Path Planning for Point Robots

Sung-Eui Yoon
(윤성의)

Course URL:
http://sglab.kaist.ac.kr/~sungeui/MPA

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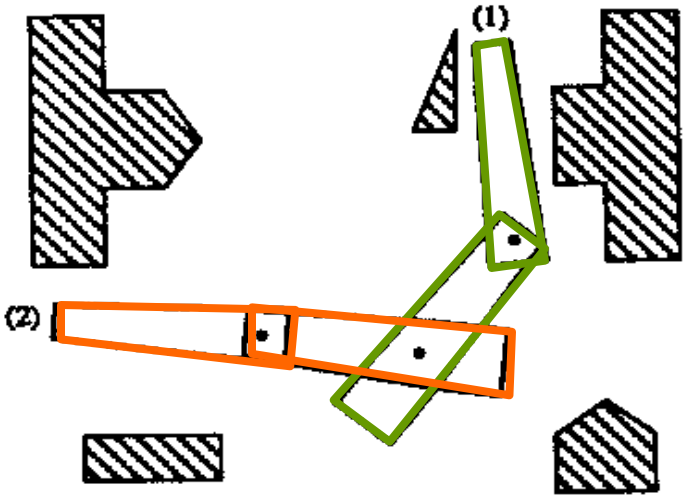
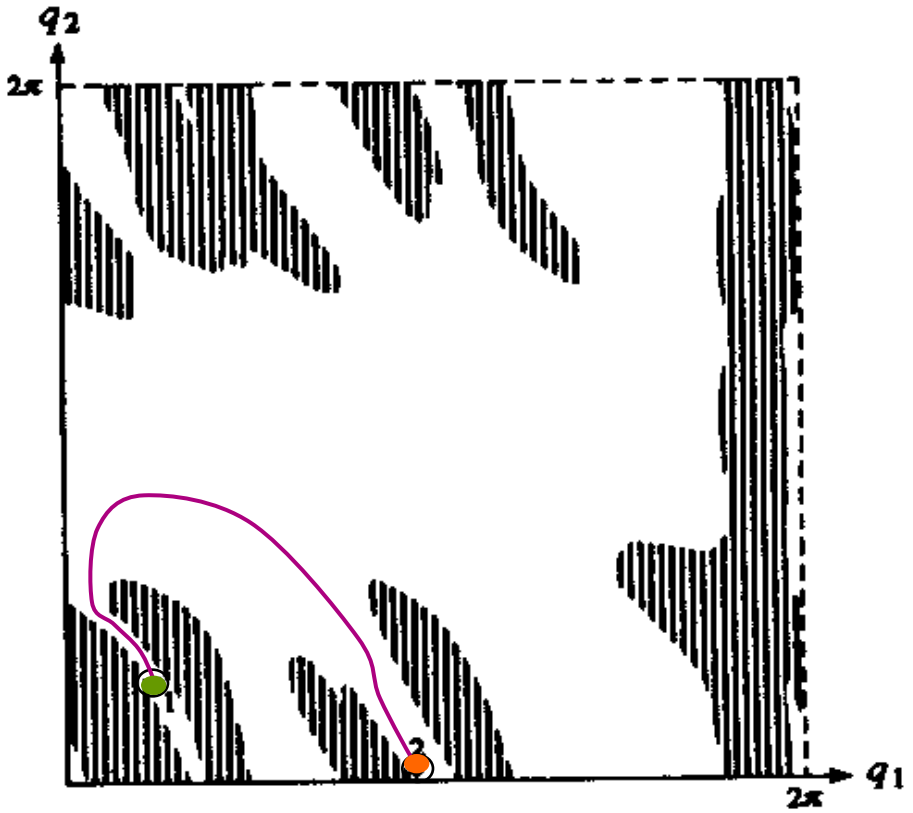
Class Objectives

