CS686:
Path Planning for Point Robots

Sung-Eui Yoon
(윤성의)

Course URL:
http://sgvr.kaist.ac.kr/~sungeui/MPA

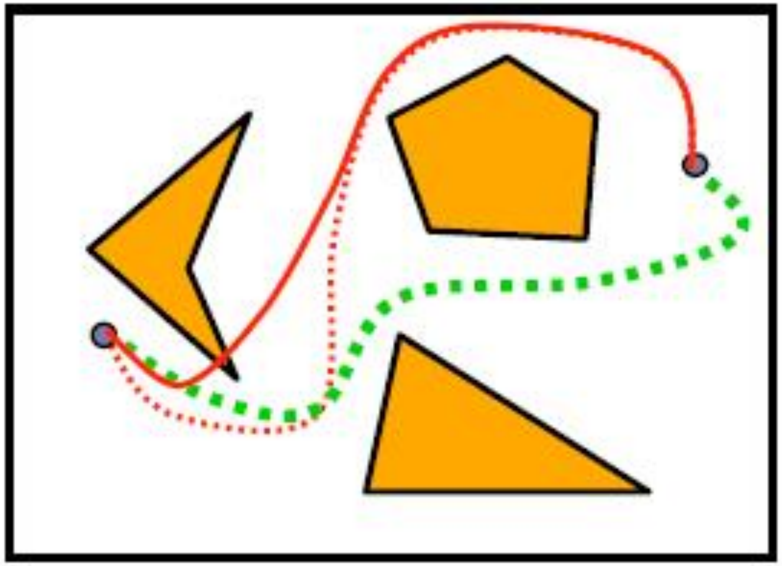
Class Objectives

- **Motion planning framework**
  - **Representations of robots and space**
  - **Discretization into a graph**
  - **Search methods**
  - **Ch. 1 of my book**

- **Last time**
  - **Class overview and grading policy w/ HWs: research oriented course**
  - **Half lectures and half presentations from students**

