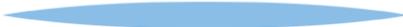

WST665/CS770A: Web-Scale Image Retrieval
Keypoint Localization

Sung-Eui Yoon
(윤성익)

Course URL:
<http://sglab.kaist.ac.kr/~sungeui/IR>

KAIST



What we will learn today?

- Local invariant features
 - Motivation
 - Requirements, invariances
- Keypoint localization
 - Harris corner detector
 - Hessian detector

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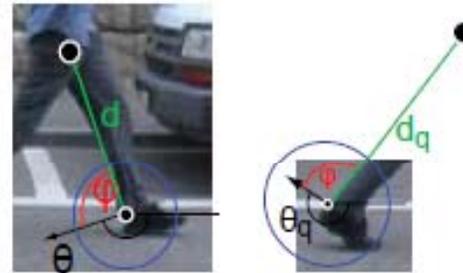
Motivation

- Global representations have major limitations
- Instead, describe and match only local regions
- Increased robustness to

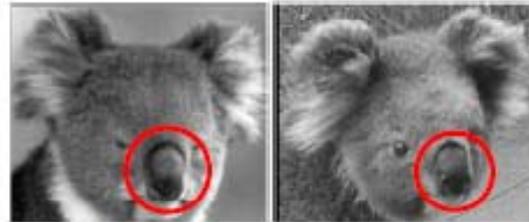
– Occlusions



– Articulation



– Intra-category variations



Application: Image Matching



by [Diva Sian](#)



by [swashford](#)

Slide credit: Steve Seitz

Harder Case



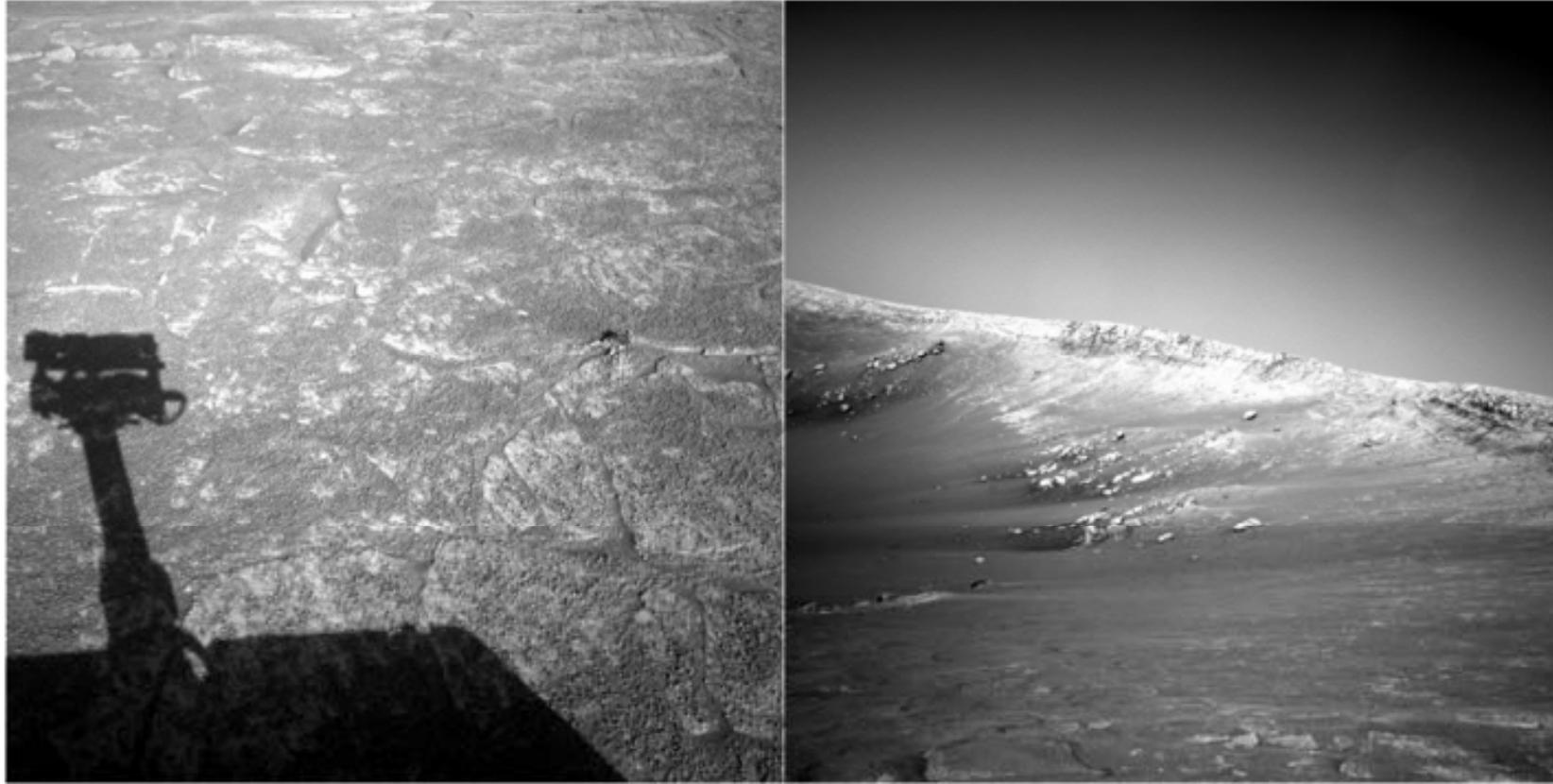
by [Diva Sian](#)



by [scgbt](#)

Slide credit: Steve Seitz

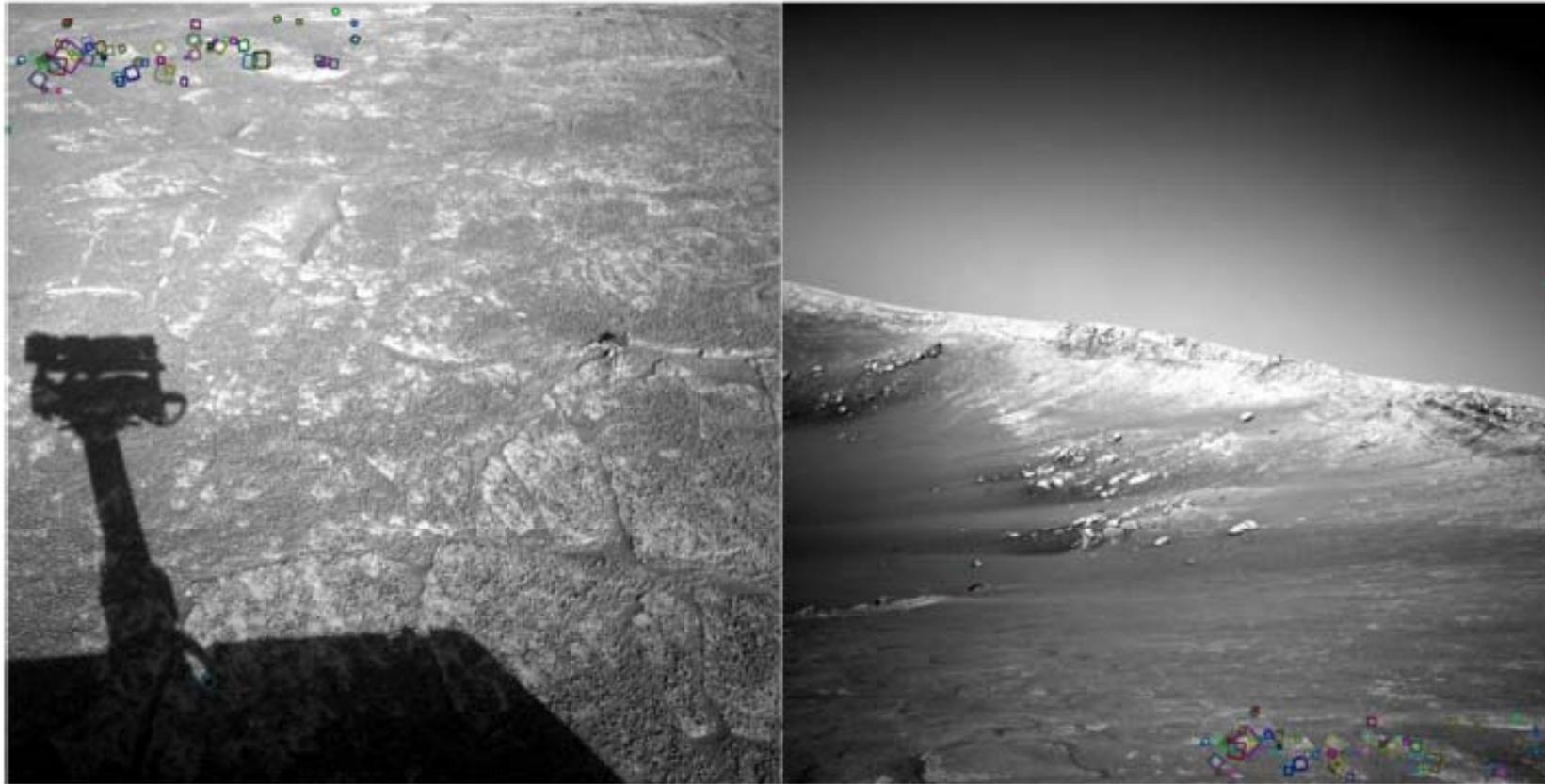
Harder Still?