- **Motion planning framework**
- **Classic motion planning approaches**

Configuration Space: Tool to Map a Robot to a Point

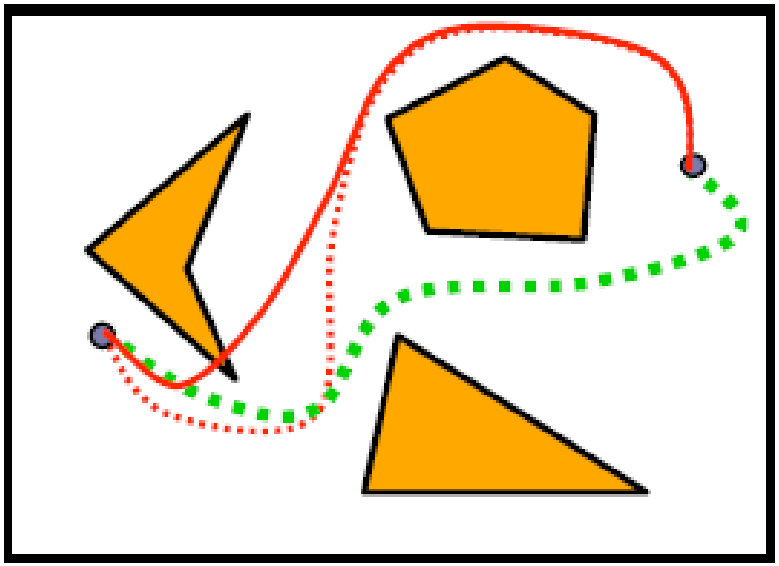




Problem

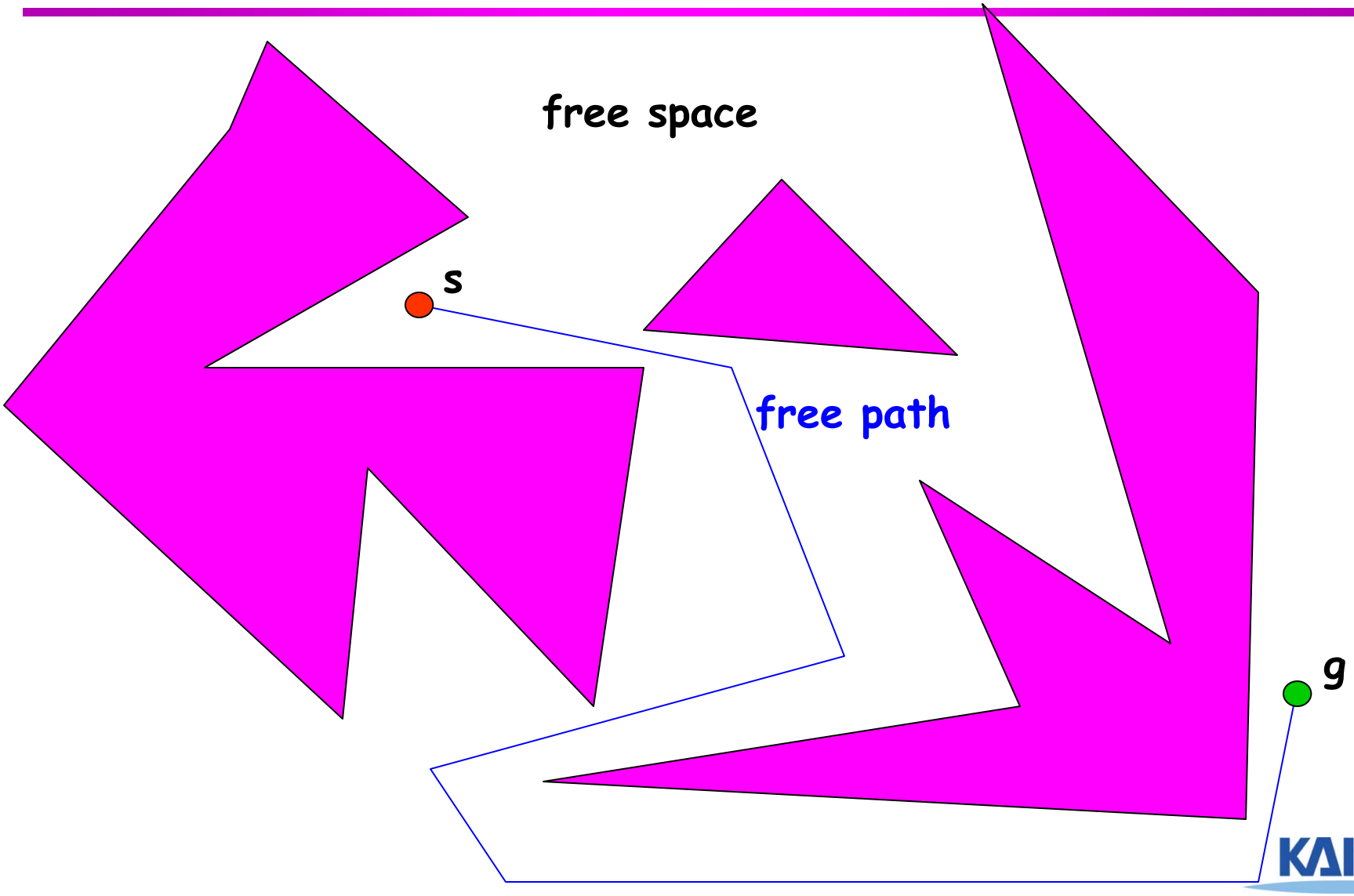
- Input
  - Robot represented as a point in the plane
  - Obstacles represented as polygons
  - Initial and goal positions
- Output