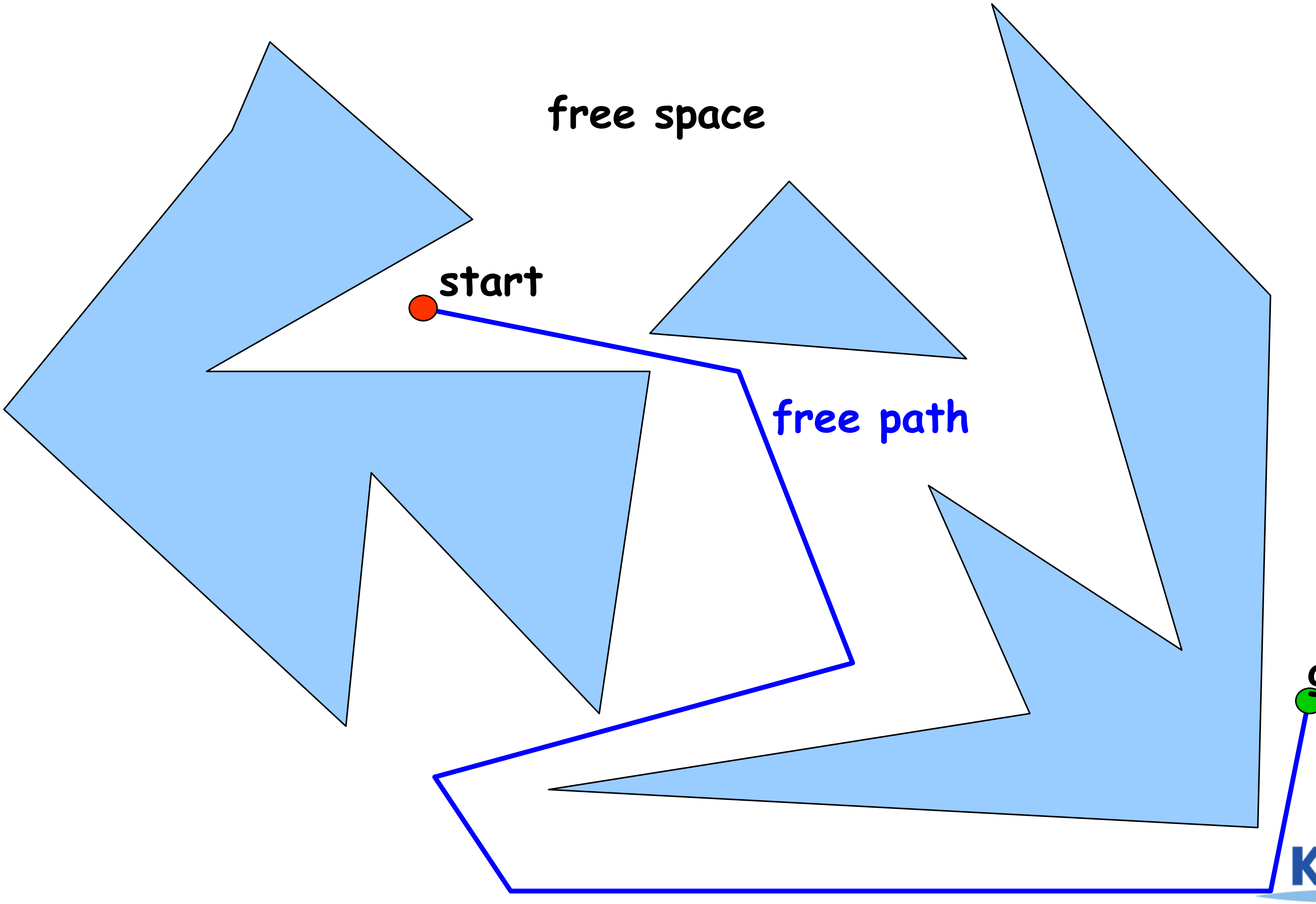
Problem

- Input
  - Robot represented as a **point** in the **plane**
  - Obstacles represented as polygons
  - Initial and goal positions
- Output
  A collision-free path between the initial and goal positions