NASA Mars Rover images

Slide credit: Steve Seitz

Answer Below (Look for tiny colored squares)



NASA Mars Rover images with SIFT feature matches
(Figure by Noah Snavely)

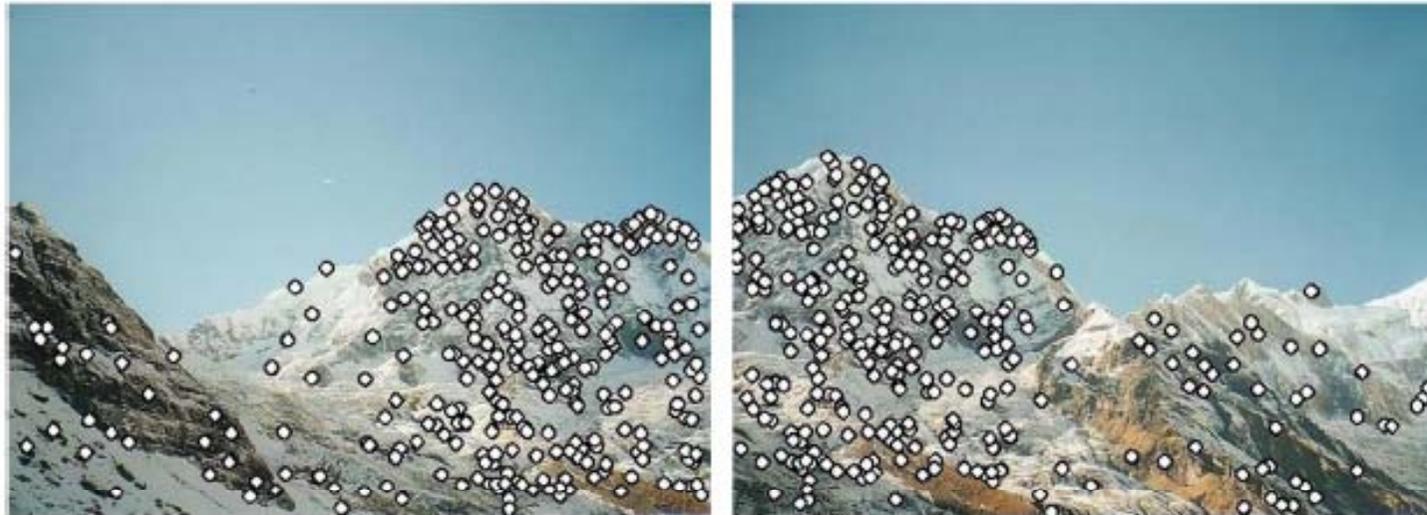
Slide credit: Steve Seitz

Application: Image Stitching



Slide credit: Darya Frolova, Denis Simakov

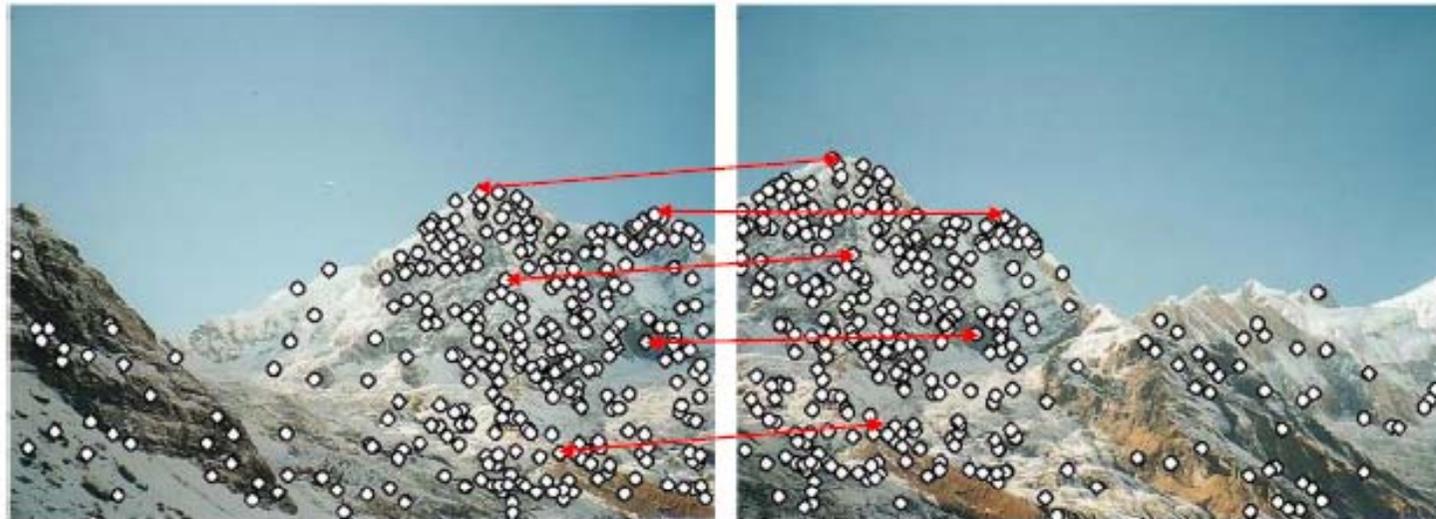
Application: Image Stitching



- Procedure:
 - Detect feature points in both images

Slide credit: Darya Frolova, Denis Simakov

Application: Image Stitching



- Procedure:
 - Detect feature points in both images
 - Find corresponding pairs

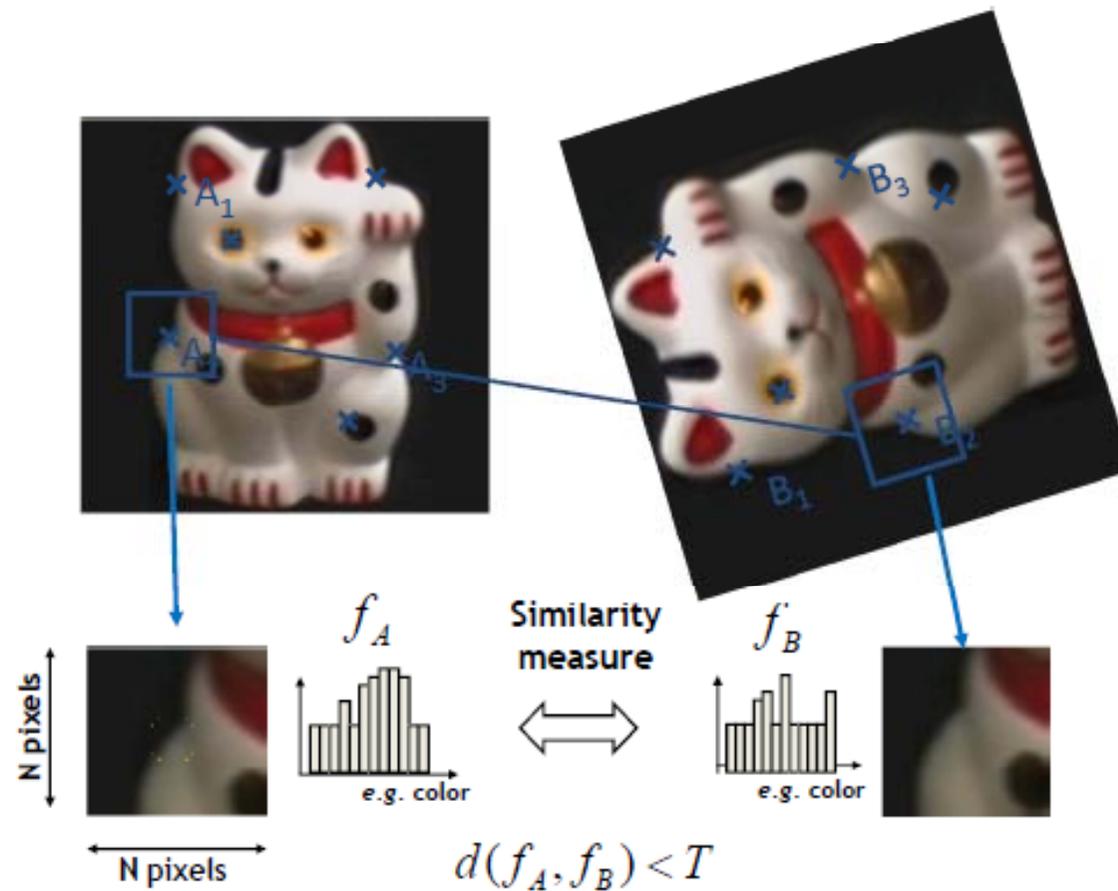
Slide credit: Darya Frolova, Denis Simakov

Application: Image Stitching



- Procedure:
 - Detect feature points in both images
 - Find corresponding pairs
 - Use these pairs to align the images

General Approach



1. Find a set of distinctive keypoints
2. Define a region around each keypoint
3. Extract and normalize the region content
4. Compute a local descriptor from the normalized region
5. Match local descriptors

Slide credit: Bastian Leibe

Common Requirements

- Problem 1:
 - Detect the same point *independently* in both images



No chance to match!