  A collision-free path between the initial and goal positions

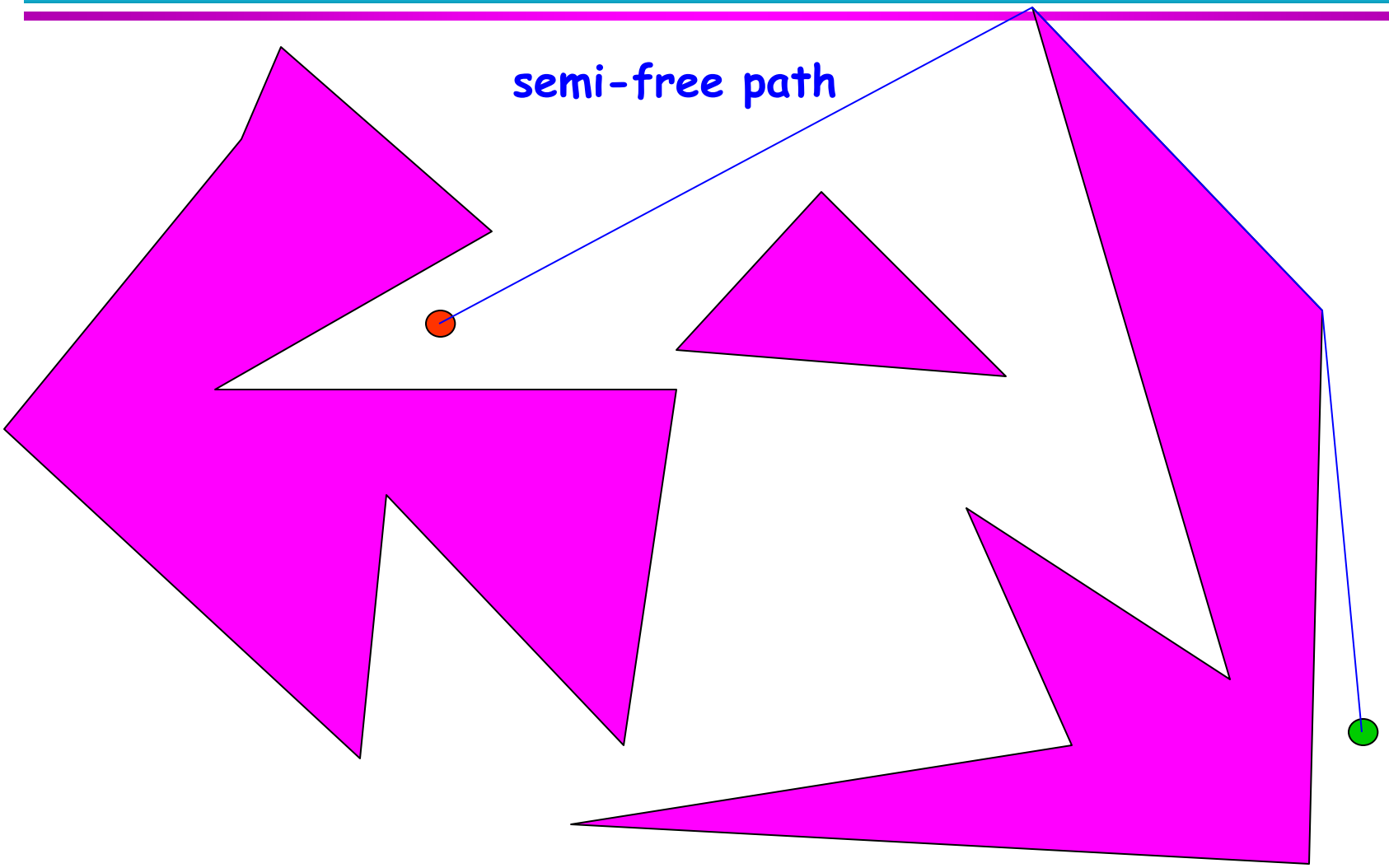


Courtesy of Prof. David Hsu

Problem



Problem



Types of Path Constraints

- **Local** constraints:
  lie in free space
- **Differential** constraints:
  have bounded curvature
- **Global** constraints:
  have minimal length

Motion-Planning Framework

Continuous representation

(configuration space formulation)

↓

Discretization

(random sampling, processing critical geometric events)

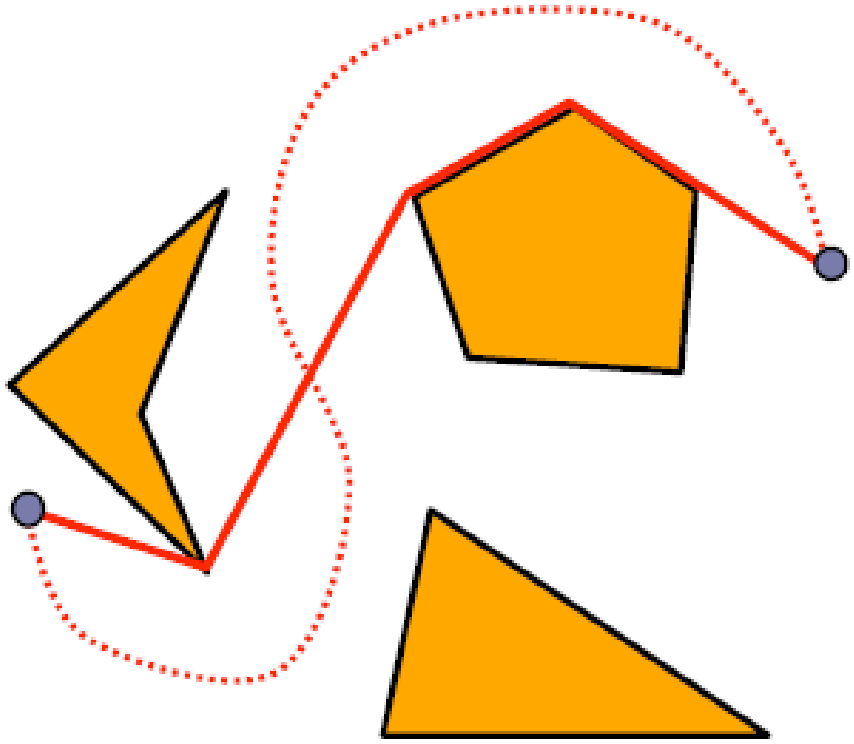
↓

Graph searching

(blind, best-first, A*)