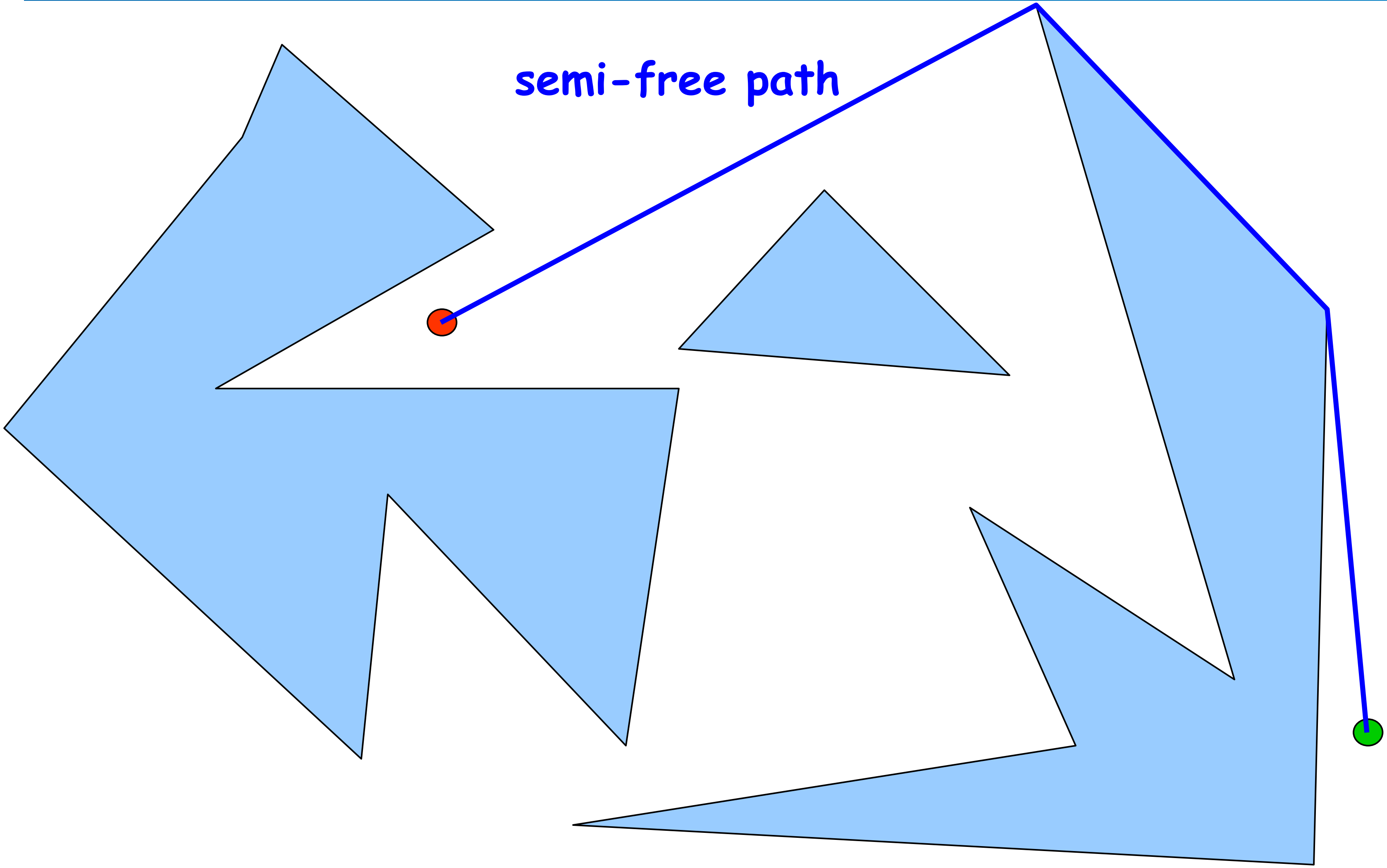


Courtesy of Prof. David Hsu

Problem

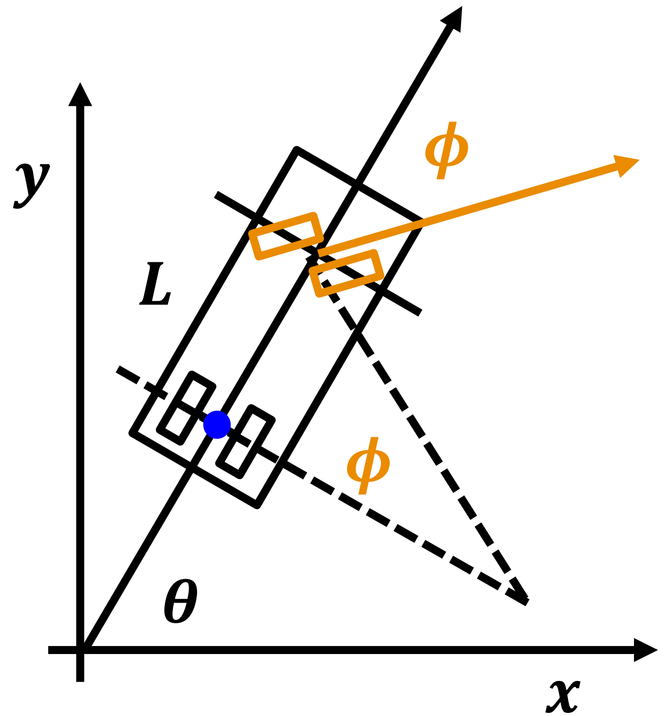


Problem



Types of Path Constraints

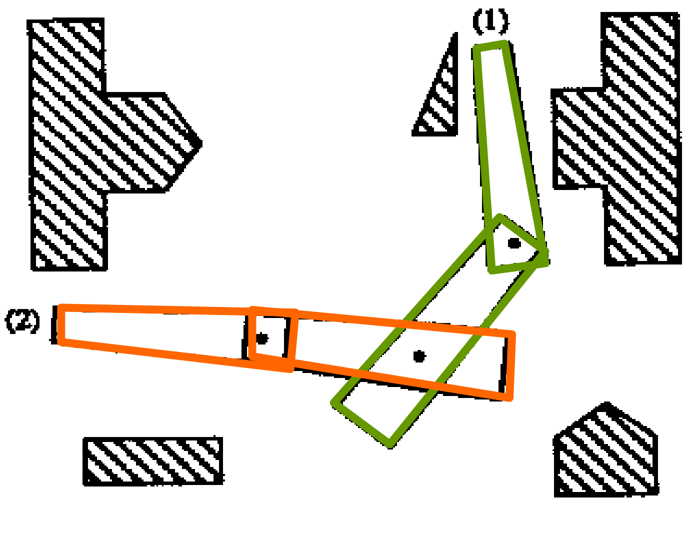
- **Local constraints**
  - **Lie in free space**
- **Global constraints**
  - **Have minimal length**
- **Differential constraints**
  - **Cannot change the car orientation instantly**



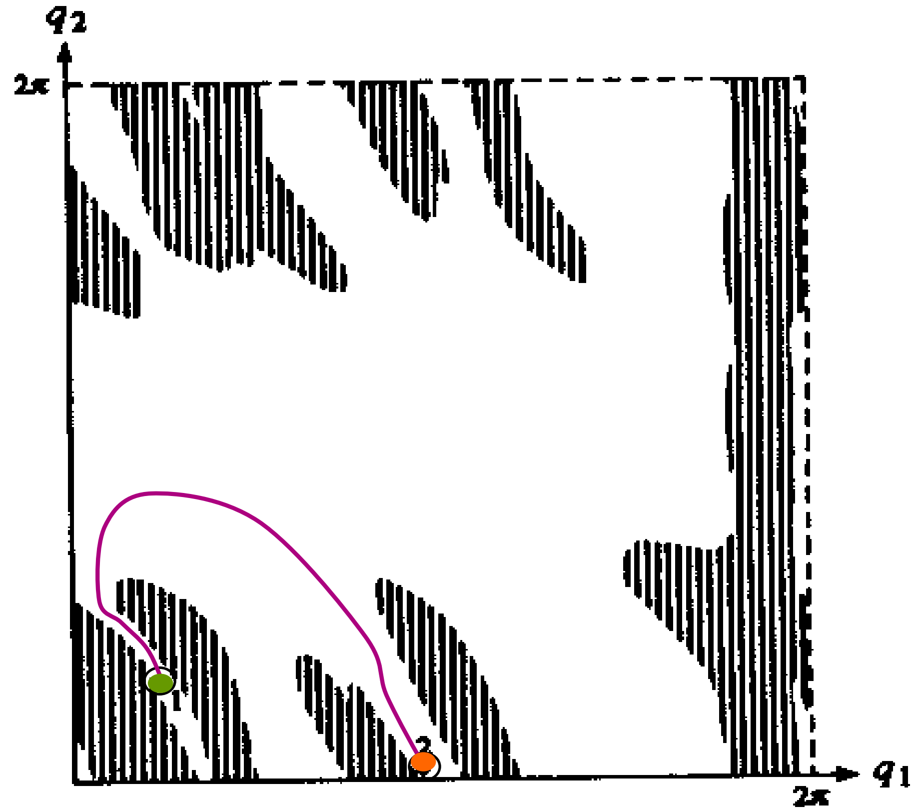
$$\frac{d\theta}{dt} = \frac{v}{L}\tan(\phi)$$

