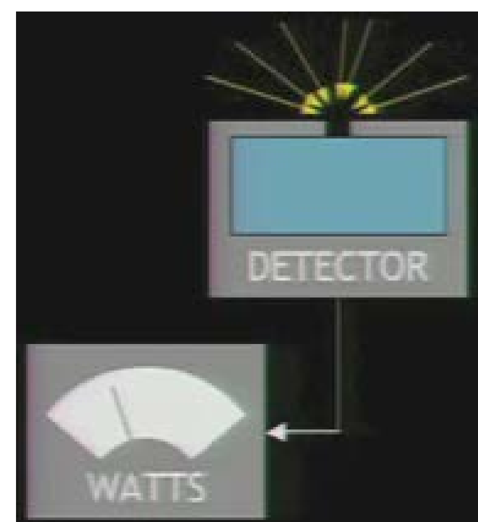
Irradiance

- Incident radiant power per unit area (dP/dA)
  - Area density of power



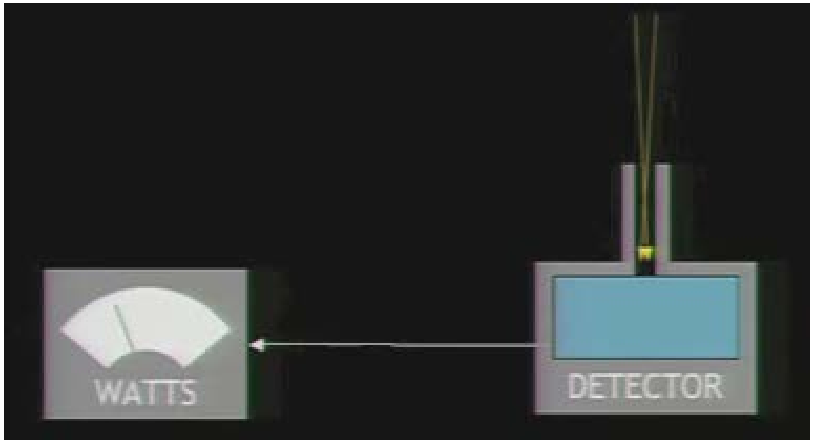


- Symbol: E, unit: W/ m^2
  - Area power density existing a surface is called radiance exitance (M) or radiosity (B)
- For example
  - A light source emitting 100 W of area 0.1 m^2
  - Its radiant exitance is 1000 W/ m^2



Radiance

- Radiant power at x in direction θ
  - $L(x \rightarrow \Theta)$: 5D function
    - Per unit area
    - Per unit solid angle

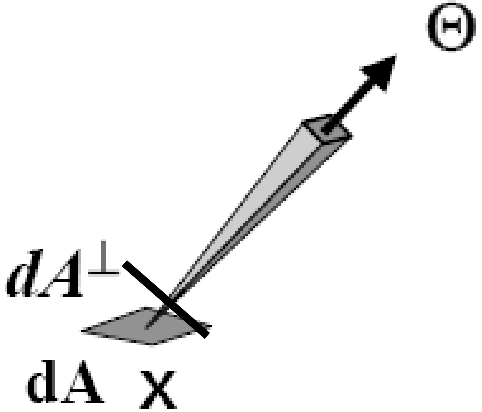


- Important quantity for rendering

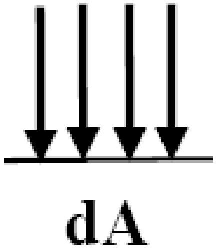
Radiance: Projected Area

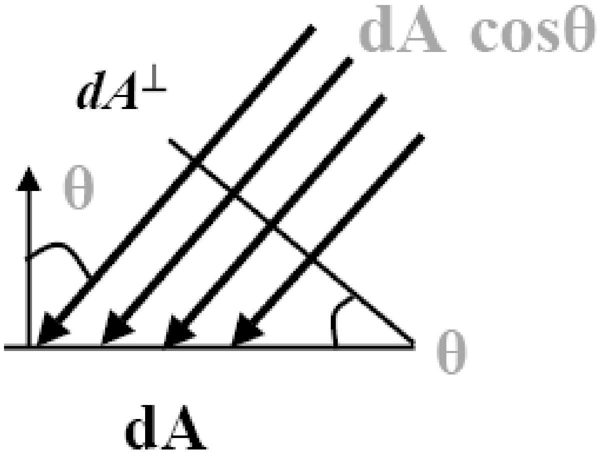
$$L(x \to \Theta) = \frac{d^2P}{dA^{\perp} d\omega_{\Theta}}$$

