

# Differentiable Monte Carlo Ray Tracing through Edge Sampling, SIGGRAPH Asia 2018 <sup>[1]</sup>

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# Agenda

1. Differentiable Rendering
2. A Pixel Example
3. Mathematical Formulation
4. Results

# Differentiable Rendering

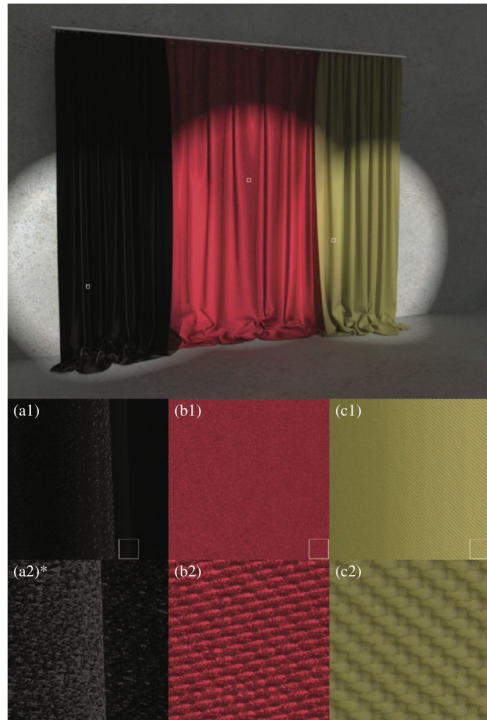
- Allows the gradients of 3D objects to be calculated and propagated through images [2]
- Crucial to optimization, inverse problem, and deep learning
- Gradient w.r.t camera parameters, light sources, scene geometry, material appearance

# Why is it challenging?

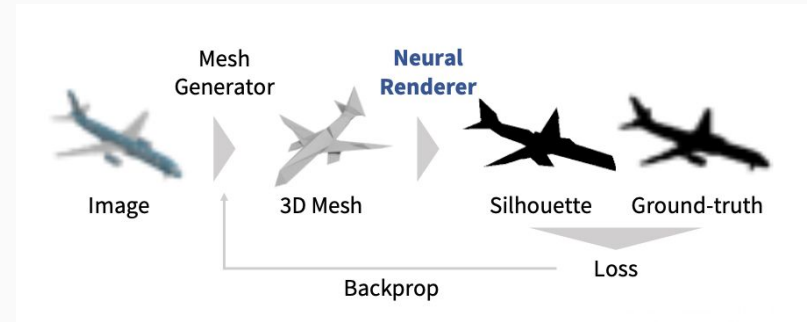
$$L_r(x \rightarrow \Theta) = \int_A L(y \rightarrow -\Psi) f_r(x, \Psi \rightarrow \Theta) \frac{\cos\theta_x \cos\theta_y}{r_{xy}^2} V(x, y) dA$$

Discontinuous!

# Previous works



1. Specialized solver for fabric [3]



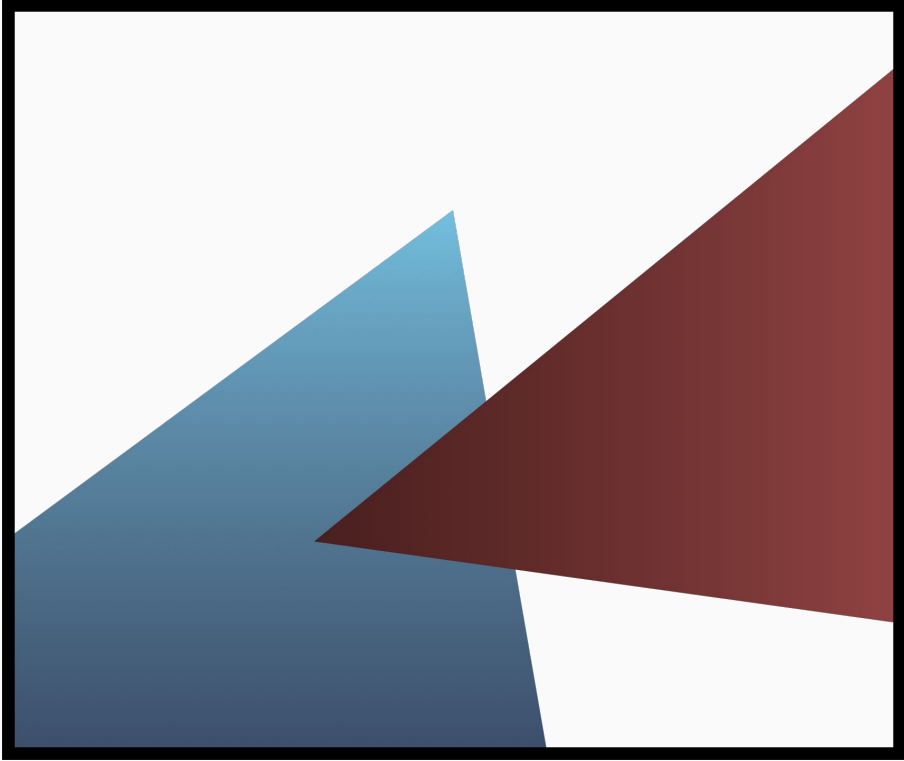
2. General renderer with limitations [4]

# Contributions

- The first comprehensive solution to derivatives w.r.t arbitrary scene parameters
- Unbiased gradients and supports arbitrary materials, shadow, and global illumination.