See Ch. 3 (Kinematic Car Model) of my draft
http://sgvr.kaist.ac.kr/~sungeui/mp/

Configuration Space: Tool to Map a Robot to a Point



Workspace



Configuration space (C-Space)

Motion-Planning Framework

Continuous representation

(configuration space formulation)

↓

Discretization

(random sampling, processing critical geometric events)

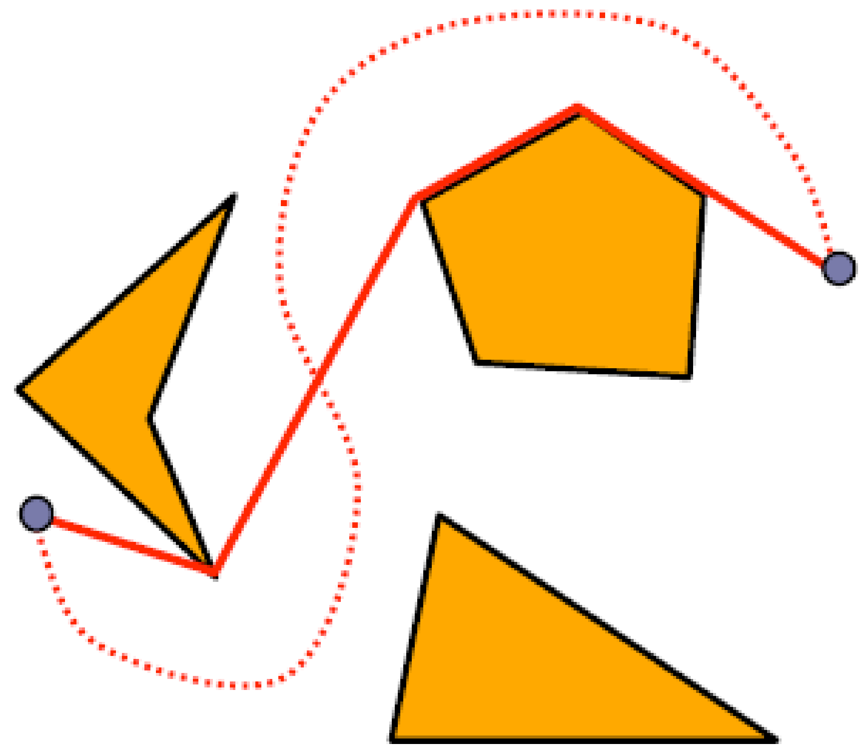
↓

Graph searching

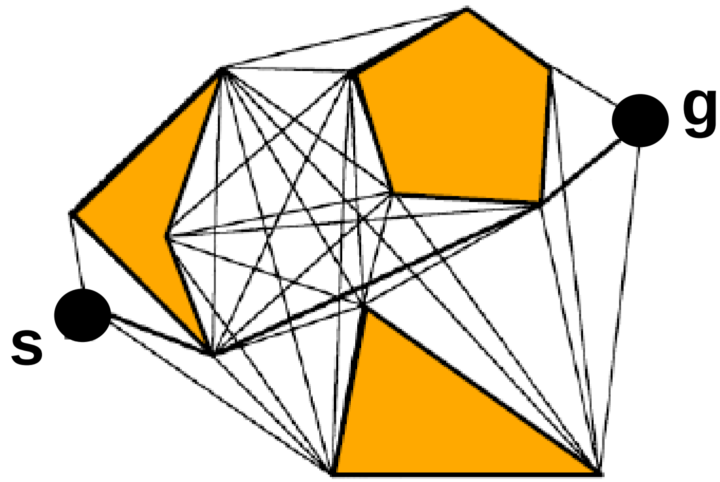
(blind, best-first, A*)

Visibility graph method

- **Observation:** If there is a a collision-free path between two points, then there is a polygonal path that bends only at the obstacles vertices.
- Why?
  Any collision-free path can be transformed into a polygonal path that bends only at the obstacle vertices.
- A **polygonal path** is a piecewise linear curve.

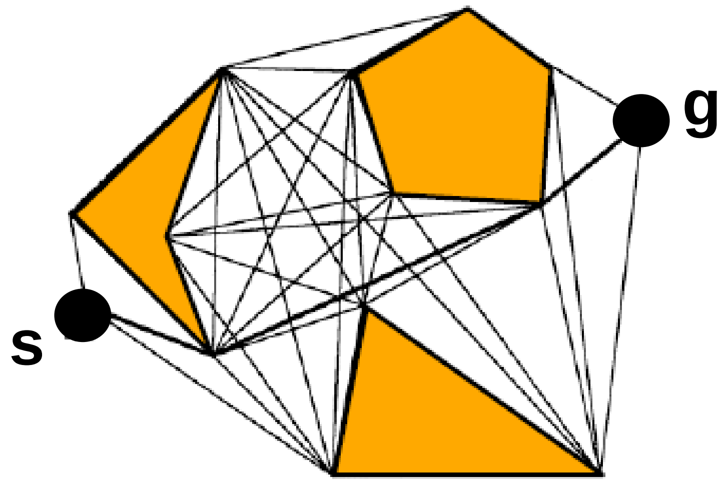


Visibility Graph



- A **visibility graph** is a graph such that
  - Nodes: s, g, or obstacle vertices
  - Edges: An edge exists between nodes u and v if the line segment between u and v is an obstacle edges or it does not intersect the obstacles