$$= \frac{d^2P}{d\omega_{\Theta} dA \cos\theta}$$



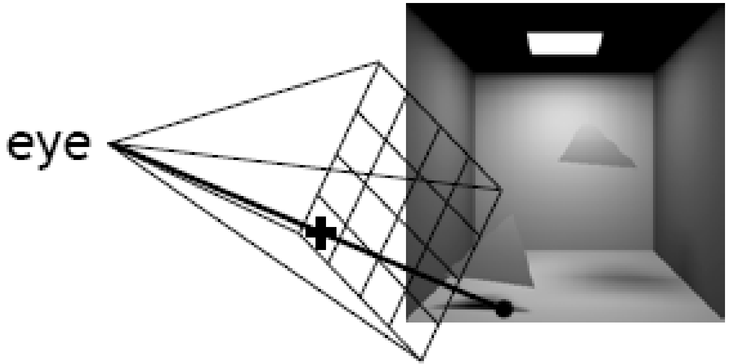
- **Why per unit projected surface area**





Sensitivity to Radiance

- **Responses of sensors (camera, human eye) is proportional to radiance**

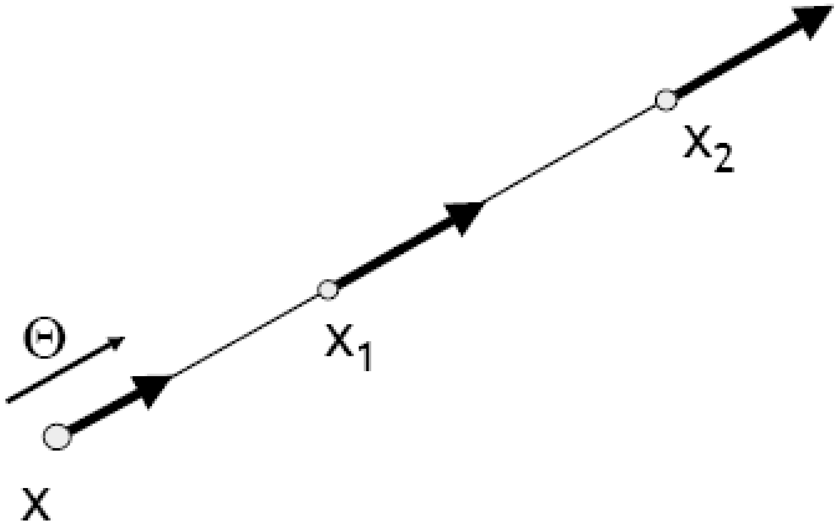


From kavita's slides

- **Pixel values in image proportional to radiance received from that direction**

Properties of Radiance

- **Invariant along a straight line (in vacuum)**



From kavita's slides

Invariance of Radiance

We can prove it based on the assumption the conservation of energy.