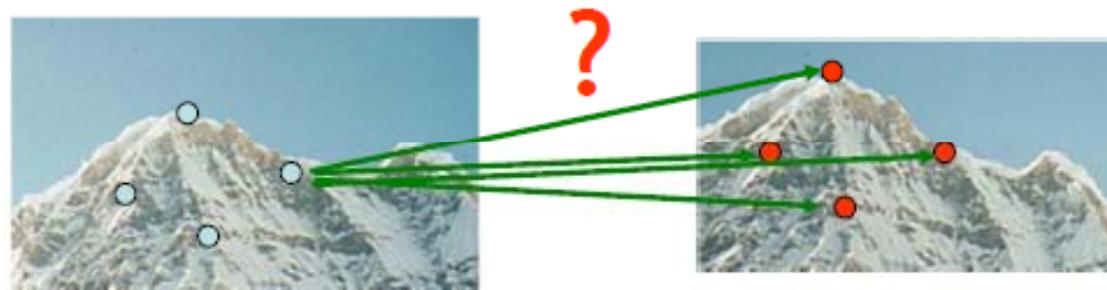
This lecture

We need a repeatable detector!

Fei-Fei Li

Common Requirements

- Problem 1:
 - Detect the same point *independently* in both images
- Problem 2:
 - For each point correctly recognize the corresponding one



Next lecture

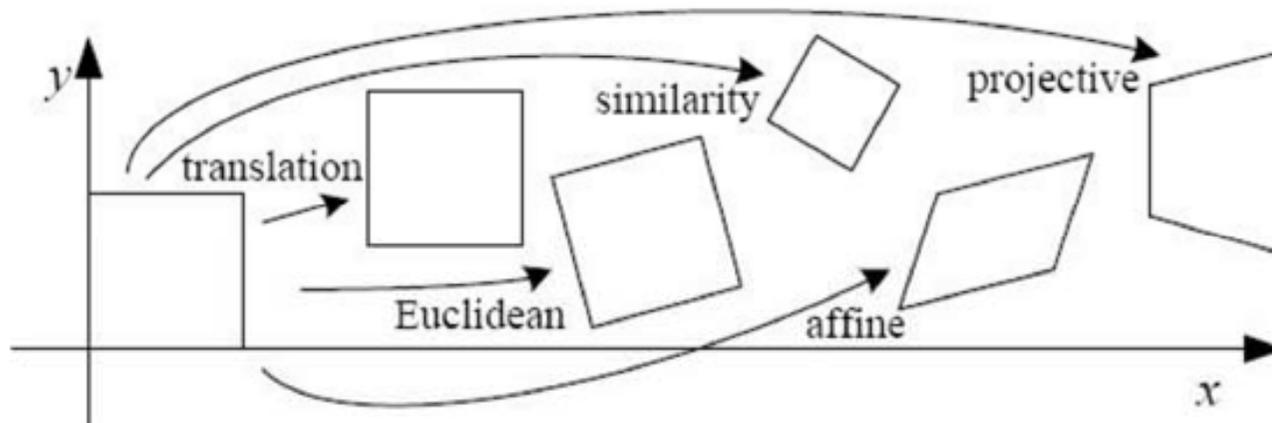
We need a reliable and distinctive descriptor!

Invariance: Geometric Transformations



Slide credit: Steve Seitz

Levels of Geometric Invariance



Slide credit: Bastian Leibe

Invariance: Photometric Transformations



- Often modeled as a linear transformation:
 - Scaling + Offset

Slide credit: Tinne Tuytelaars

Requirements

- Region extraction needs to be **repeatable** and **accurate**
 - **Invariant** to translation, rotation, scale changes
 - **Robust** or **covariant** to out-of-plane (\approx affine) transformations
 - **Robust** to lighting variations, noise, blur, quantization
- **Locality**: Features are local, therefore robust to occlusion and clutter.
- **Quantity**: We need a sufficient number of regions to cover the object.
- **Distinctiveness** : The regions should contain “interesting” structure.
- **Efficiency**: Close to real-time performance.

Many Existing Detectors Available

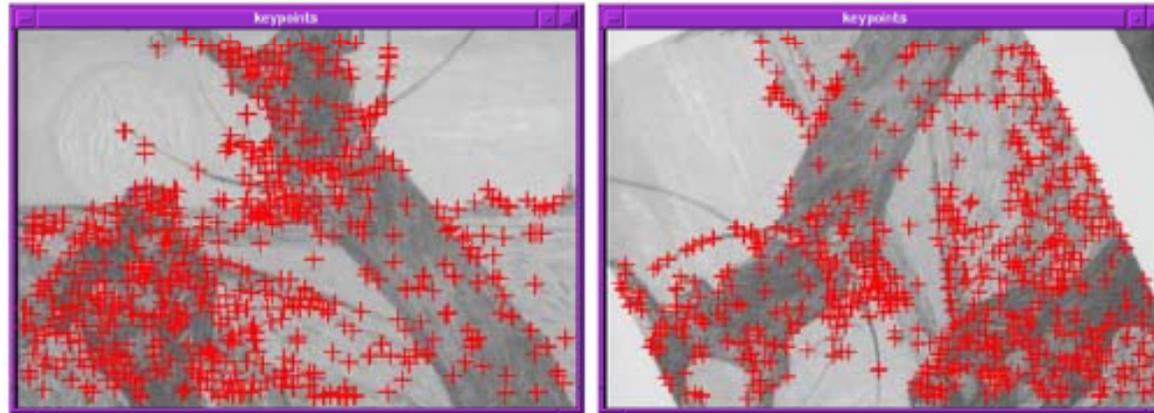
- Hessian & Harris [Beaudet '78], [Harris '88]
 - Laplacian, DoG [Lindeberg '98], [Lowe '99]
 - Harris-/Hessian-Laplace [Mikolajczyk & Schmid '01]
 - Harris-/Hessian-Affine [Mikolajczyk & Schmid '04]
 - EBR and IBR [Tuytelaars & Van Gool '04]
 - MSER [Matas '02]
 - Salient Regions [Kadir & Brady '01]
 - Others...
- *Those detectors have become a basic building block for many recent applications in Computer Vision.*

Keypoint Localization



- Goals:
 - Repeatable detection
 - Precise localization
 - Interesting content
- ⇒ *Look for two-dimensional signal changes*

Finding Corners

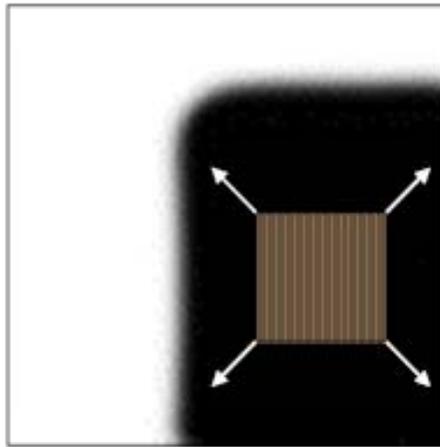


- Key property:
 - In the region around a corner, image gradient has two or more dominant directions
- Corners are *repeatable* and *distinctive*

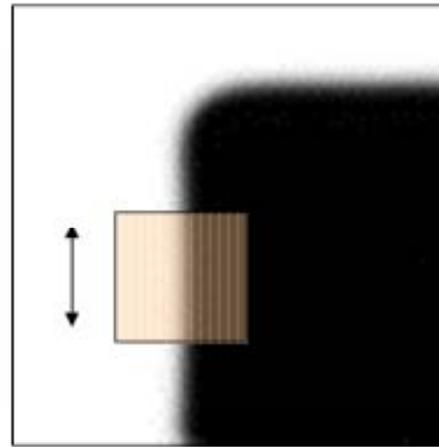
C.Harris and M.Stephens. "[A Combined Corner and Edge Detector.](#)"
Proceedings of the 4th Alvey Vision Conference, 1988.

Corners as Distinctive Interest Points

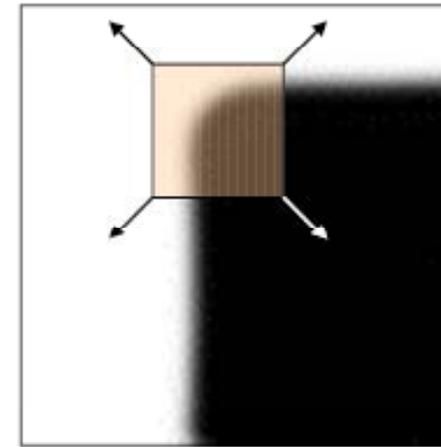
- Design criteria
 - We should easily recognize the point by looking through a small window (*locality*)
 - Shifting the window in *any direction* should give a large change in intensity (*good localization*)



“flat” region:
no change in all directions



“edge”:
no change along the edge direction



“corner”:
significant change in all directions

Slide credit: Alyosha Efros

Harris Detector Formulation

- Change of intensity for the shift $[u,v]$:

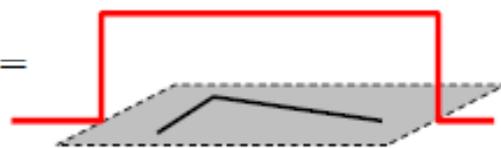
$$E(u, v) = \sum_{x,y} w(x, y) [I(x+u, y+v) - I(x, y)]^2$$