Visibility Graph



- A **visibility graph**
  - Introduced in the late 60s
  - Can produce shortest paths in 2-D configuration spaces

Simple Algorithm

- **Input: s, q, polygonal obstacles**
- **Output: visibility graph G**

```
1: for every pair of nodes u, v
2:   if segment (u, v) is an obstacle edge then
3:     insert edge (u, v) into G;
4:   else
5:     for every obstacle edge e
6:       if segment (u, v) intersects e      // check collisions
7:         go to (1);
8:     insert edge (u, v) into G;
9: Search a path with G using A*
```

Computation Efficiency

1: **for** every pair of nodes u, v $O(n^2)$
2:   **if** segment (u, v) is an obstacle edge **then** $O(n)$
3:     insert edge (u, v) into G;
4:   **else**
5:     **for** every obstacle edge e $O(n)$
6:       **if** segment (u, v) intersects e
7:         go to (1);
8:     insert edge (u, v) into G;

- **Simple algorithm: $O(n^3)$ time**
- **More efficient algorithms**
  - **Rotational sweep $O(n^2 \log n)$ time, etc.**
- **$O(n^2)$ space**

Motion-Planning Framework

Continuous representation

(configuration space formulation)

↓

Discretization

(random sampling, processing critical geometric events)

↓

Graph searching

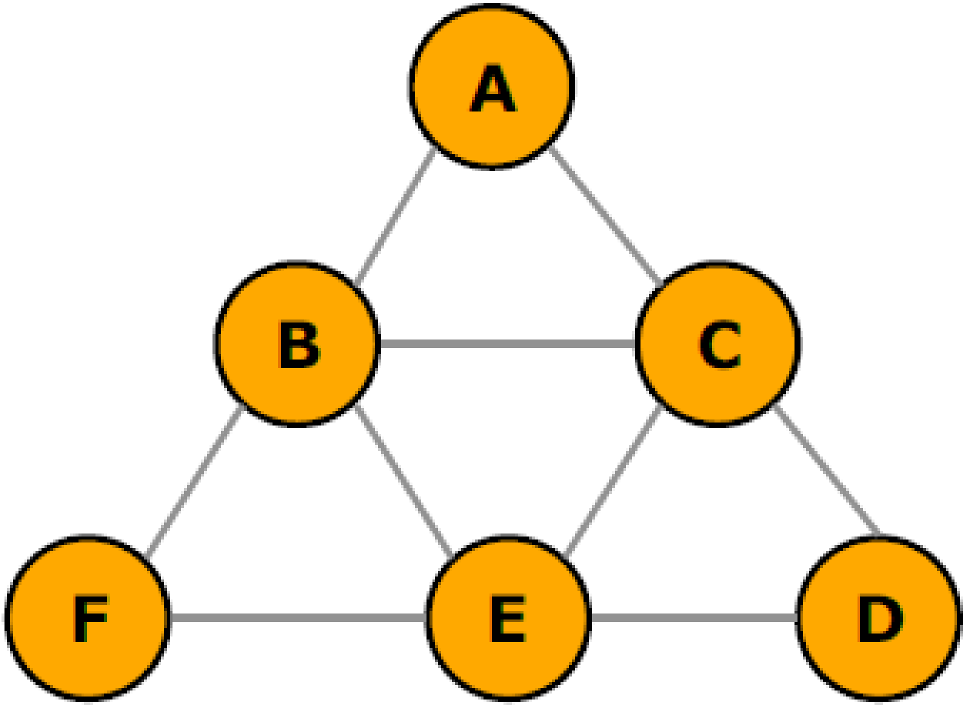
(blind, best-first, A*)