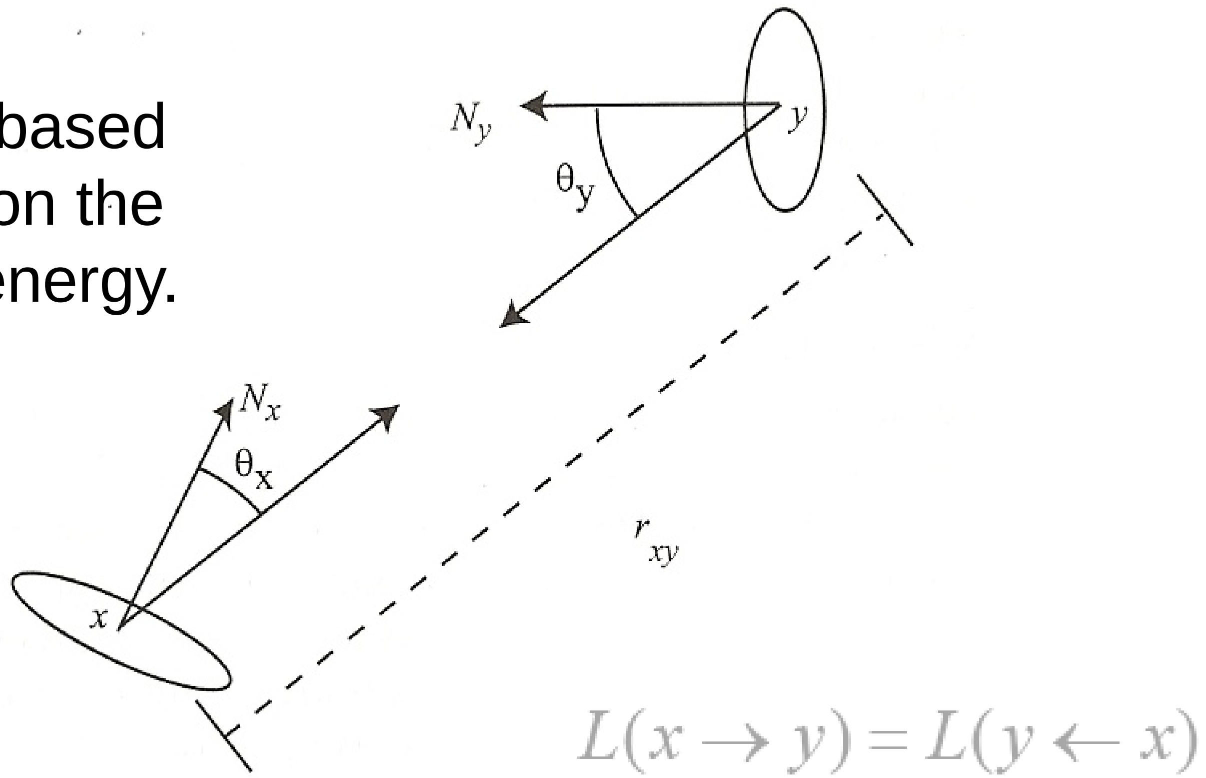
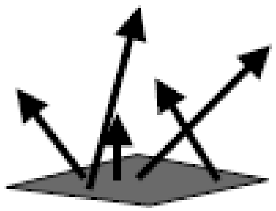
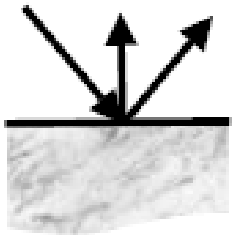


Figure 2.3. Invariance of radiance.