Idea - A Pixel Example

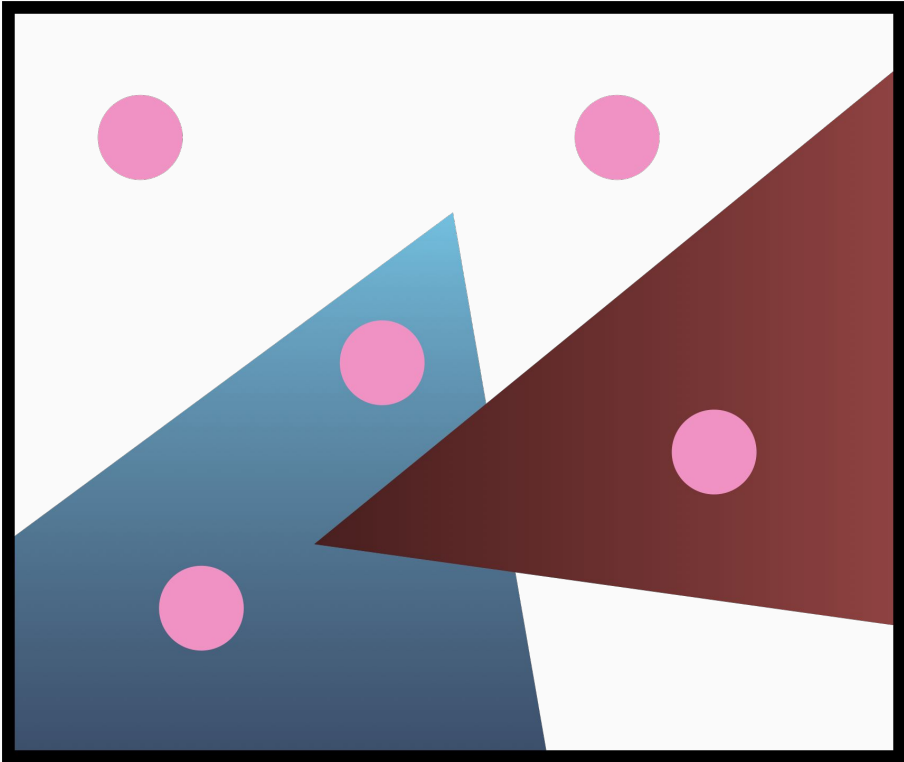
# A Pixel Example



From Tzu-Mao Li's slides



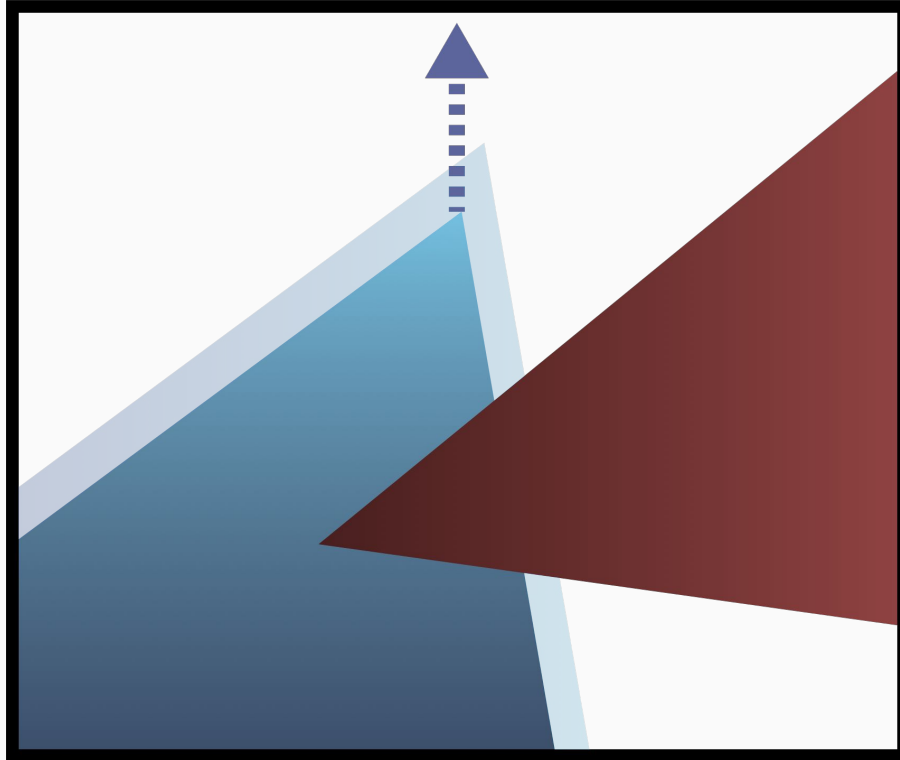
# A Pixel Example



# A Pixel Example

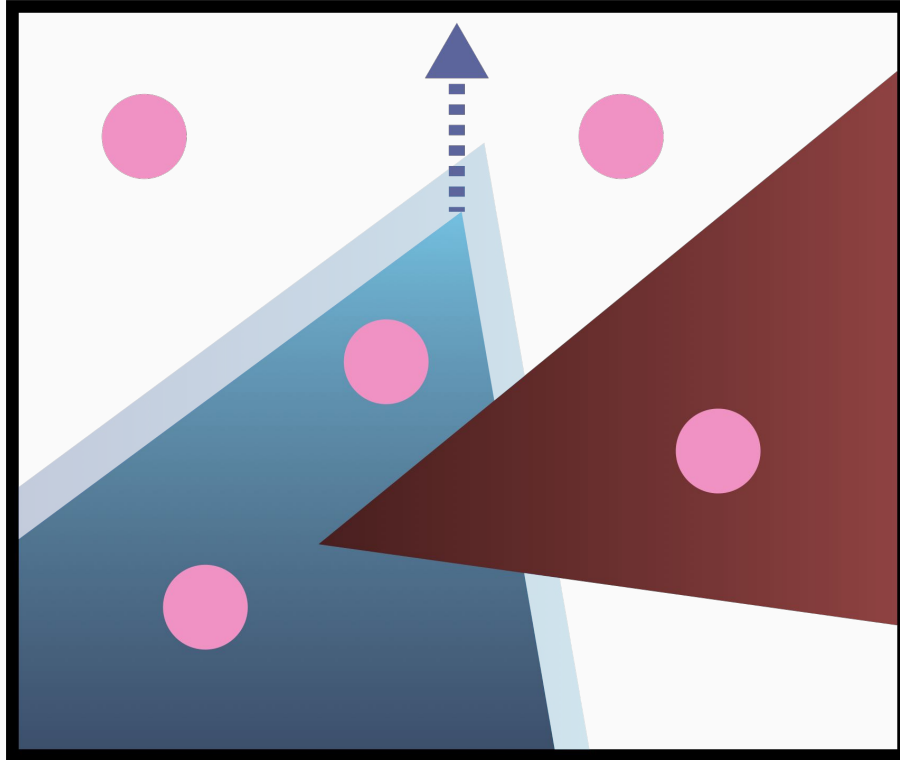
Q. Color change  
when blue  
triangle moves  
up?