Window
function

Shifted
intensity

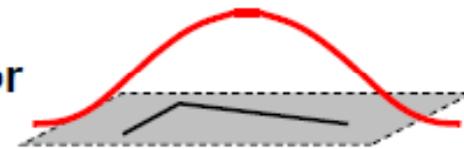
Intensity

Window function $w(x,y) =$



1 in window, 0 outside

or



Gaussian

Slide credit: Rick Szeliski

Harris Detector Formulation

- This measure of change can be approximated by:

$$E(u, v) \approx [u \ v] M \begin{bmatrix} u \\ v \end{bmatrix}$$

where M is a 2×2 matrix computed from image derivatives:

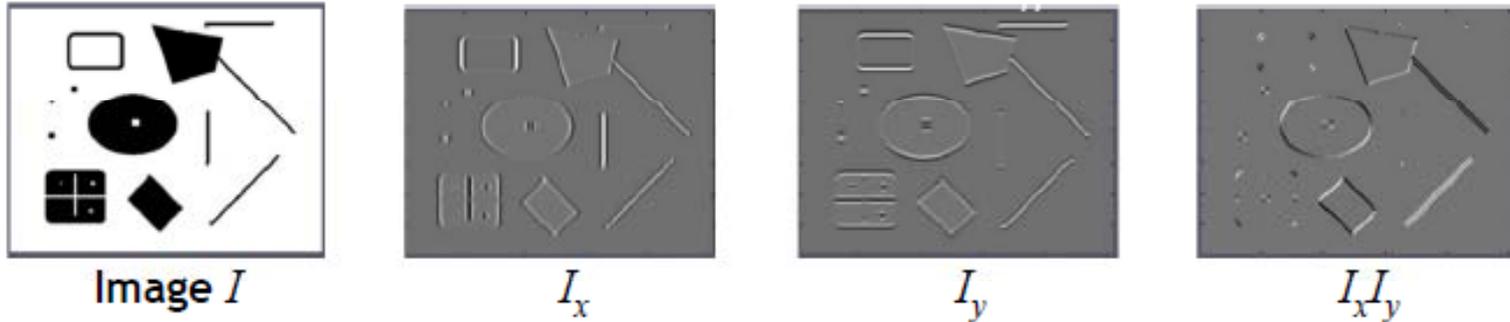
$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Sum over image region – the area we are checking for corner

Gradient with respect to x , times gradient with respect to y

$$M = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [I_x \ I_y]$$

Harris Detector Formulation



where M is a 2×2 matrix computed from image derivatives:

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

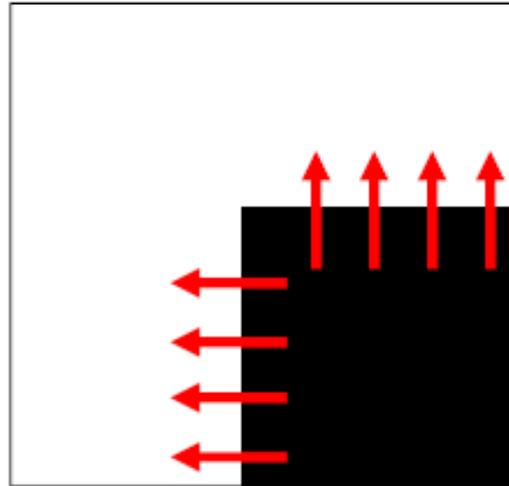
Gradient with respect to x , times gradient with respect to y

↑
Sum over image region – the area we are checking for corner

$$M = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [I_x \ I_y]$$

What Does This Matrix Reveal?

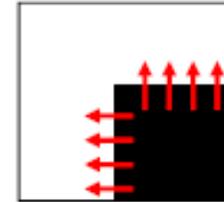
- First, let's consider an axis-aligned corner:



What Does This Matrix Reveal?

- First, let's consider an axis-aligned corner:

$$M = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$



- This means:
 - Dominant gradient directions align with x or y axis
 - If either λ is close to 0, then this is not a corner, so look for locations where both are large.
- What if we have a corner that is not aligned with the image axes?

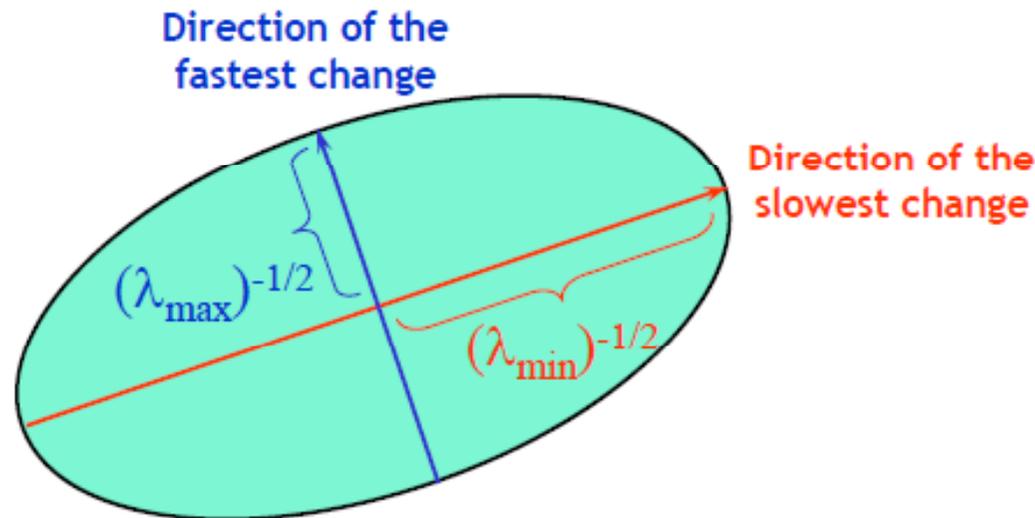
Slide credit: David Jacobs

General Case

- Since M is symmetric, we have $M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$

(Eigenvalue decomposition)

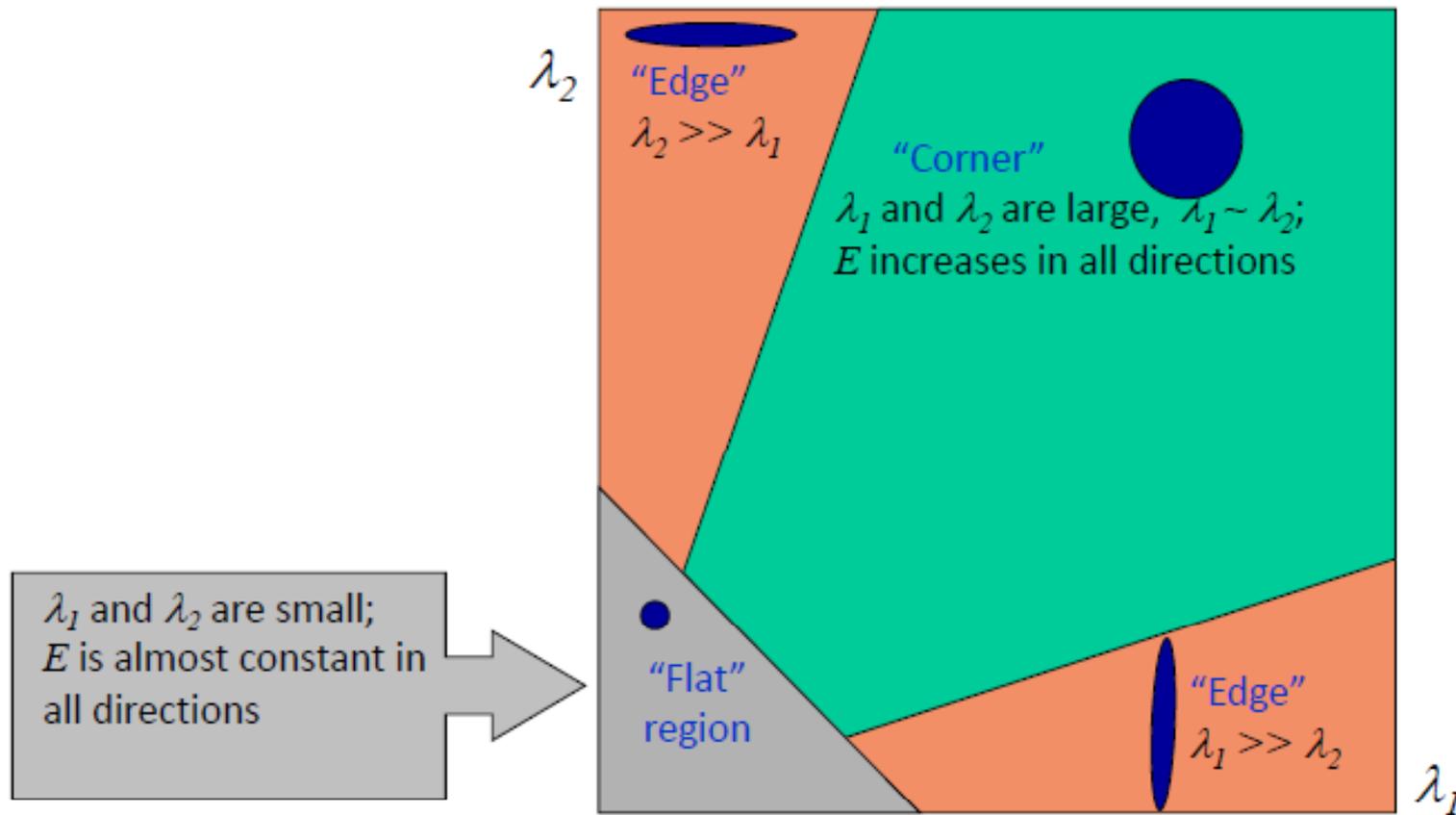
- We can visualize M as an ellipse with axis lengths determined by the eigenvalues and orientation determined by R