Light and Material Interactions

- Physics of light



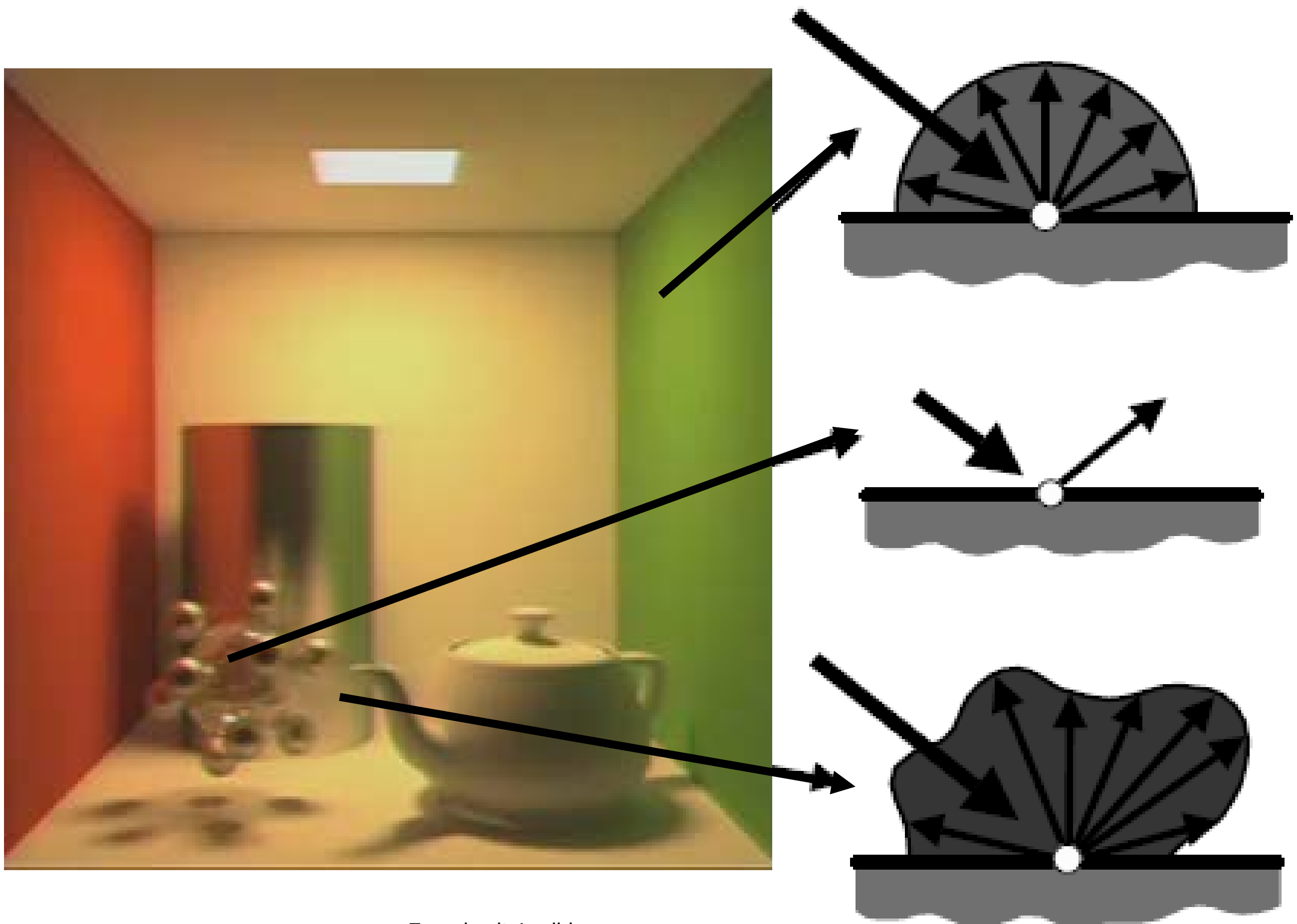


- Radiometry
- **Material properties**

From kavita's slides

- Rendering equation

Materials



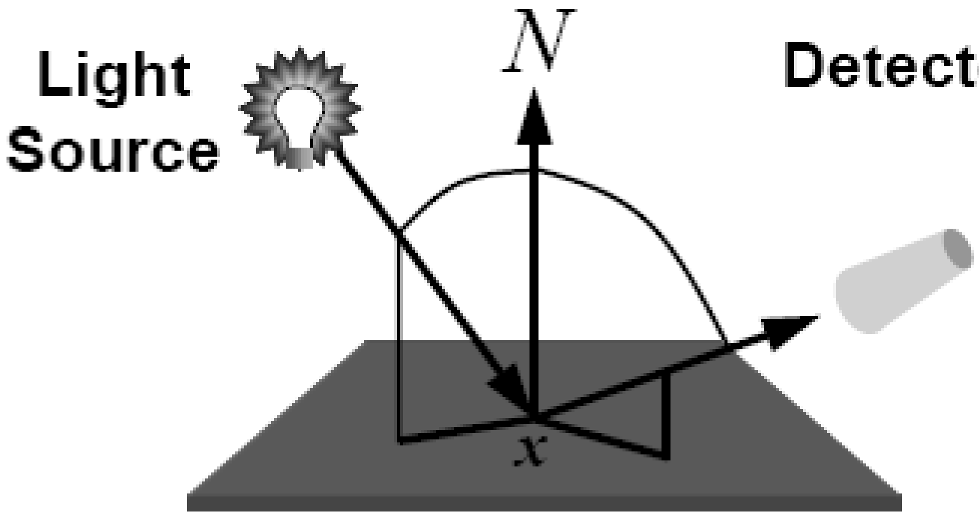
Ideal diffuse (Lambertian)