A. More blue  
less white



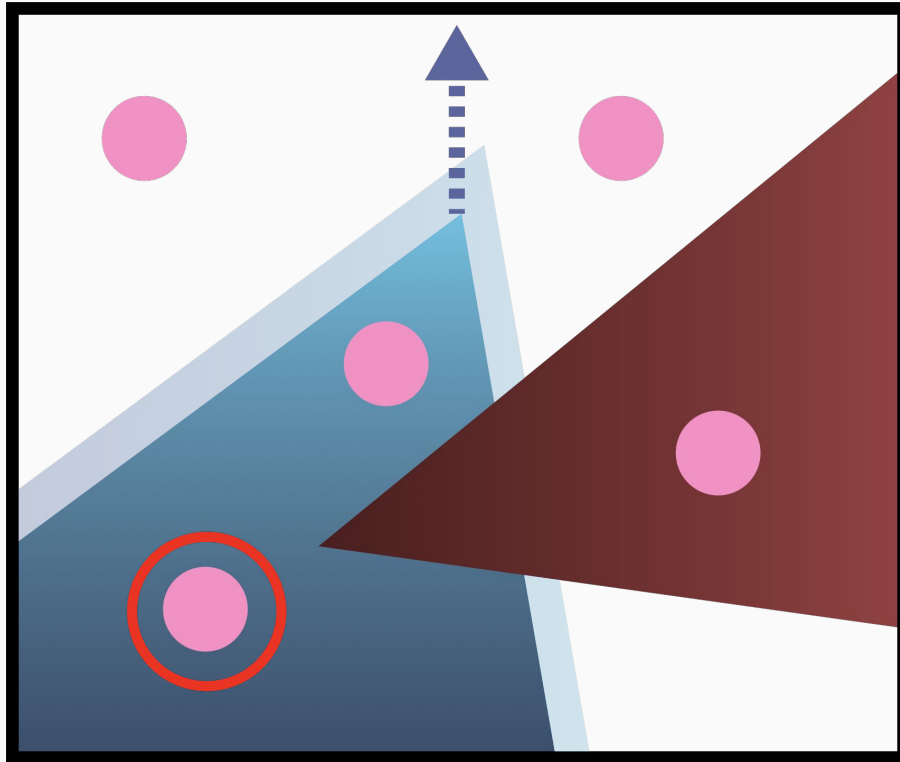
# A Pixel Example

Q. Color change  
when blue  
triangle moves  
up?



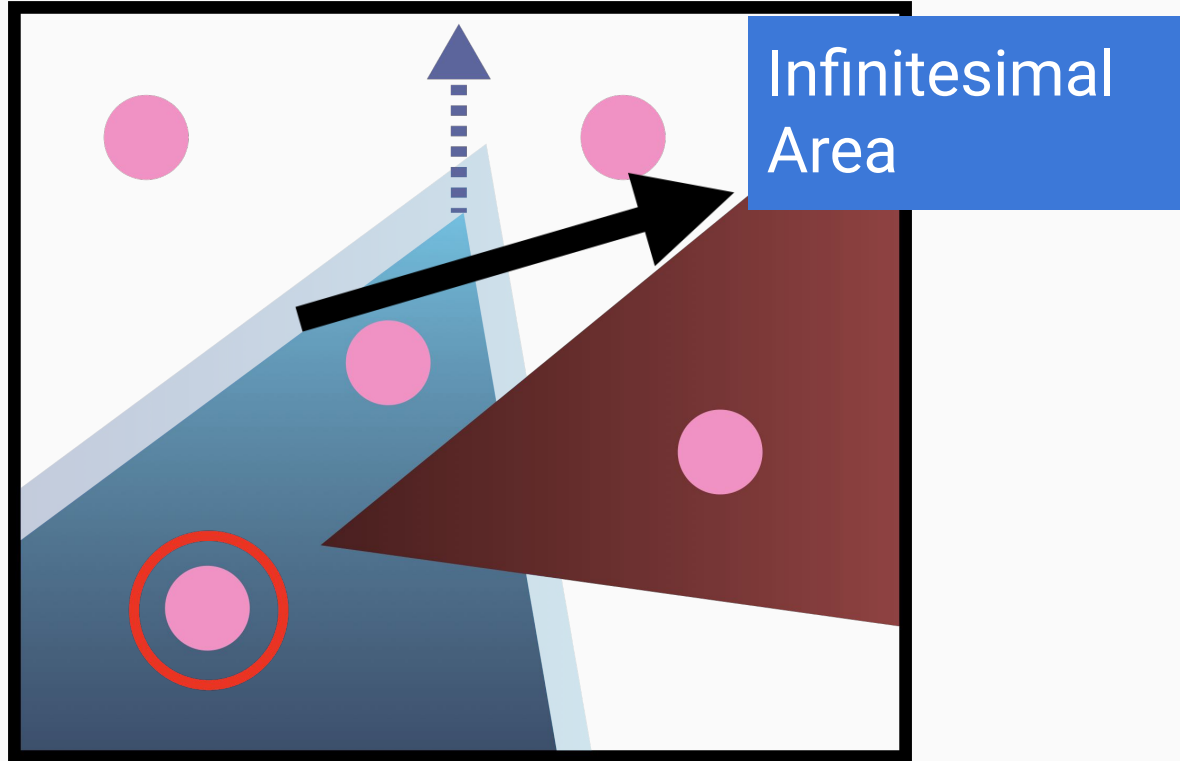
# A Pixel Example

Q. Color change  
when blue  