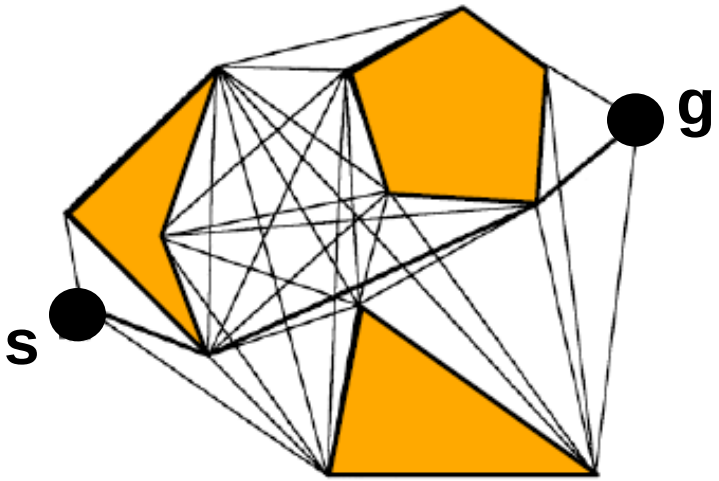
Visibility graph method

- **Observation**: If there is a a collision-free path between two points, then there is a polygonal path that bends only at the obstacles vertices.
- Why?
  Any collision-free path can be transformed into a polygonal path that bends only at the obstacle vertices.



- A **polygonal path** is a piecewise linear curve.

Visibility Graph



- A visibility graph is a graph such that
  - Nodes: s, g, or obstacle vertices
  - Edges: An edge exists between nodes u and v if the line segment between u and v is an obstacle edges or it does not intersect the obstacles

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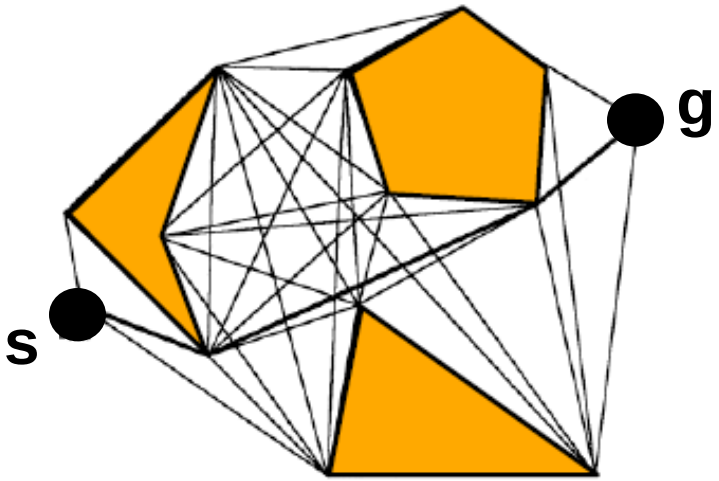
Visibility Graph



- A **visibility graph**
  - Introduced in the late 60s
  - Can produce shortest paths in 2-D configuration spaces

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Simple Algorithm

- **Input: s, q, polygonal obstacles**
- **Output: visibility graph G**

```
1: for every pair of nodes u, v
2:   if segment (u, v) is an obstacle then
3:     insert edge (u, v) into G;
4:   else
5:     for every obstacle edge e
6:       if segment (u, v) intersects e
7:         go to (1);
8:     insert edge (u, v) into G;
9: Search a path with G using A*
```

Computation Efficiency

1: **for** every pair of nodes u, v	**O(n^2)**
2: **if** segment (u, v) is an obstacle **then**	**O(n)**
3: insert edge (u, v) into G;	
4: **else**	
5: **for** every obstacle edge e	**O(n)**
6: **if** segment (u, v) intersects e	
7: go to (1);	
8: insert edge (u, v) into G;	

- **Simple algorithm: O(n^3) time**
- **More efficient algorithms**
  - **Rotational sweep O(n^2 log n) time, etc.**
- **O(n^2) space**

Motion-Planning Framework

Continuous representation

(configuration space formulation)

Discretization

(random sampling, processing critical geometric events)