Ideal specular

Glossy

From kavita's slides

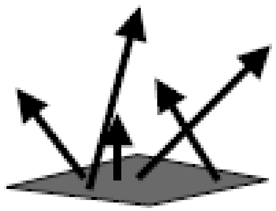
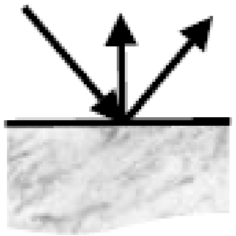
Bidirectional Reflectance Distribution Function (BRDF)



$$f_r(x, \Psi \to \Theta) = \frac{dL(x \to \Theta)}{dE(x \leftarrow \Psi)} = \frac{dL(x \to \Theta)}{L(x \leftarrow \Psi)\cos(N_x, \Psi)d\omega_\Psi}$$

Light and Material Interactions

- Physics of light

- Radiometry
- Material properties

From kavita's slides

- Rendering equation

Light Transport

- Goal
  - Describe steady-state radiance distribution in the scene
- Assumptions
  - Geometric optics
  - Achieves steady state instantaneously

Rendering Equation

- Describes energy transport in the scene
- Input
  - Light sources
  - Surface geometry
  - Reflectance characteristics of surfaces

- Output
  - Value of radiances at all surface points in all directions

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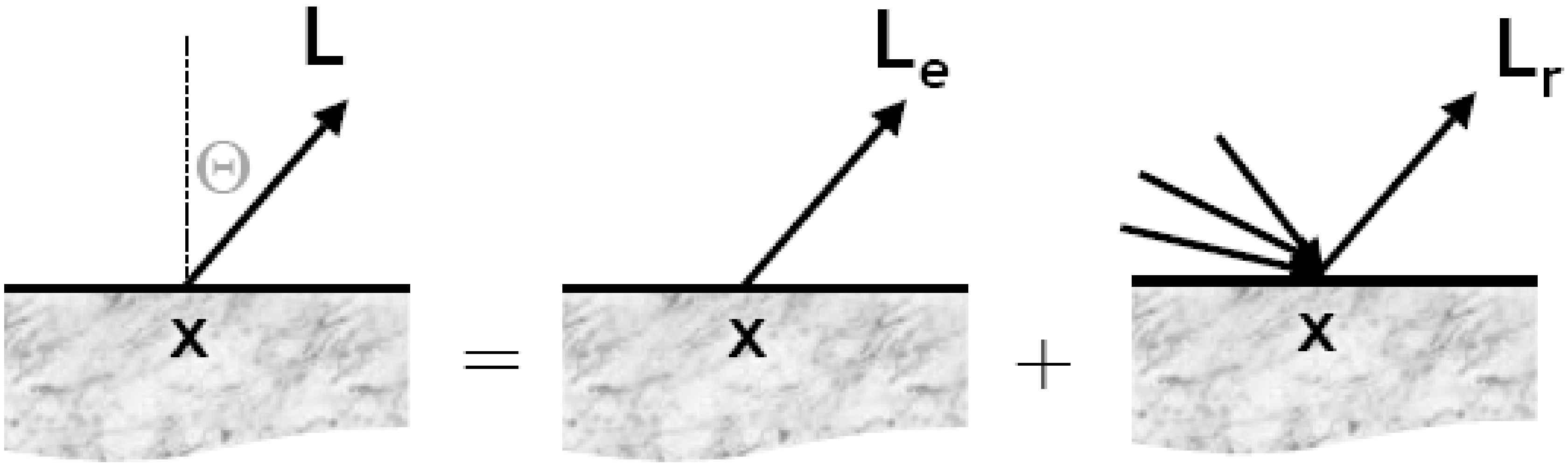
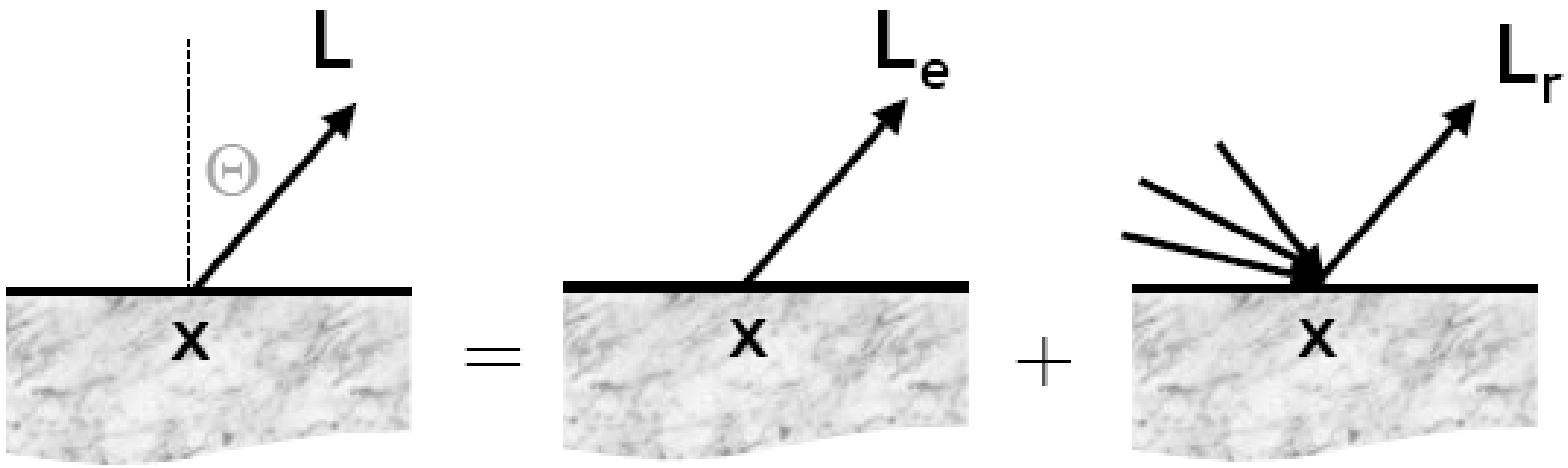
Rendering Equation