Interpreting the Eigenvalues

- Classification of image points using eigenvalues of M :

Slide credit: Kristen Grauman



Corner Response Function

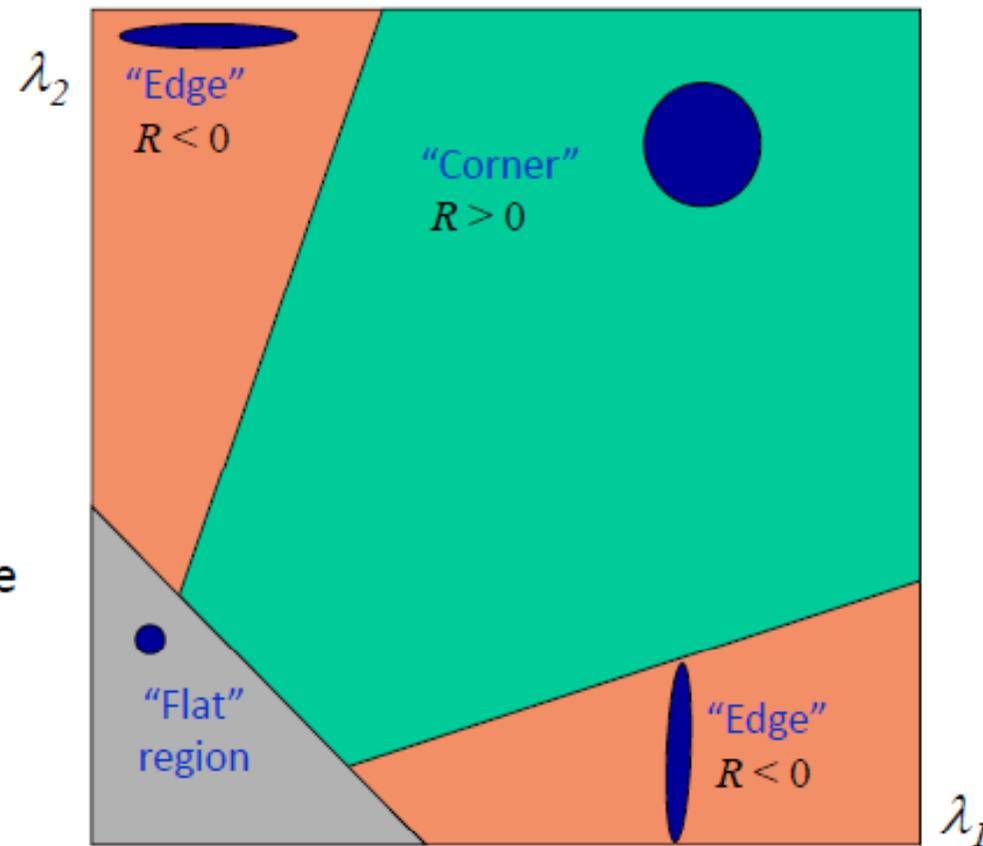
$$R = \det(M) - \alpha \text{trace}(M)^2 = \lambda_1 \lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

$$M = \begin{bmatrix} A & B \\ B & C \end{bmatrix},$$

$$\text{trace}(M) = A + C,$$

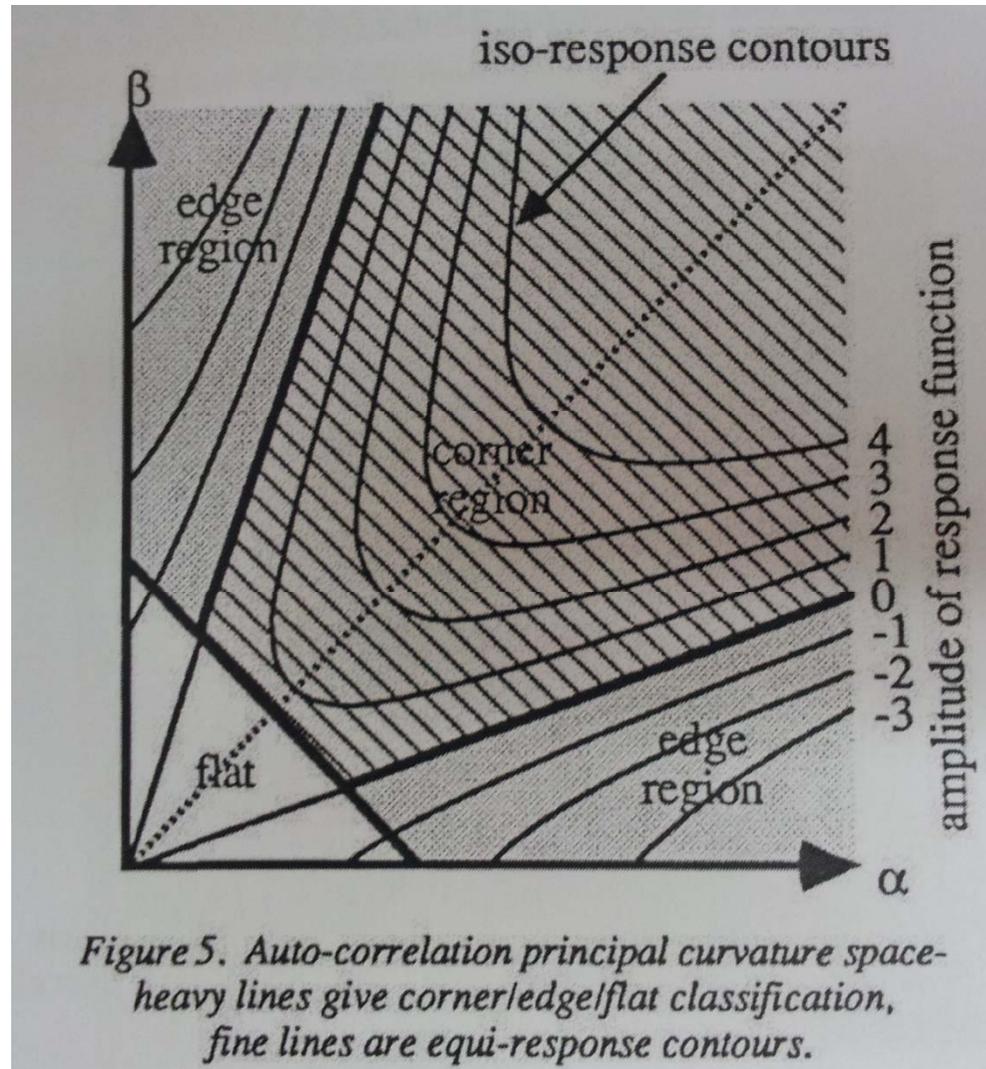
$$\det(M) = AC - B^2$$

- Fast approximation
 - Avoid computing the eigenvalues
 - α : constant (0.04 to 0.06)



Slide credit: Kristen Grauman

Corner Response Function



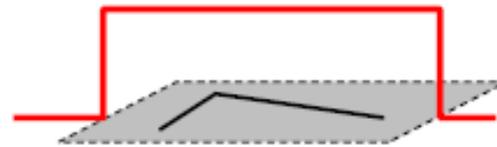
Window Function $w(x,y)$

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

- Option 1: uniform window
 - Sum over square window

$$M = \sum_{x,y} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

- Problem: not rotation invariant



1 in window, 0 outside

Window Function $w(x,y)$

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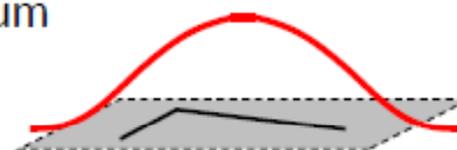


1 in window, 0 outside

- Option 2: Smooth with Gaussian
 - Gaussian already performs weighted sum

$$M = g(\sigma) * \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

- Result is rotation invariant



Gaussian

Summary: Harris Detector [Harris88]

- Compute second moment matrix (autocorrelation matrix)

$$M(\sigma_I, \sigma_D) = g(\sigma_I) * \begin{bmatrix} I_x^2(\sigma_D) & I_x I_y(\sigma_D) \\ I_x I_y(\sigma_D) & I_y^2(\sigma_D) \end{bmatrix}$$

1. Image derivatives



2. Square of derivatives



3. Gaussian filter $g(\sigma_I)$



4. Cornerness function - two strong eigenvalues

$$\begin{aligned} R &= \det[M(\sigma_I, \sigma_D)] - \alpha [\text{trace}(M(\sigma_I, \sigma_D))]^2 \\ &= g(I_x^2)g(I_y^2) - [g(I_x I_y)]^2 - \alpha [g(I_x^2) + g(I_y^2)]^2 \end{aligned}$$