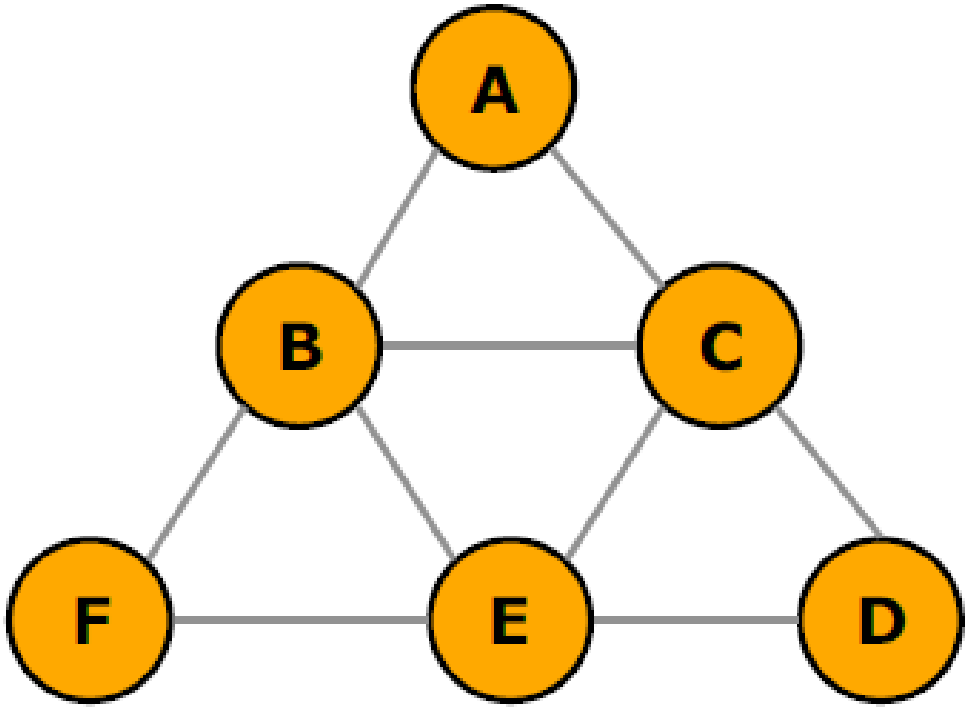
Graph searching

(blind, best-first, A*)