$$L(x \to \Theta) = L_e(x \to \Theta) + L_r(x \to \Theta)$$

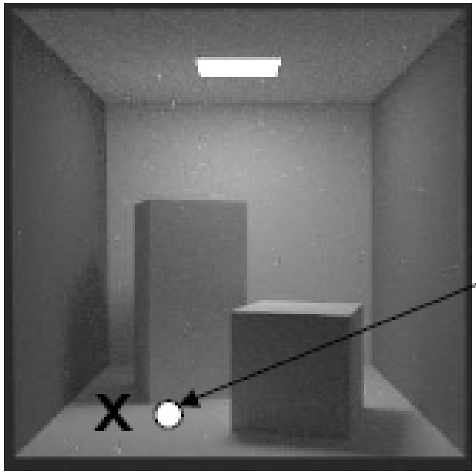
Rendering Equation



$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{hemisphere} L(x \leftarrow \Psi)\, f_r(x, \Psi \leftrightarrow \Theta) \cos(N_x, \Psi)\, d\omega_\Psi$$

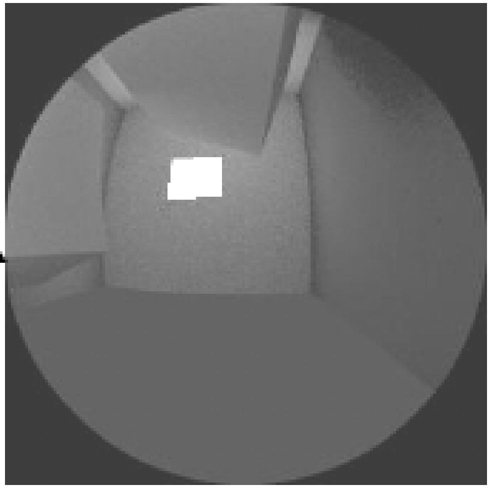
- Applicable for each wavelength

Rendering Equation



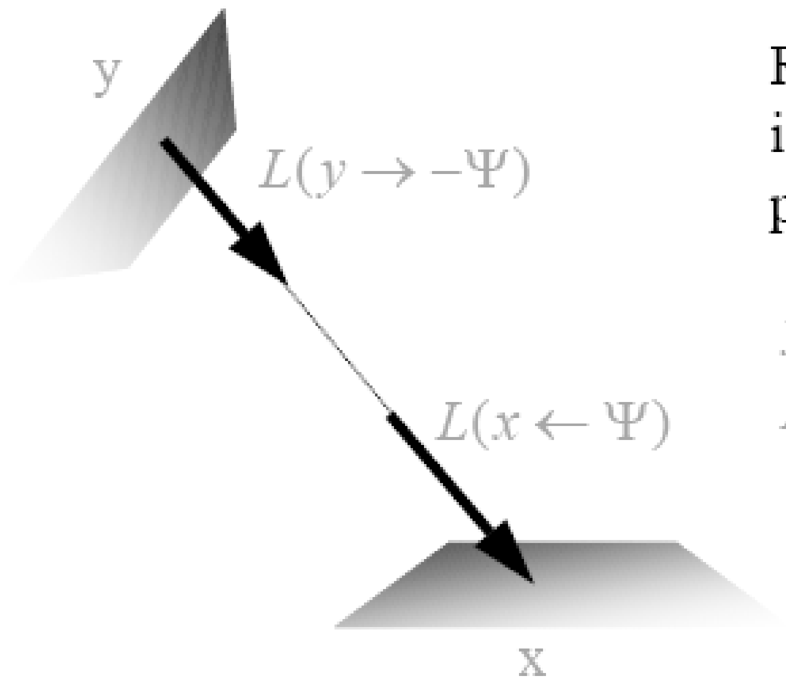
$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{hemisphere} L(x \leftarrow \Psi) f_r(x, \Psi \leftrightarrow \Theta) \cos(N_x, \Psi) d\omega_\Psi$$

incoming radiance



Rendering Equation: Area Formulation

$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos\theta_x \cdot d\omega_\Psi$$



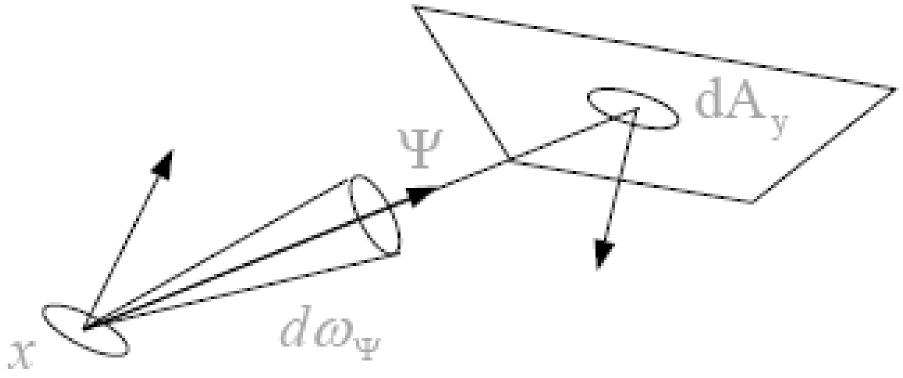
Ray-casting function: what is the nearest visible surface point seen from x in direction Ψ?

$$y = vp(x, \Psi)$$

$$L(x \leftarrow \Psi) = L(vp(x,\Psi) \to -\Psi)$$

Rendering Equation

$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos\theta_x \cdot d\omega_\Psi$$



$$y = vp(x, \Psi)$$

$$L(x \leftarrow \Psi) = L(vp(x, \Psi) \to -\Psi)$$

$$d\omega_\Psi = \frac{dA_y \cos\theta_y}{r_{xy}^2}$$

Rendering Equation: Visible Surfaces

$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos\theta_x \cdot d\omega_\Psi$$

Coordinate transform

$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{\substack{y \text{ on} \\ \text{all surfaces}}} f_r(\Psi \leftrightarrow \Theta) \cdot L(y \to -\Psi)\ \cos\theta_x \cdot \frac{\cos\theta_y}{r_{xy}^2} \cdot dA_y$$