5. Perform non-maximum suppression



Harris Detector: Workflow

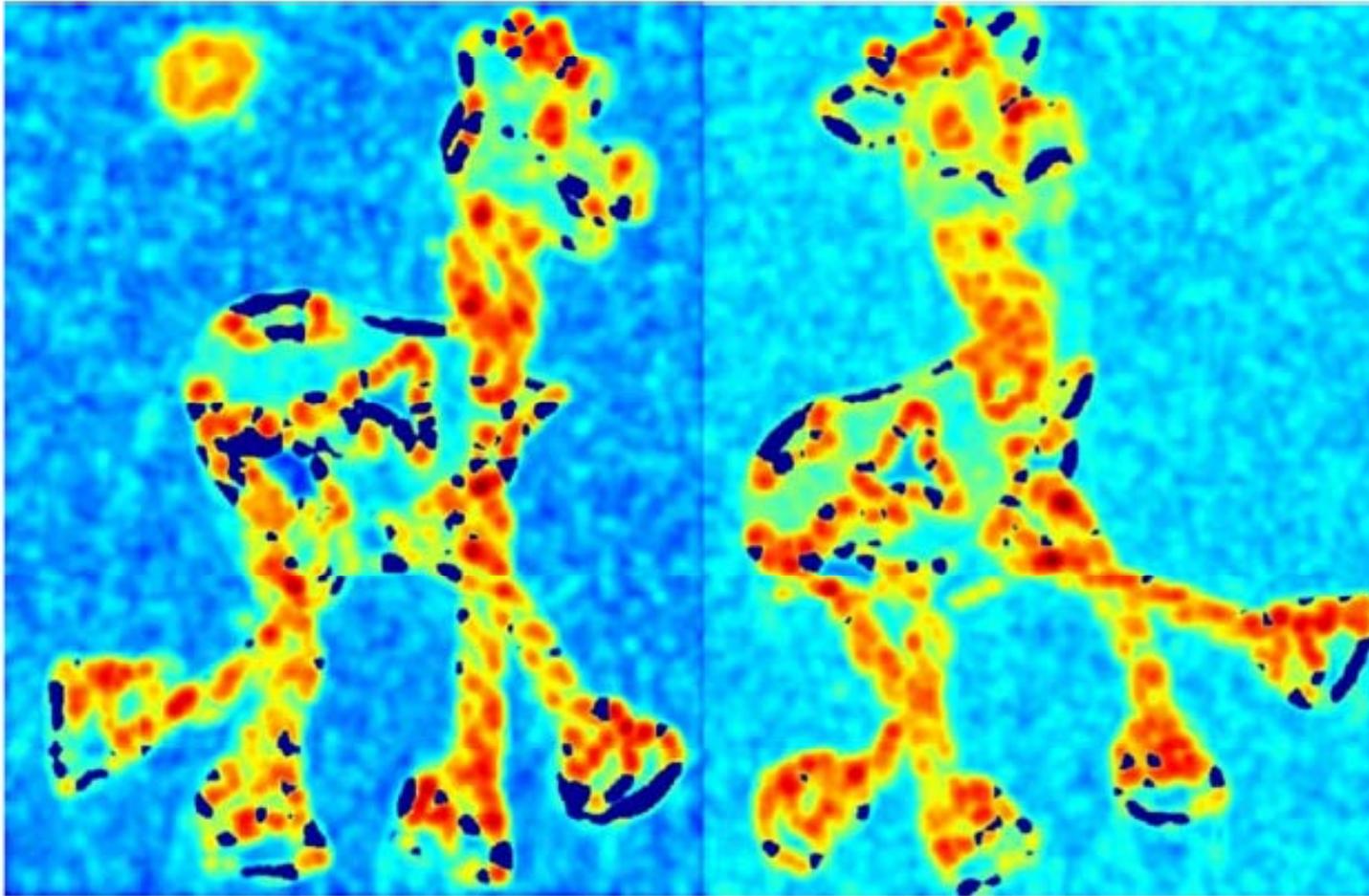


Fei-Fei Li

Slide adapted from Darya Frolova, Denis Simakov

Harris Detector: Workflow

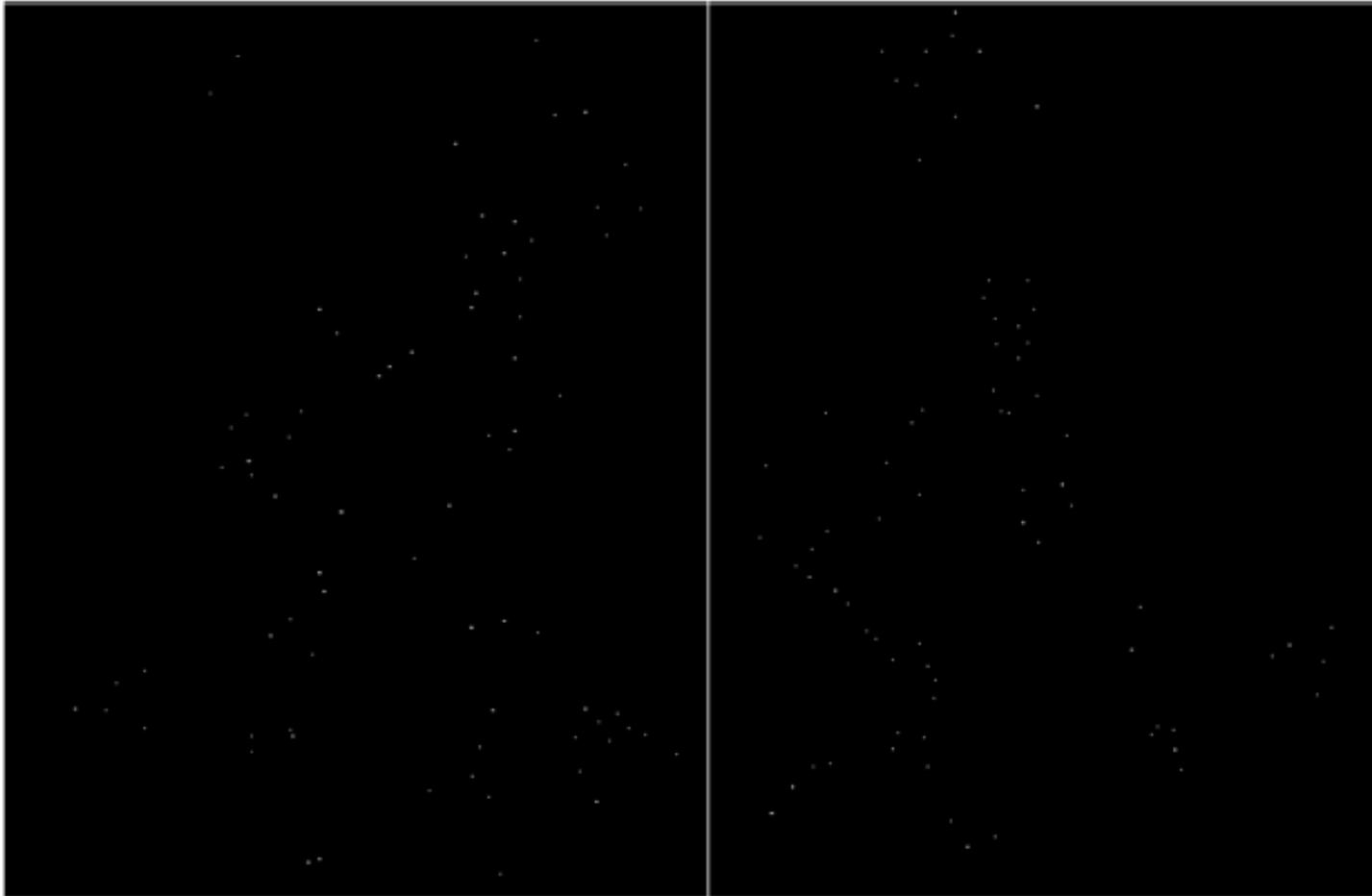
- computer corner responses R



Slide adapted from Darya Frolova, Denis Simakov

Harris Detector: Workflow

- Take only the local maxima of R , where $R > \text{threshold}$



Slide adapted from Darya Frolova, Denis Simakov

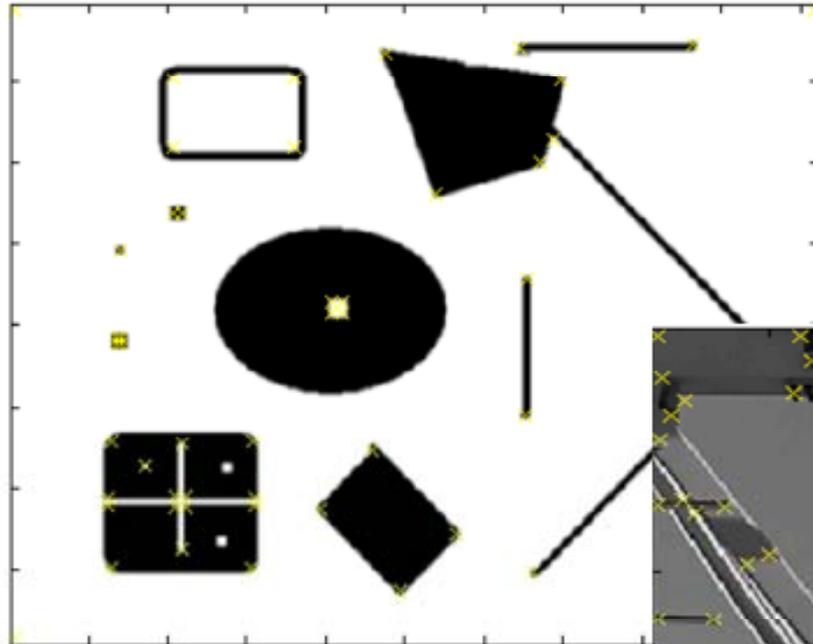
Harris Detector: Workflow

- Resulting Harris points

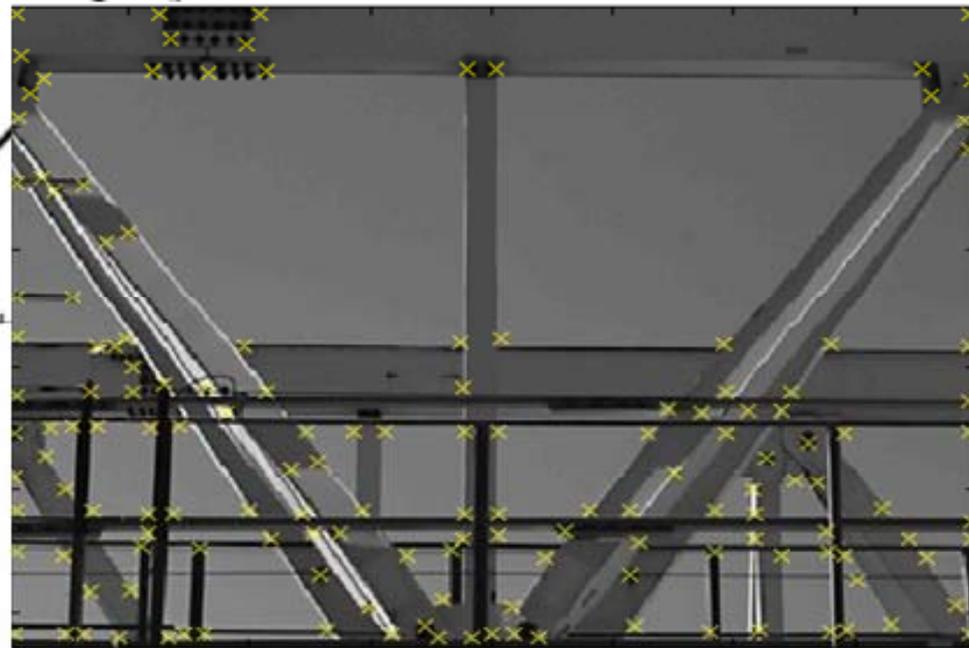


Slide adapted from Darya Frolova, Denis Simakov