KAIST

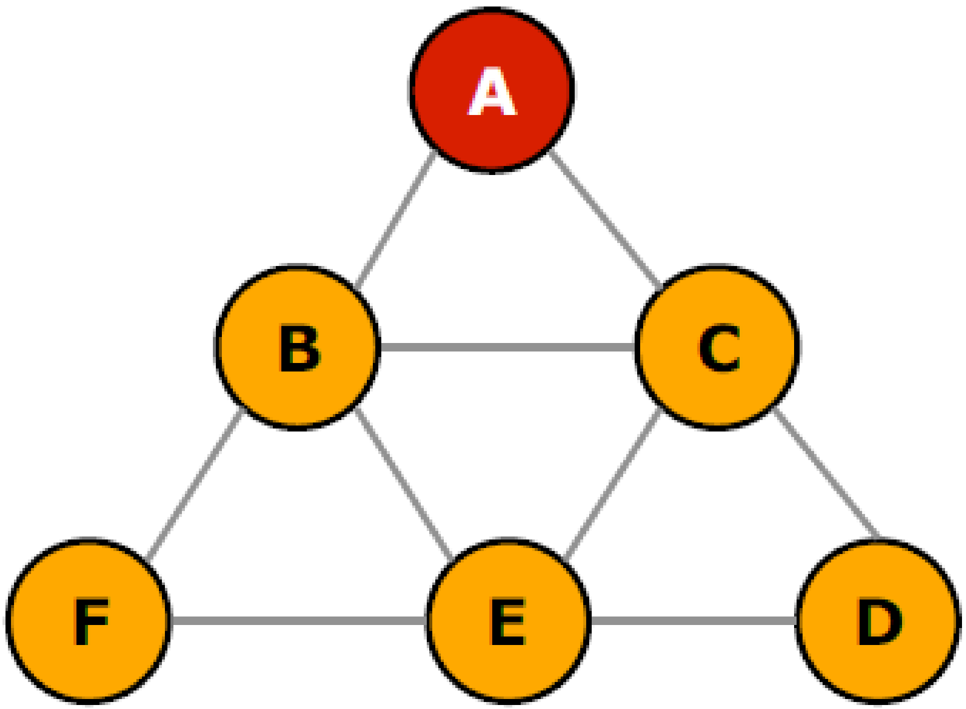
Graph Search Algorithms

- **Breadth, depth-first, best-first**
- **Dijkstra's algorithm**
- **A***

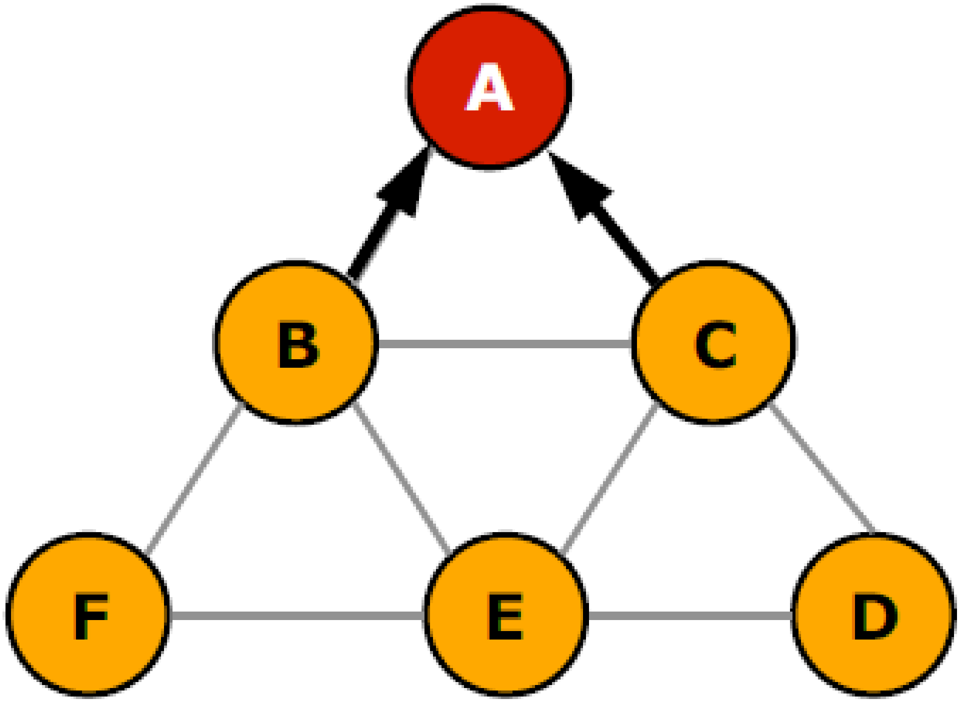
Breadth-first search



Breadth-first search

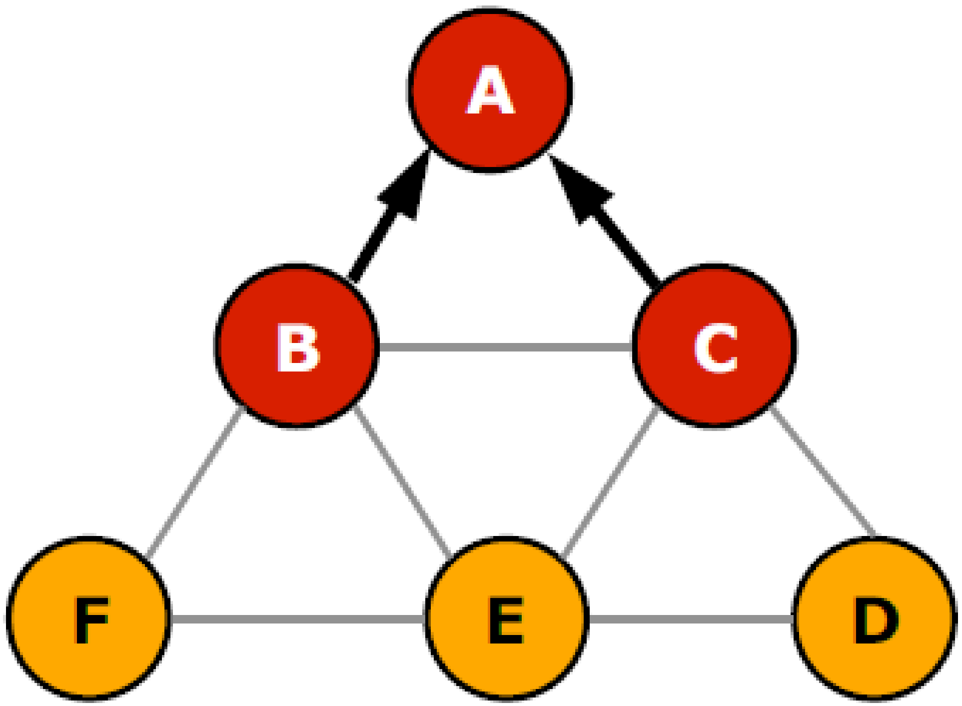


Breadth-first search



Breadth-first search

Traverse the graph by using the queue, resulting in the level-by-level traversal



Dijkstra's Shortest Path Algorithm

- Given a (non-negative) weighted graph, two vertices, s and g:
  - Find a path of minimum total weight between them
  - Also, find minimum paths to other vertices
  - Has O (|V| lg|V| + |E|), where V & E refer vertices & edges

Dijkstra's Shortest Path Algorithm