triangle moves  
up?

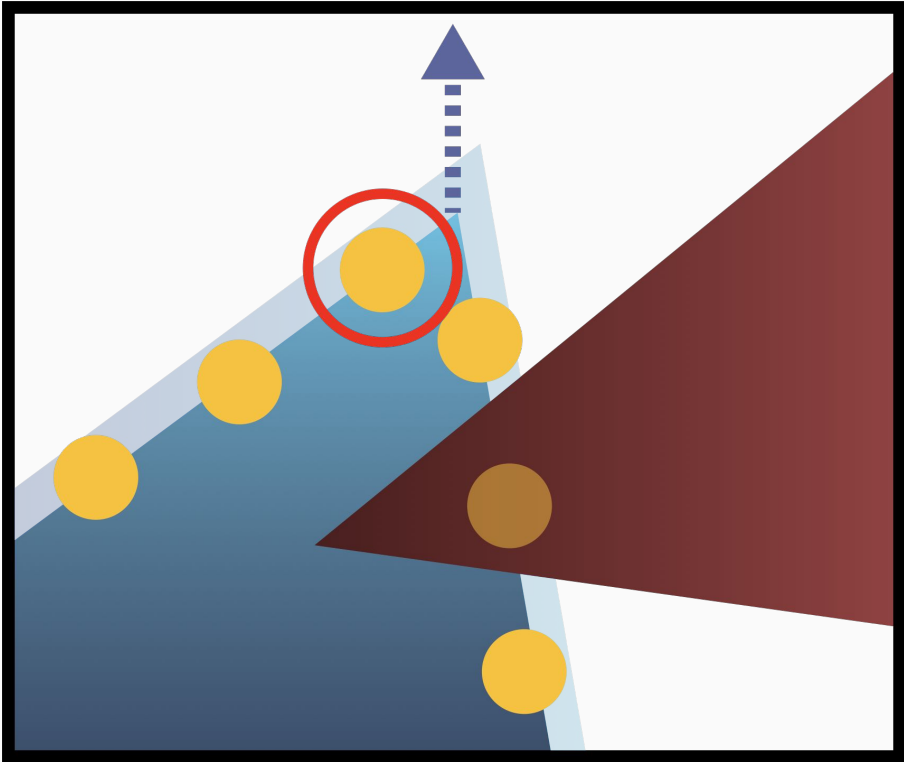


# A Pixel Example

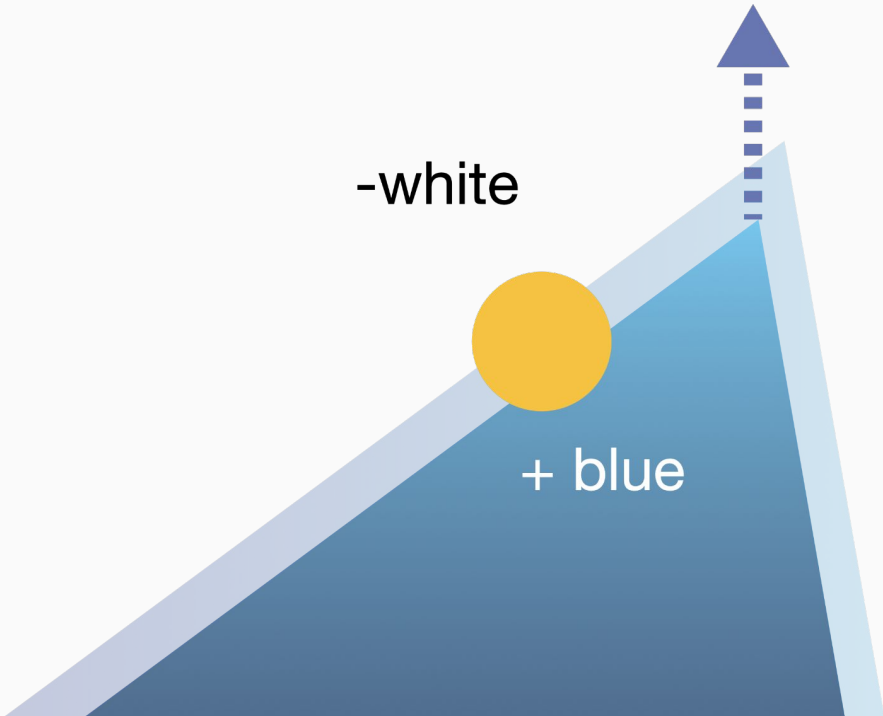
Q. Color change  
when blue  
triangle moves  
up?



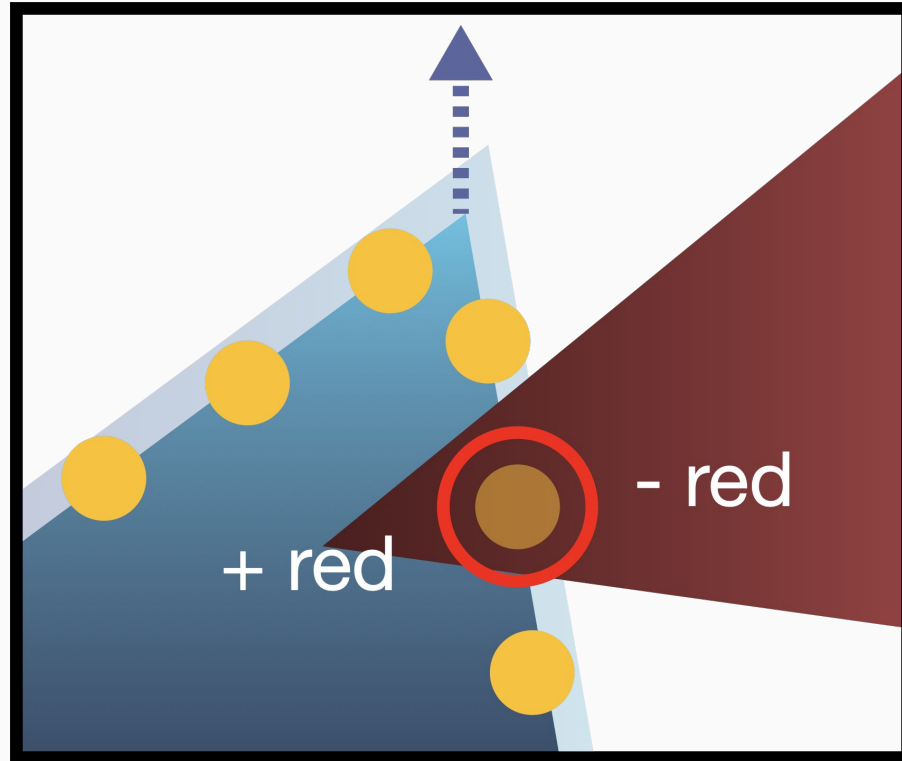
# A Pixel Example



# A Pixel Example



# A Pixel Example



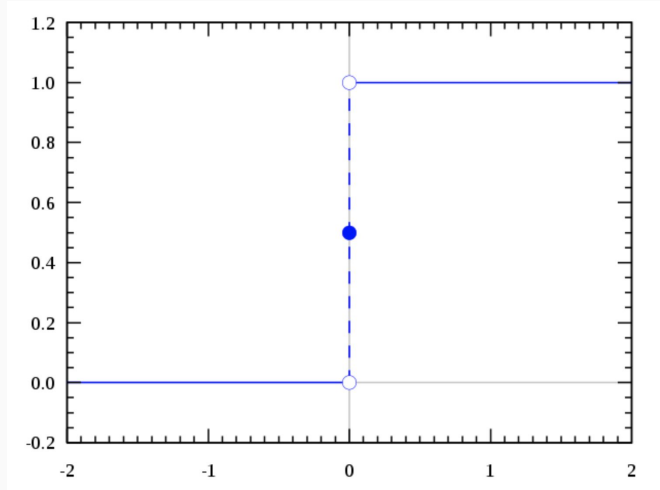


# A Pixel Example


- Both primary and secondary visibility can be modeled using this idea.
- Correctly models the global illumination effects.