Harris Detector – Responses [Harris88]



Effect: A very precise corner detector.



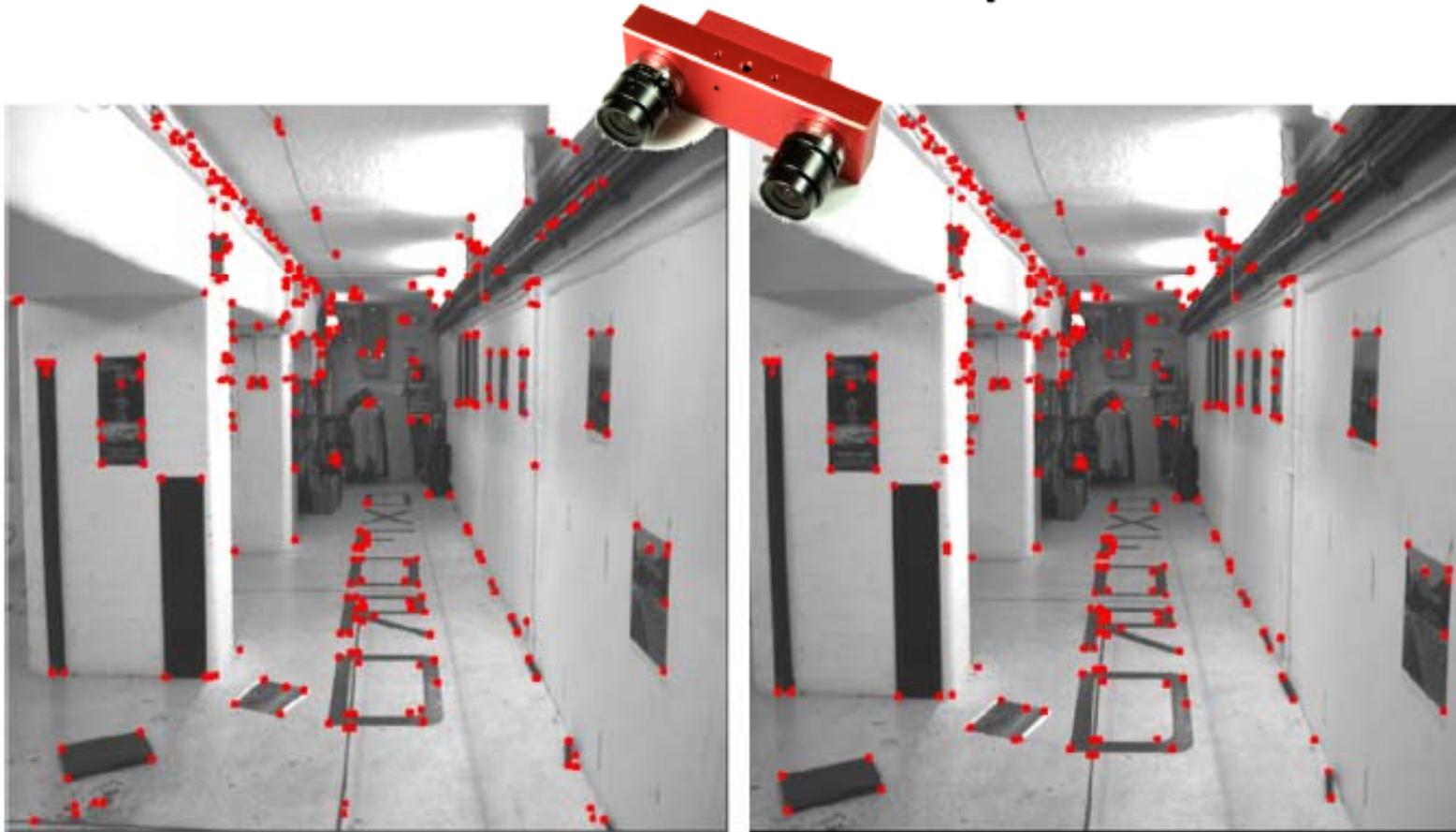
Slide credit: Krystian Mikolajczyk

Harris Detector – Responses [Harris88]



Slide credit: Krystian Mikolajczyk

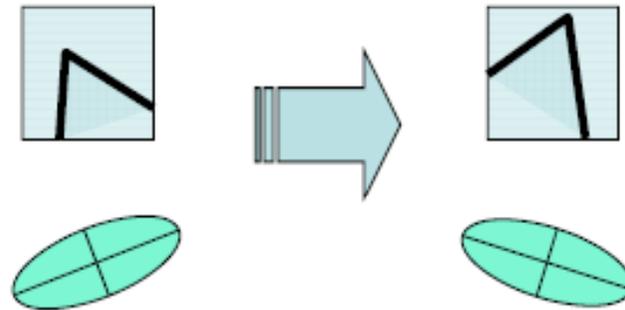
Harris Detector – Responses [Harris88]



- Results are well suited for finding stereo correspondences

Harris Detector: Properties

- Rotation invariance?



Ellipse rotates but its shape (i.e. eigenvalues) remains the same

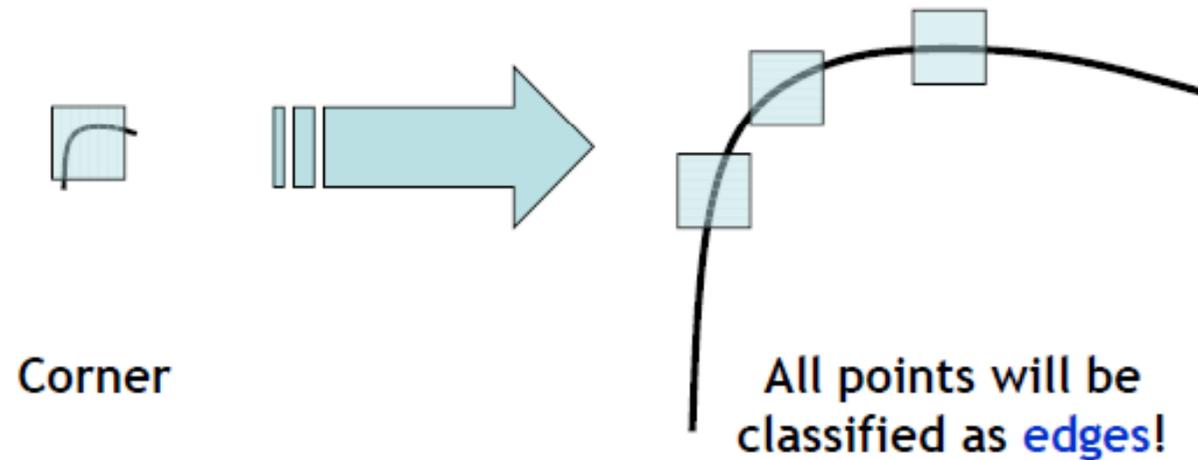
Corner response R is invariant to image rotation

Harris Detector: Properties

- Rotation invariance
- Scale invariance?