- **Set S**
  - **Contains vertices whose final shortest-path cost has been determined**

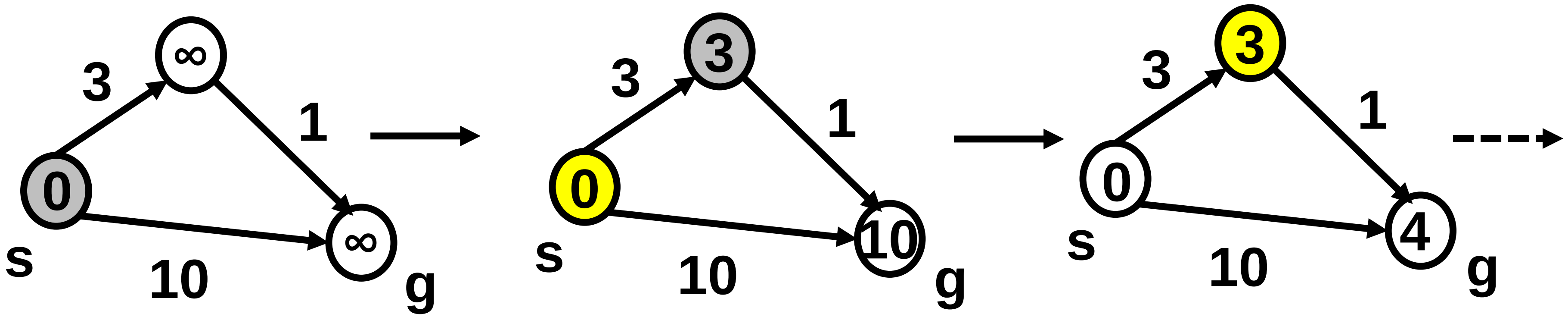
- **DIJKSTRA (G, s):**

  Input: G is an input graph, s is the source

  1. Initialize-Single-Source (G, s)
  2. S ← empty
  3. Queue ← Vertices of G
  4. **While** Queue is not empty
  5.   **Do** u ← min-cost from Queue
  6.   S ← union of S and {u}
  7.   **for** each vertex v in Adj [u]
  8.   **do** RELAX (u, v)

Dijkstra’s Shortest Path Algorithm

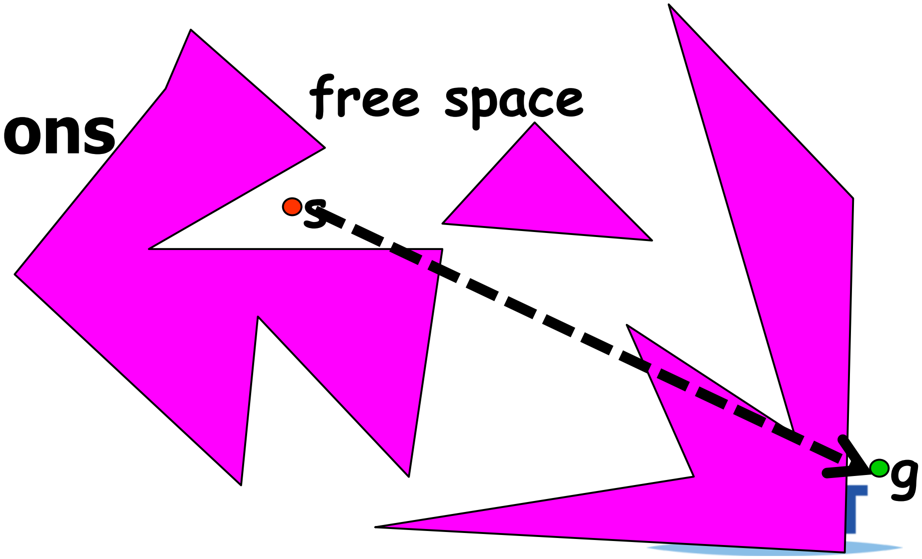
Compute optimal cost-to-come at each iteration



Yellow vertices are in a set with shortest costs
White vertices are in the queue.
Shaded one is chosen for relaxation.