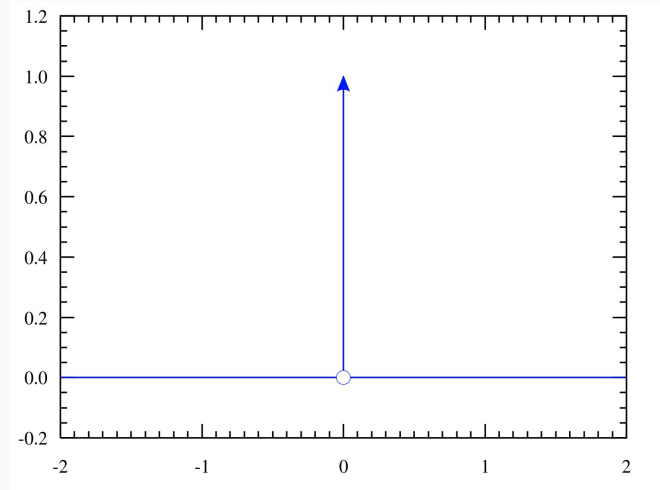
# Mathematical Formulation

# Heaviside Step Function



$\theta(x)$  : Heaviside step function [5]

$$\frac{d}{dx}$$




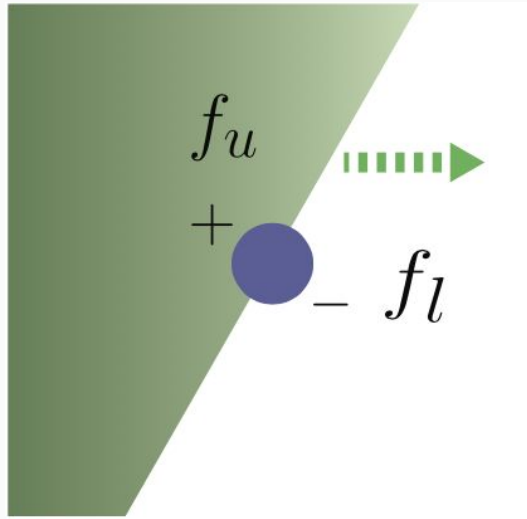
$\delta(x)$  : Dirac delta function [6]

# Mathematical Formulation

$f$  : Product of radiance and visibility term

Intensity of a pixel:  $I = \int \int f(x, y) dx dy$

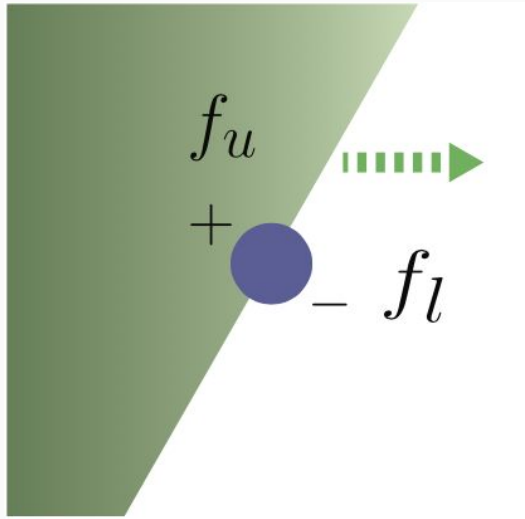
# Mathematical Formulation



$$\alpha(x, y) = Ax + By + C$$

: Equation of an edge

# Mathematical Formulation



$$f(x, y)$$



$$\theta(\alpha(x, y))f_u(x, y) + \theta(-\alpha(x, y))f_l(x, y)$$

$$= \begin{cases} f_u(x, y), & \text{if } \alpha(x, y) > 0 \\ f_l(x, y), & \text{otherwise} \end{cases}$$

# Mathematical Formulation

- We can generalize the idea to  $n$  edges in a pixel

$$I = \int \int f(x, y) dx dy$$

$$= \sum_i \int \int \theta(\alpha_i(x, y)) f_i(x, y) dx dy$$

# Mathematical Formulation

- We can analytically differentiate each term in the sum.
- Estimate the gradient using two Monte Carlo estimators

$$\nabla \int \int \theta(\alpha_i(x, y)) f_i(x, y) dx dy$$

$$= \int \int \delta(\alpha_i(x, y)) \nabla \alpha_i(x, y) f_i(x, y) dx dy$$

$$+ \int \int \theta(\alpha_i(x, y)) \nabla f_i(x, y) dx dy$$

Gradients on edge  
samples  
(yellow sample points)

Gradients on area  
samples  
(pink sample points)



# Importance sampling the edges

- Only few edges are important for a given viewport.
- Hierarchical sampling for silhouette edges
- Several factors (eg. distance, material response, ...)

# Recap

- Model edge using step function
- Analytical differentiation -> two Monte Carlo estimators
- Importance sampling of edges

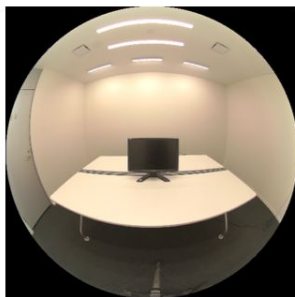
# Results

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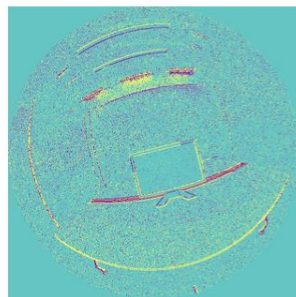
- Gradients w.r.t. various scene parameters