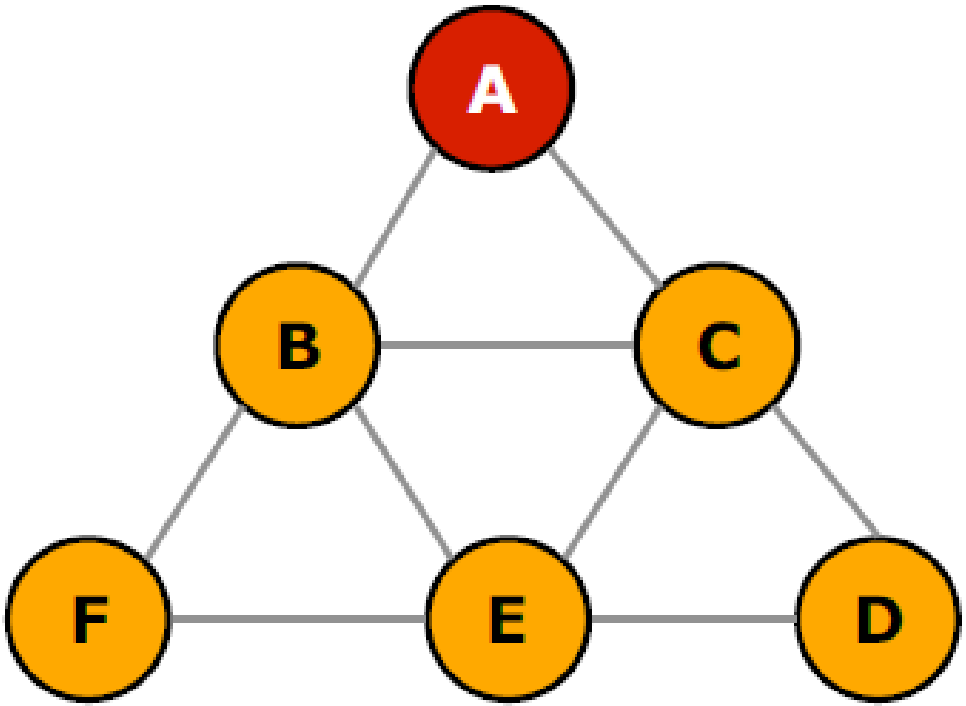
Graph Search Algorithms

- **Breadth, depth-first, best-first**
- **Dijkstra's algorithm**
- **A***

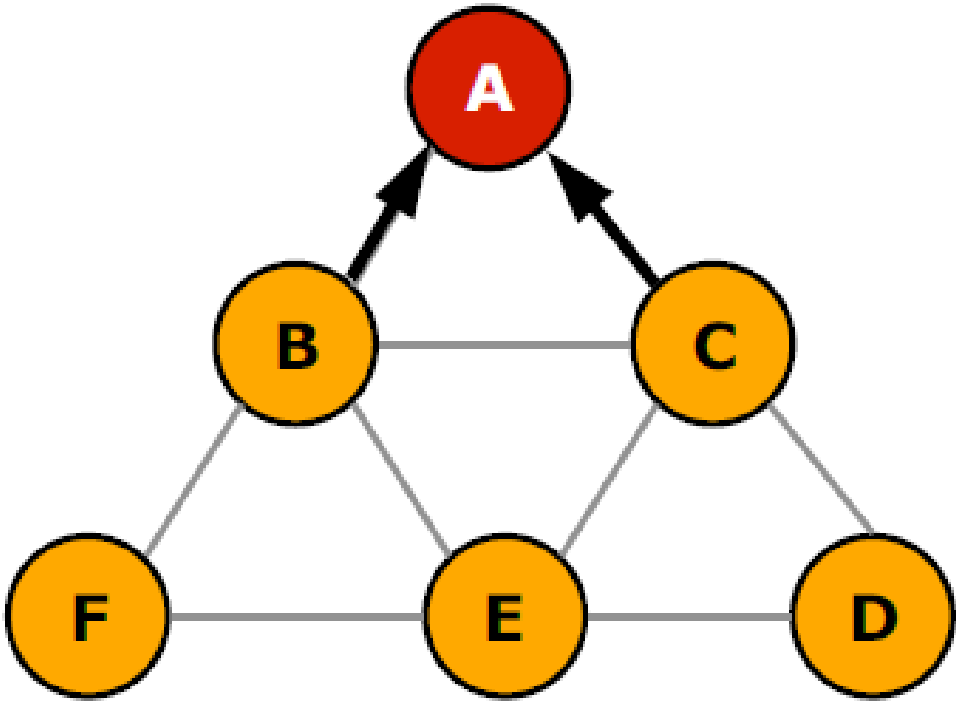
Breadth-first search



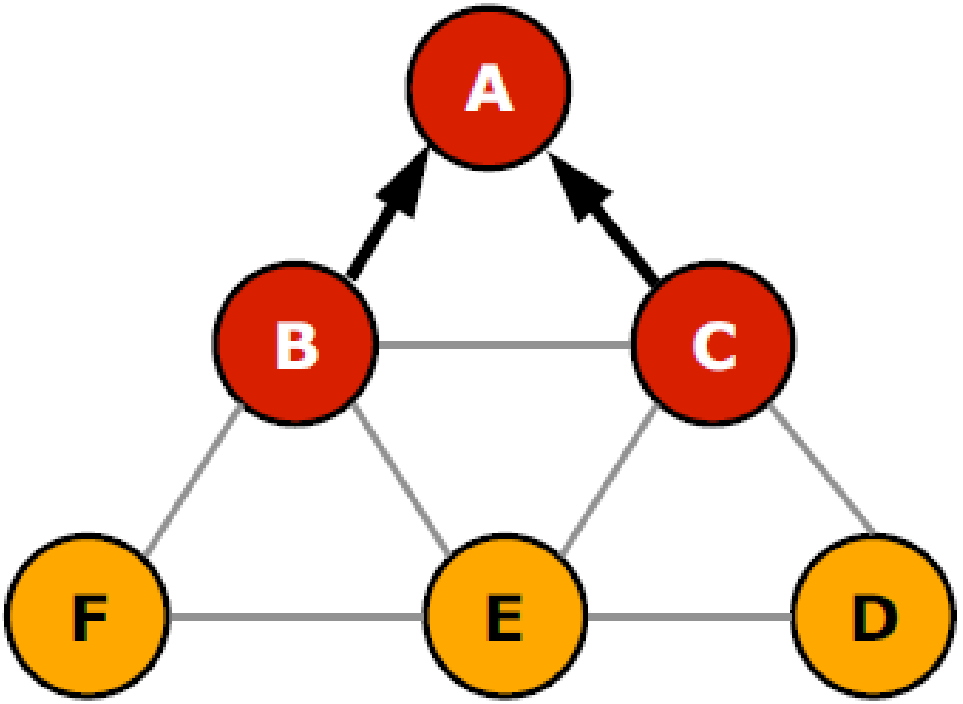
Breadth-first search



Breadth-first search



Breadth-first search



Dijkstra's Shortest Path Algorithm

- Given a (non-negative) weighted graph, two vertices, s and g:
  - Find a path of minimum total weight between them
  - Also, find minimum paths to other vertices
  - Has O ($|V| \lg |V| + |E|$)