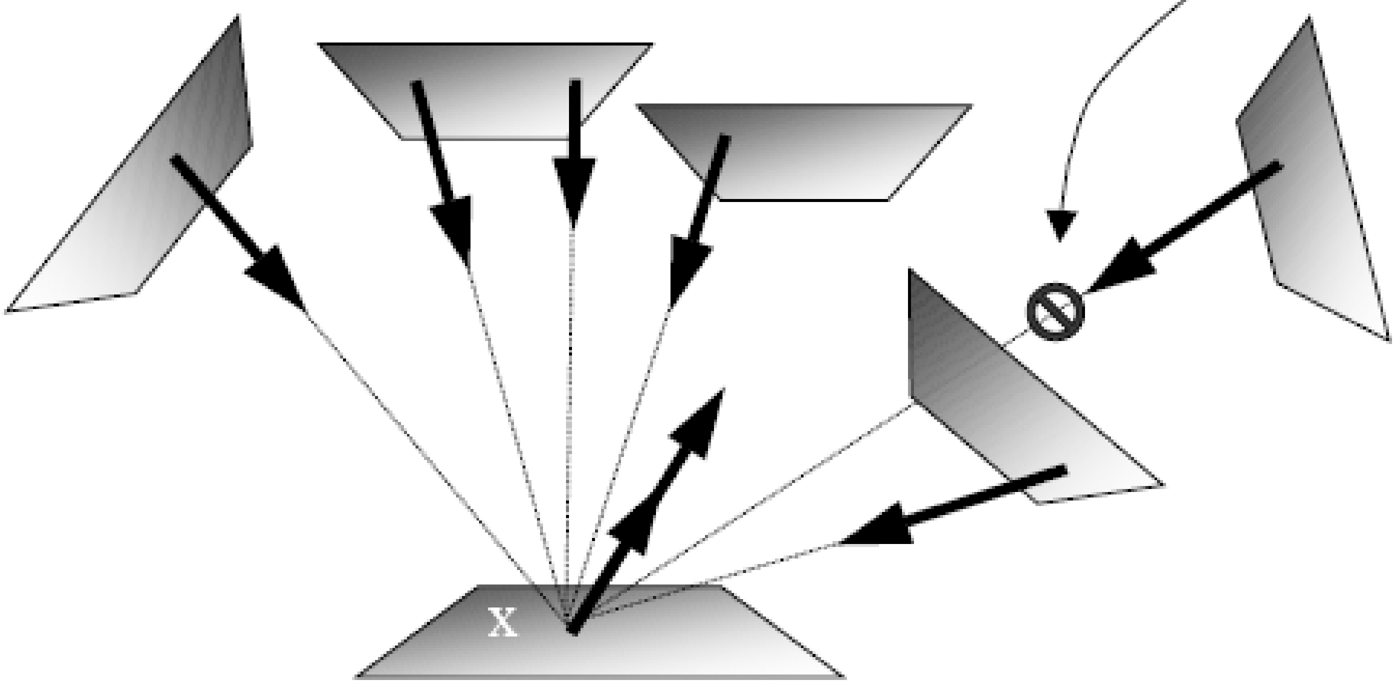
$$y = vp(x, \Psi)$$

Integration domain = visible surface points y

- Integration domain extended to ALL surface points by including visibility function

Rendering Equation: All Surfaces

$$L(x \rightarrow \Theta) = L_e(\ldots) + \int_A f_r(\ldots) \cdot L(y \rightarrow -\Psi) \cdot \frac{\cos\theta_x \cdot \cos\theta_y}{r_{xy}^2} \cdot V(x,y)\, dA_y$$



Two Forms of the Rendering Equation

- **Hemisphere integration**

$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos\theta_x \cdot d\omega_\Psi$$

- **Area integration (used as the form factor)**

$$L(x \to \Theta) = L_e(x \to \Theta) + \int_A f_r(\Psi \leftrightarrow \Theta) \cdot L(y \to -\Psi) \cdot \frac{\cos\theta_x \cdot \cos\theta_y}{r_{xy}^2} \cdot V(x,y) \cdot dA_y$$

Class Objectives (Ch. 12 & 13) were:

- Know terms of:
  - Hemispherical coordinates and integration
  - Various radiometric quantities (e.g., radiance)
  - Basic material function, BRDF
  - Understand the rendering equation

Next Time

- **Monte Carlo rendering methods**

Homework

- **Go over the next lecture slides before the class**
- **Watch two videos or go over papers, and submit your summaries every Tue. class**
  - **Just one paragraph for each summary**

Example:

Title: XXX XXXX XXXX
Abstract: this video is about accelerating the performance of ray tracing. To achieve its goal, they design a new technique for reordering rays, since by doing so, they can improve the ray coherence and thus improve the overall performance.

Any Questions?

- **Submit four times in Sep./Oct.**
- **Come up with one question on what we have discussed in the class and submit at the end of the class**
  - **1 for typical questions**
  - **2 for questions that have some thoughts or surprise me**