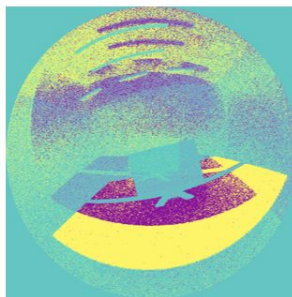
(a) initial guess



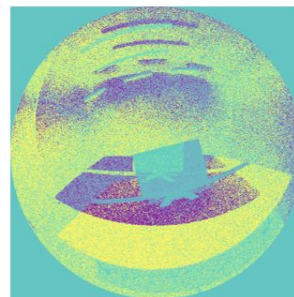
(b) real photograph



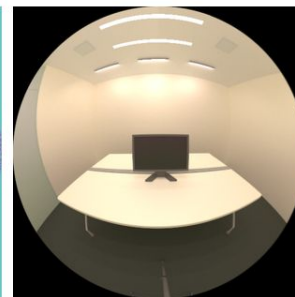
(c) camera gradient  
(per-pixel contribution)



(d) table albedo gradient  
(per-pixel contribution)



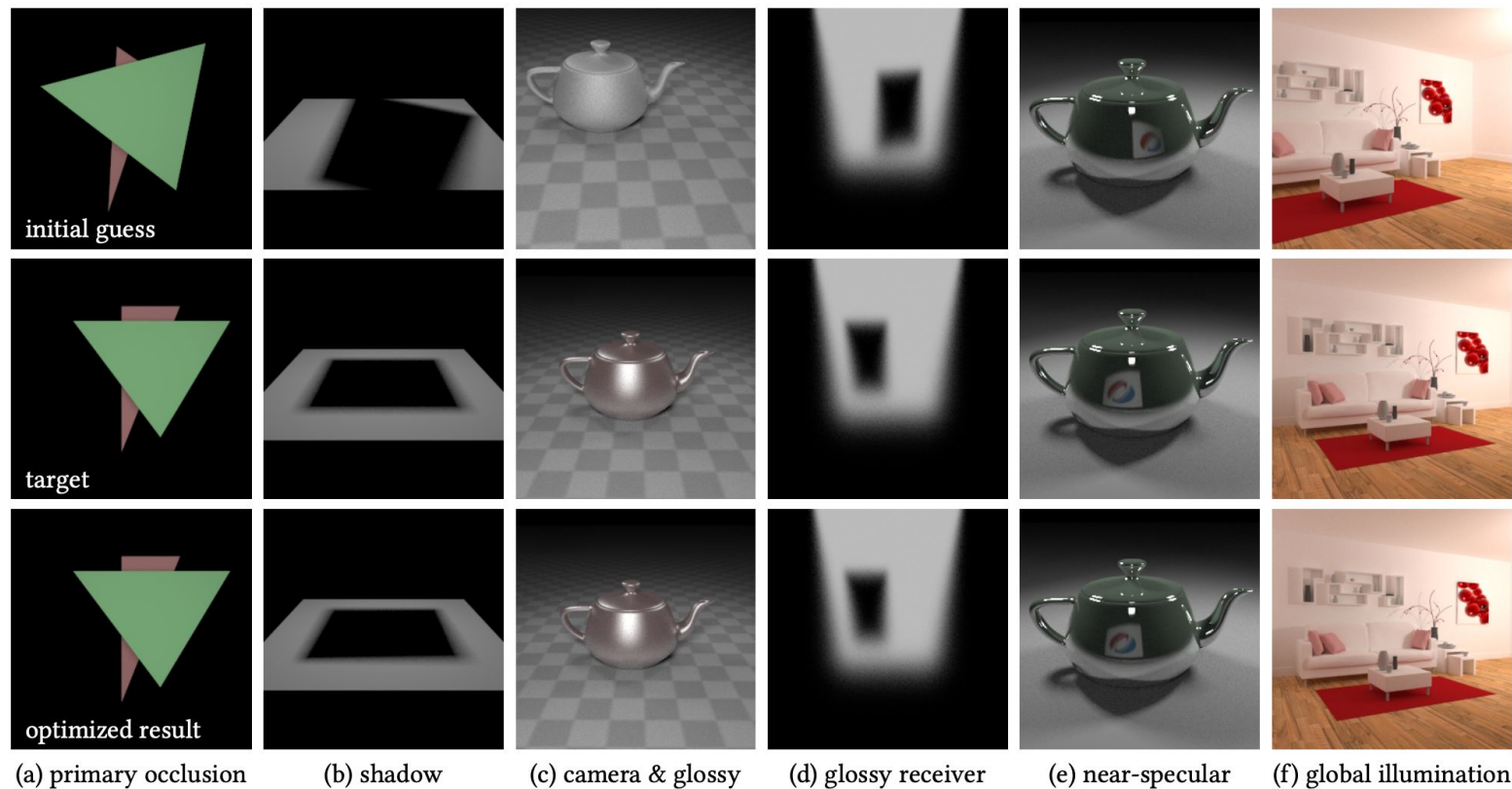
(e) light gradient  
(per-pixel contribution)