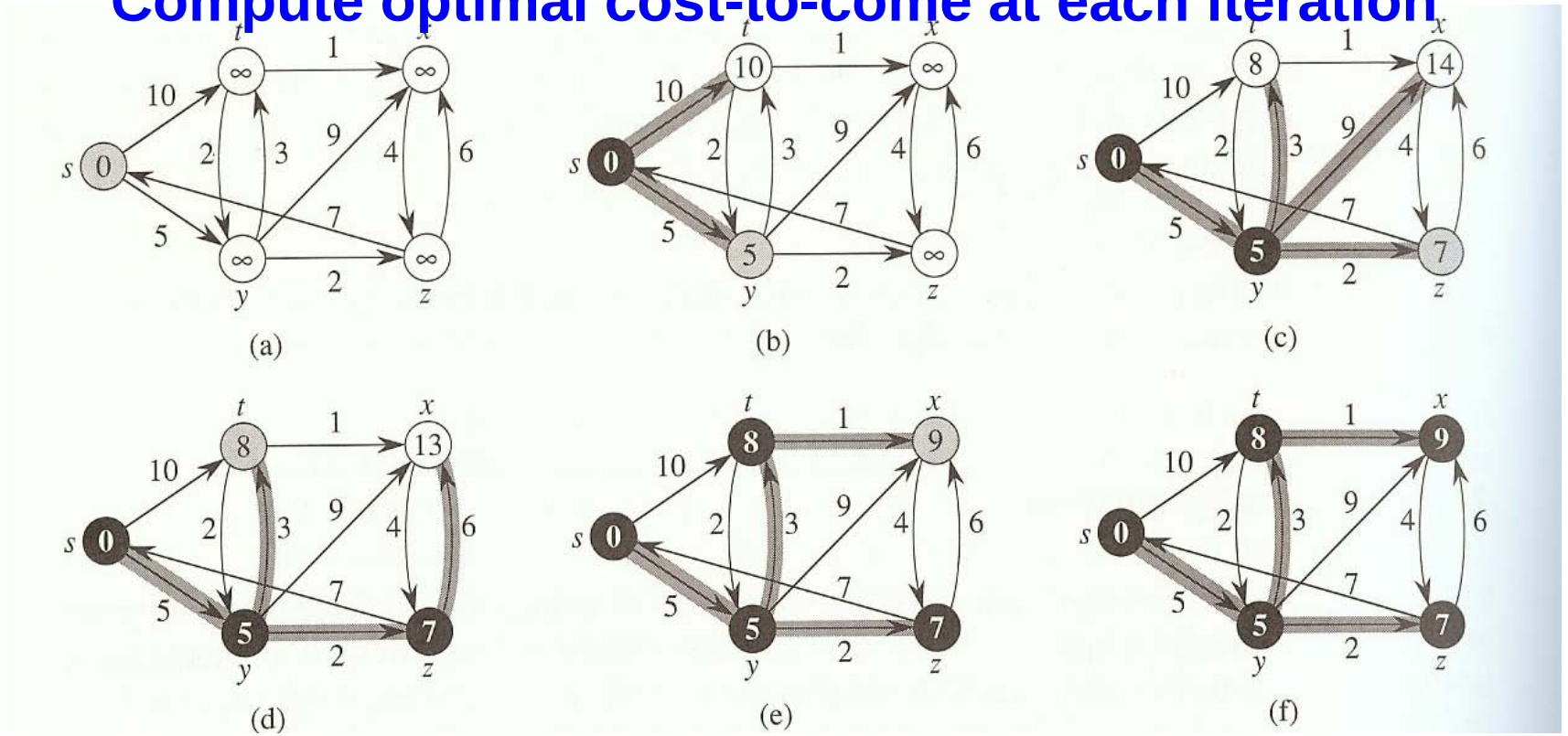
Dijkstra's Shortest Path Algorithm

- **A set S**
  - **Contains vertices whose final shortest-path cost has been determined**

- **DIJKSTRA (G, s)**
  1. Initialize-Single-Source (G, s)
  2. S ← empty
  3. Queue ← Vertices of G
  4. **While** Queue is not empty
  5. **Do** u ← min-cost from Queue
  6. S ← union of S and {u}
  7. **for** each vertex v in Adj [u]
  8. **do** RELAX (u, v)

Dijkstra's Shortest Path Algorithm

Compute optimal cost-to-come at each iteration



Black vertices are in the set.
White vertices are in the queue.
Shaded one is chosen for relaxation.

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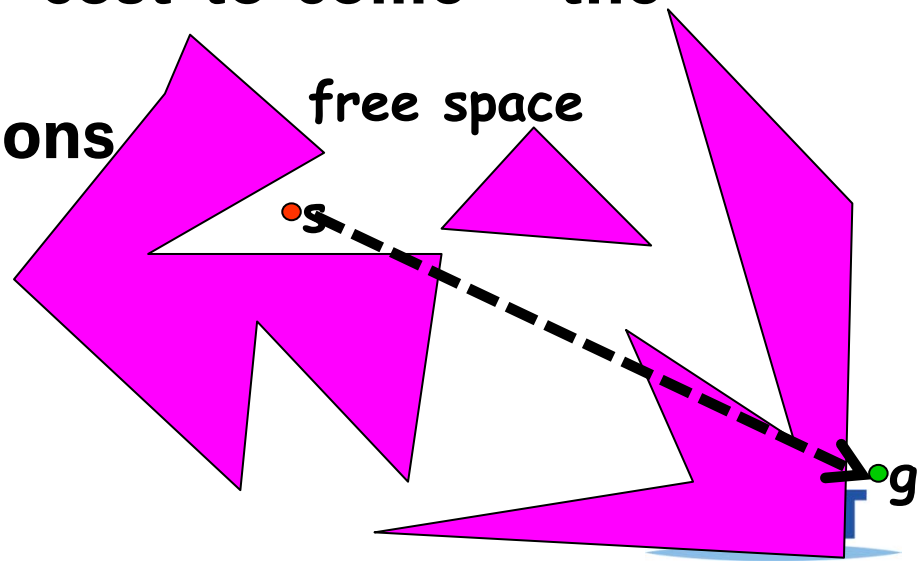
A* Search Algorithm

- An extension of Dijkstra's algorithm based on a heuristic estimate
  - Conservatively estimate the cost-to-go from a vertex to the goal
  - The estimate should not be greater than the optimal cost-to-go
  - Sort vertices based on "cost-to-come + the estimated cost-to-go"
  - Can find optimal solutions with fewer steps