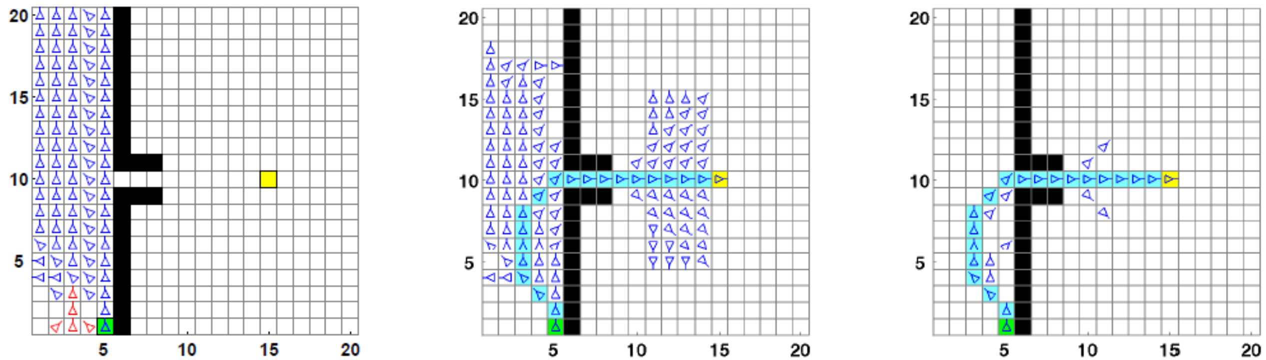
A* Search Algorithm

- An extension of Dijkstra’s algorithm based on a heuristic estimate
  - Conservatively estimate the cost-to-go from a vertex to the goal
  - The estimate should not be greater than the optimal cost-to-go
  - Sort vertices based on “cost-to-come + the estimated cost-to-go”
  - Can find optimal solutions with fewer steps



K* Algorithm (Video)

- **Recursive Path Planning Using Reduced States for Car-like Vehicles on Grid Maps**
  - **IEEE Transactions on Intelligent Transportation System**



(a) H_{free} (b) H_{obs} (c) H_{free} & H_{obs}

- **A* and its variants are quite commonly used for its optimality and high performance**

Framework

continuous representation

↓

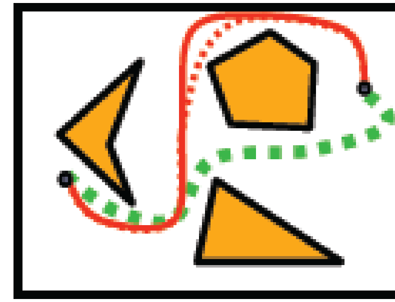
discretization

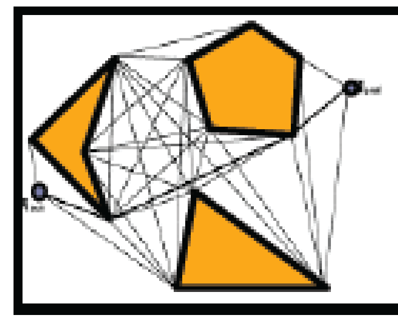
construct visibility graph

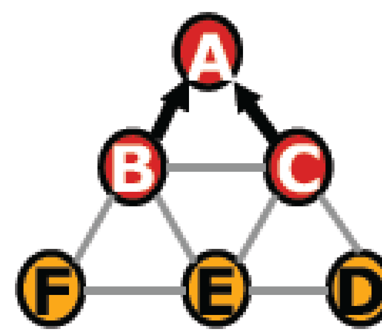
↓

graph searching

breadth-first search







Computational Efficiency

- **Running time $O(n^3)$**
  - **Compute the visibility graph**
  - **Search the graph**
- **Space $O(n^2)$**

- **Can we do better?**
  - **Lead to classical approaches such as roadmap**

Class Objectives were:

- **Motion planning framework**
  - **Representations of robots and space**
  - **Discretization into a graph**
  - **Search methods**
  - **Ch. 1 of my book**

Homework

- **Browse 2 ICRA/IROS/RSS/CoRL/WAFR/TRO/IJRR papers**
  - **Submit it online before the Mon. Class**

- **Example of a summary (just a paragraph)**

  Title: XXX XXXX XXXX
  Conf./Journal Name: ICRA, 2023
  Summary: this paper is about accelerating the performance of collision detection. To achieve its goal, they design a new technique for reordering nodes, since by doing so, they can improve the coherence and thus improve the overall performance.

Valid Papers for Paper Presentation

- Related to the course theme

- Top-tier conf/journals
  - No arxiv paper, unless it has meaningful citation counts (say, 10 per year)

- Recent ones
  - Published at 2019~2023

Homework for Every Class

- **Go over the next lecture slides**
- **Come up with one question on what we have discussed today and submit at the end of the class**
  - **1 for typical questions**
  - **2 for questions with thoughts or that surprised me**
- **Write a question two times before the mid-term exam; submit at the course webpage**

Next Time….

- **Classic path planning algorithms**