(f) our fitted result

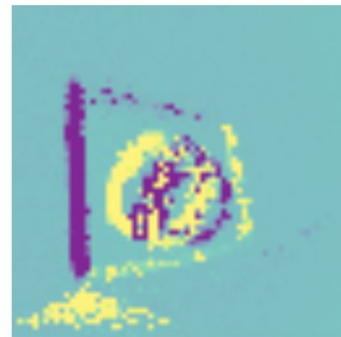
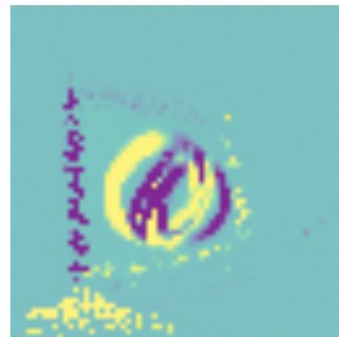
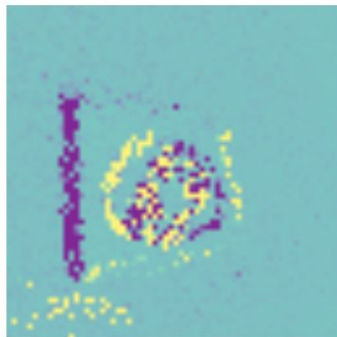
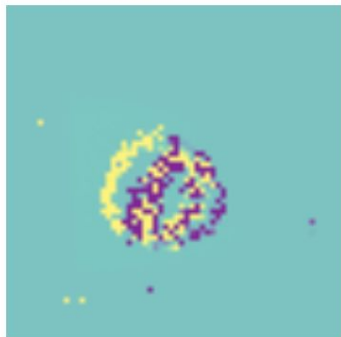
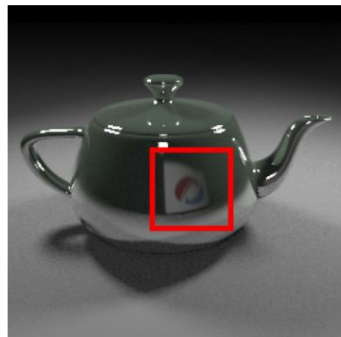
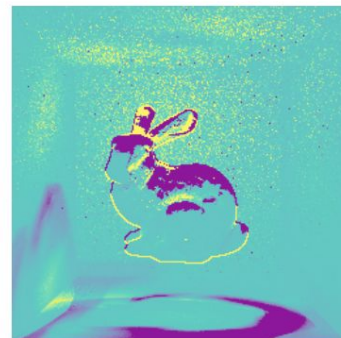
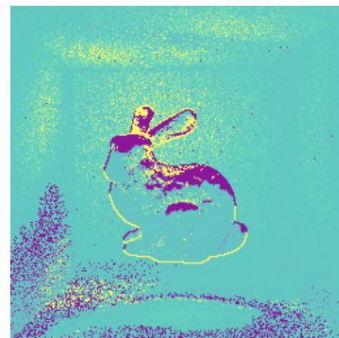
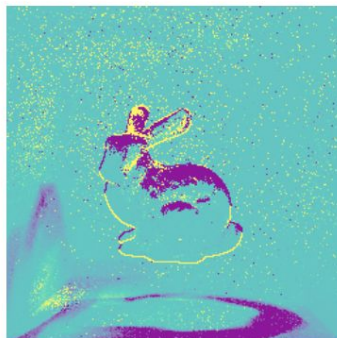
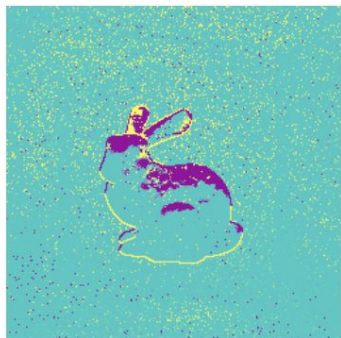
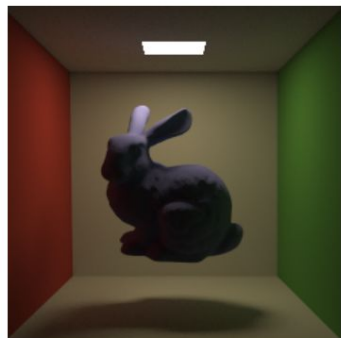
# Results

- Various materials and global illumination effects



# Results

- Importance sampling of edges reduces variance



scenes

10s, w/o importance samp. 10s, w/ importance samp. 350s, w/o importance samp. 350s, w/ importance samp.

# Contributions

- The first comprehensive solution to compute derivatives of scalar functions over a rendered image w.r.t arbitrary scene parameters
- Unbiased gradients and supports arbitrary materials, shadow, and global illumination.

# Limitations

- Performance
- Other light transport phenomena (e.g. motion blur)
- Interpenetrating geometries



# References

- [1] Li, Tzu-Mao, et al. "Differentiable monte carlo ray tracing through edge sampling." ACM Transactions on Graphics (TOG) 37.6 (2018): 1-11.
- [2] Kato, Hiroharu, et al. "Differentiable rendering: A survey." arXiv preprint arXiv:2006.12057 (2020).
- [3] Khungurn, Pramook, et al. "Matching Real Fabrics with Micro-Appearance Models." ACM Trans. Graph. 35.1 (2015): 1-1.
- [4] Kato, Hiroharu, Yoshitaka Ushiku, and Tatsuya Harada. "Neural 3d mesh renderer." Proceedings of the IEEE conference on computer vision and pattern recognition. 2018.
- [5] Wikipedia contributors. "Heaviside step function." Wikipedia, The Free Encyclopedia. Wikipedia, The Free Encyclopedia, 6 Nov. 2021. Web. 6 Nov. 2021.
- [6] Wikipedia contributors. "Dirac delta function." Wikipedia, The Free Encyclopedia. Wikipedia, The Free Encyclopedia, 2 Nov. 2021. Web. 6 Nov. 2021.

Thank you! :)