Harris Detector: Properties

- Rotation invariance
- Scale invariance?



Not invariant to image scale!

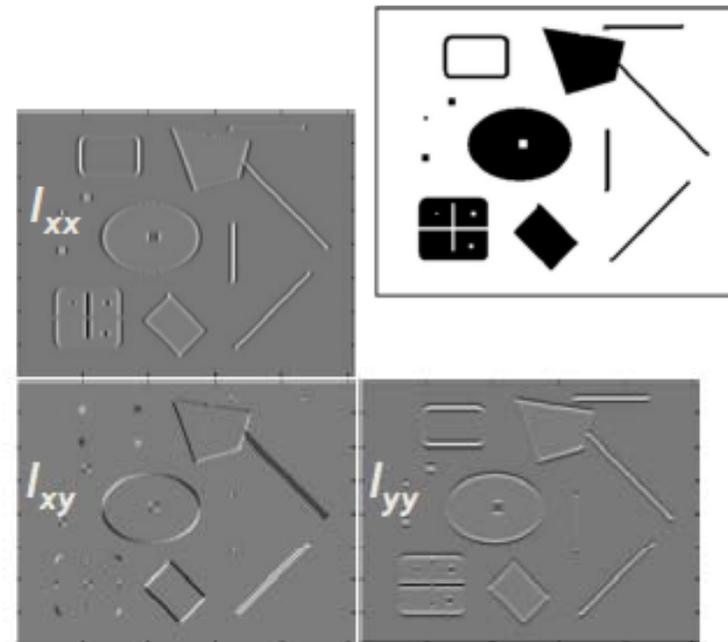
Hessian Detector [Beaudet78]

- Hessian determinant

$$\text{Hessian}(I) = \begin{bmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{bmatrix}$$

Note: these are 2nd derivatives!

Intuition: Search for strong derivatives in two orthogonal directions



Slide credit: Krystian Mikołajczyk

Hessian Detector [Beaudet78]

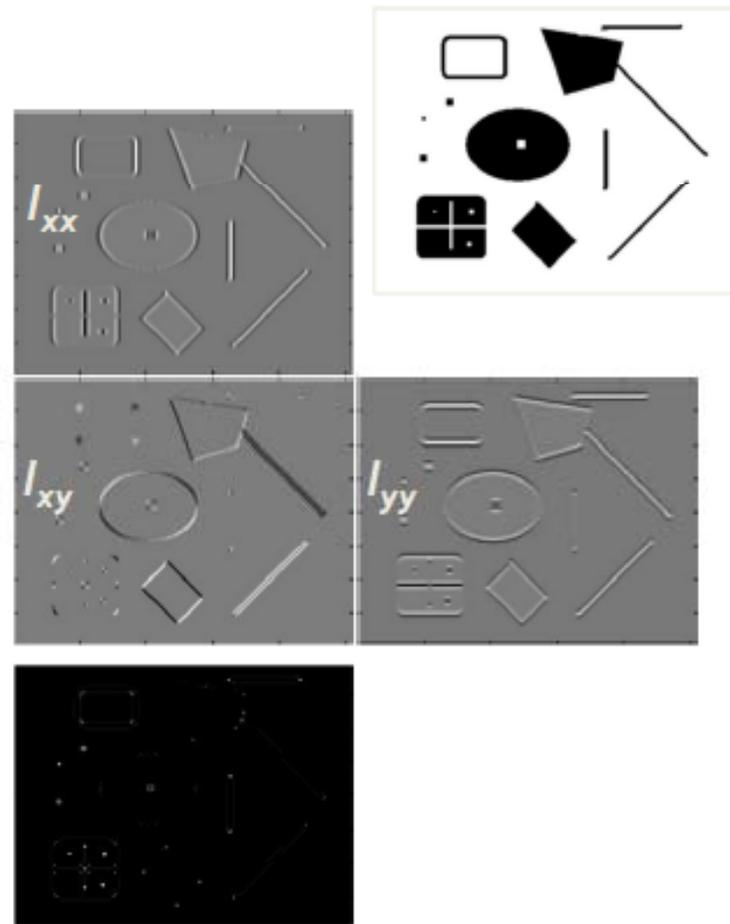
- Hessian determinant

$$\text{Hessian}(I) = \begin{bmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{bmatrix}$$

$$\det(\text{Hessian}(I)) = I_{xx}I_{yy} - I_{xy}^2$$

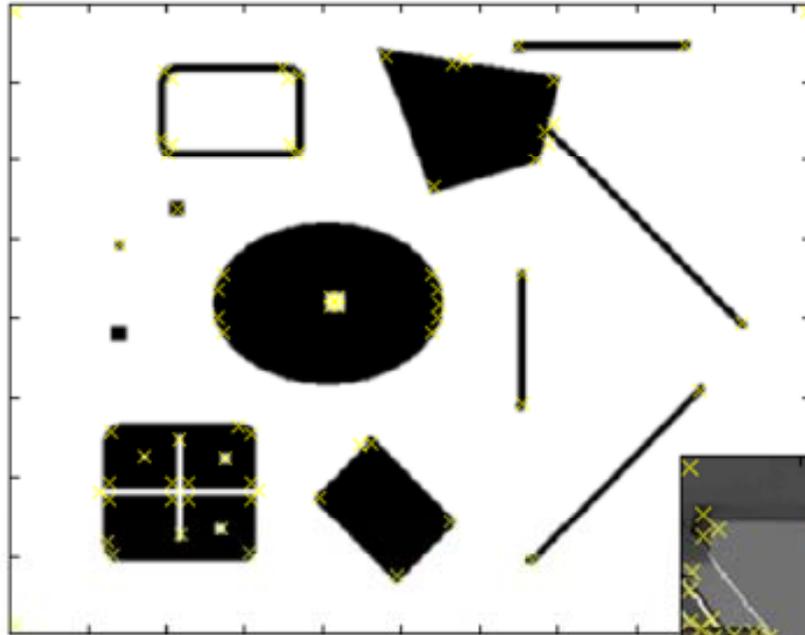
In Matlab:

$$I_{xx} \cdot I_{yy} - (I_{xy})^2$$

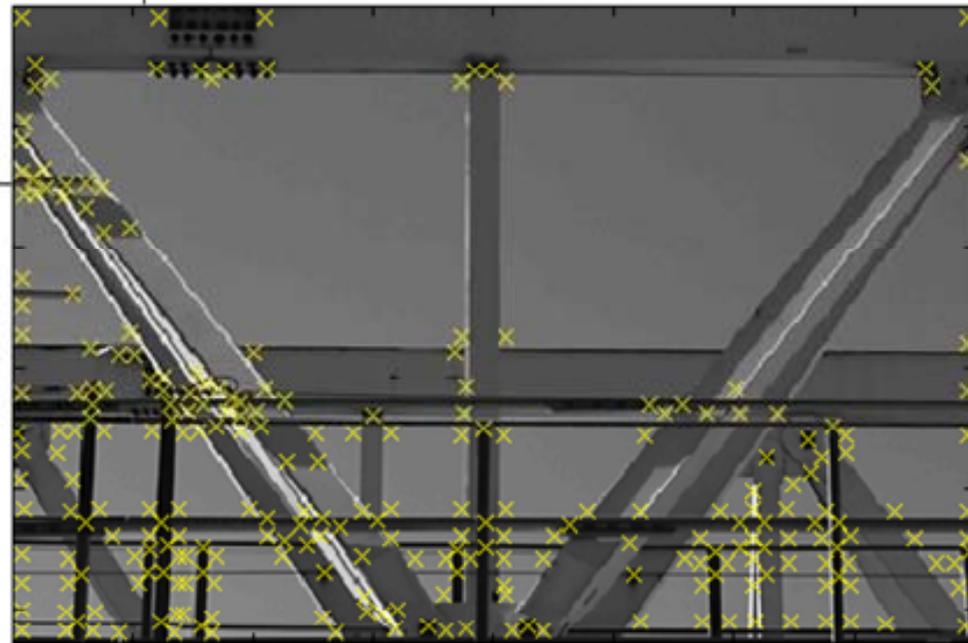


Slide credit: Krystian Mikołajczyk

Hessian Detector – Responses [Beaudet78]



Effect: Responses mainly on corners and strongly textured areas.



Slide credit: Krystian Mikołajczyk

Hessian Detector – Responses [Beaudet78]



Slide credit: Krystian Mikołajczyk

Next Time..

- **Scale invariant region selection**

Homework for Every Class

- **Go over the next lecture slides**
- **Come up with one question on what we have discussed today and submit at the beginning of the next class**
 - **0 for no questions**
 - **2 for typical questions**
 - **3 for questions with thoughts**
 - **4 for questions that surprised me**