Best-First Search

- **Pick a next node based on an estimate of the optimal cost-to-go cost**
  - **Greedily finds solutions that look good**
  - **Solutions may not be optimal**
  - **Can find solutions quite fast, but can be also very slow**

Framework

continuous representation

↓

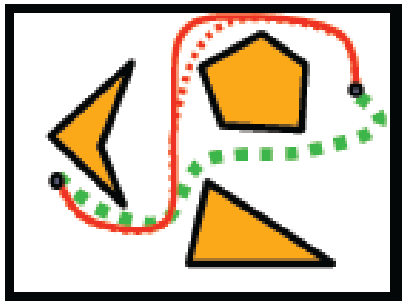
discretization

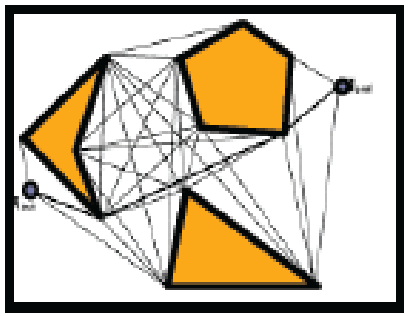
construct visibility graph

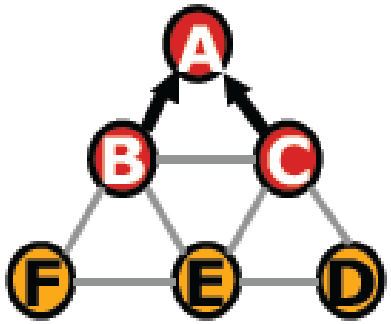
↓

graph searching

breadth-first search







Computational Efficiency

- Running time $O(n^3)$
  - Compute the visibility graph
  - Search the graph
- Space $O(n^2)$

- Can we do better?

Classic Path Planning Approaches

- **Roadmap**
  - **Represent the connectivity of the free space by a network of 1-D curves**
- **Cell decomposition**
  - **Decompose the free space into simple cells and represent the connectivity of the free space by the adjacency graph of these cells**
- **Potential field**
  - **Define a function over the free space that has a global minimum at the goal configuration and follow its steepest descent**

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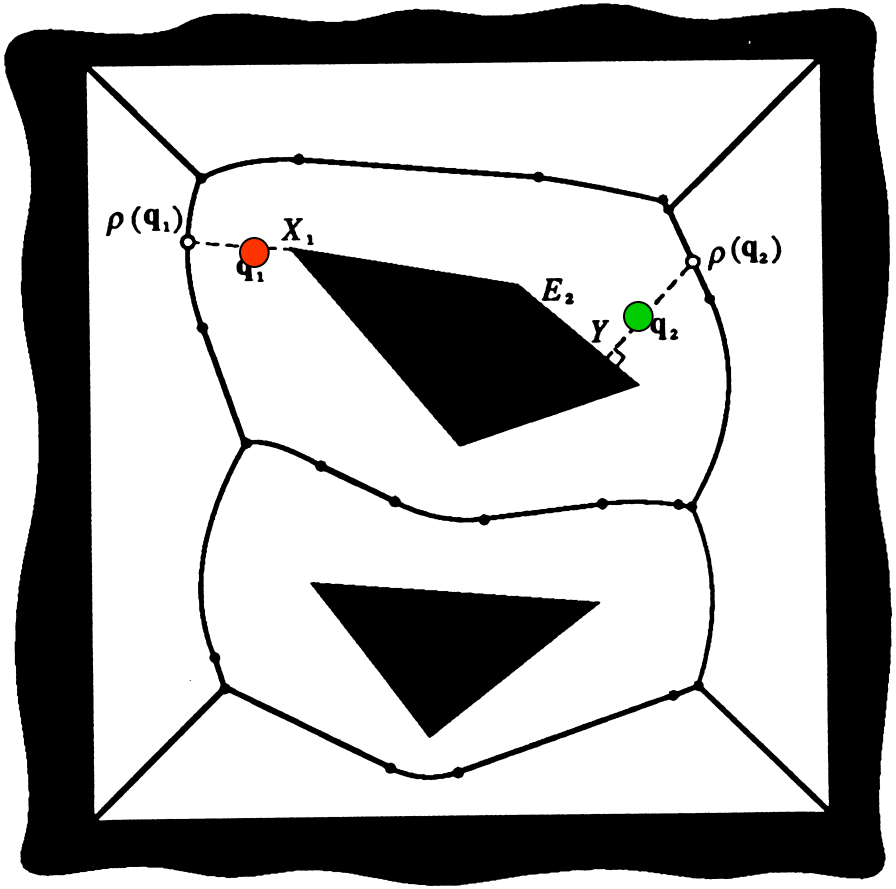
Roadmap Methods

- Visibility Graph
  - Shakey project, SRI [Nilsson 69]
- **Voronoi diagram**
  - **Introduced by computational geometry researchers**
  - **Generate paths that maximize clearance**
  - **O(n log n) time and O(n) space**



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Other Roadmap Methods

- Visibility graph
- Voronoi diagram
- Silhouette
  - First complete general method that applies to spaces of any dimension and is singly exponential in # of dimensions [Canny, 87]
- Probabilistic roadmaps

Classic Path Planning Approaches

- **Roadmap**
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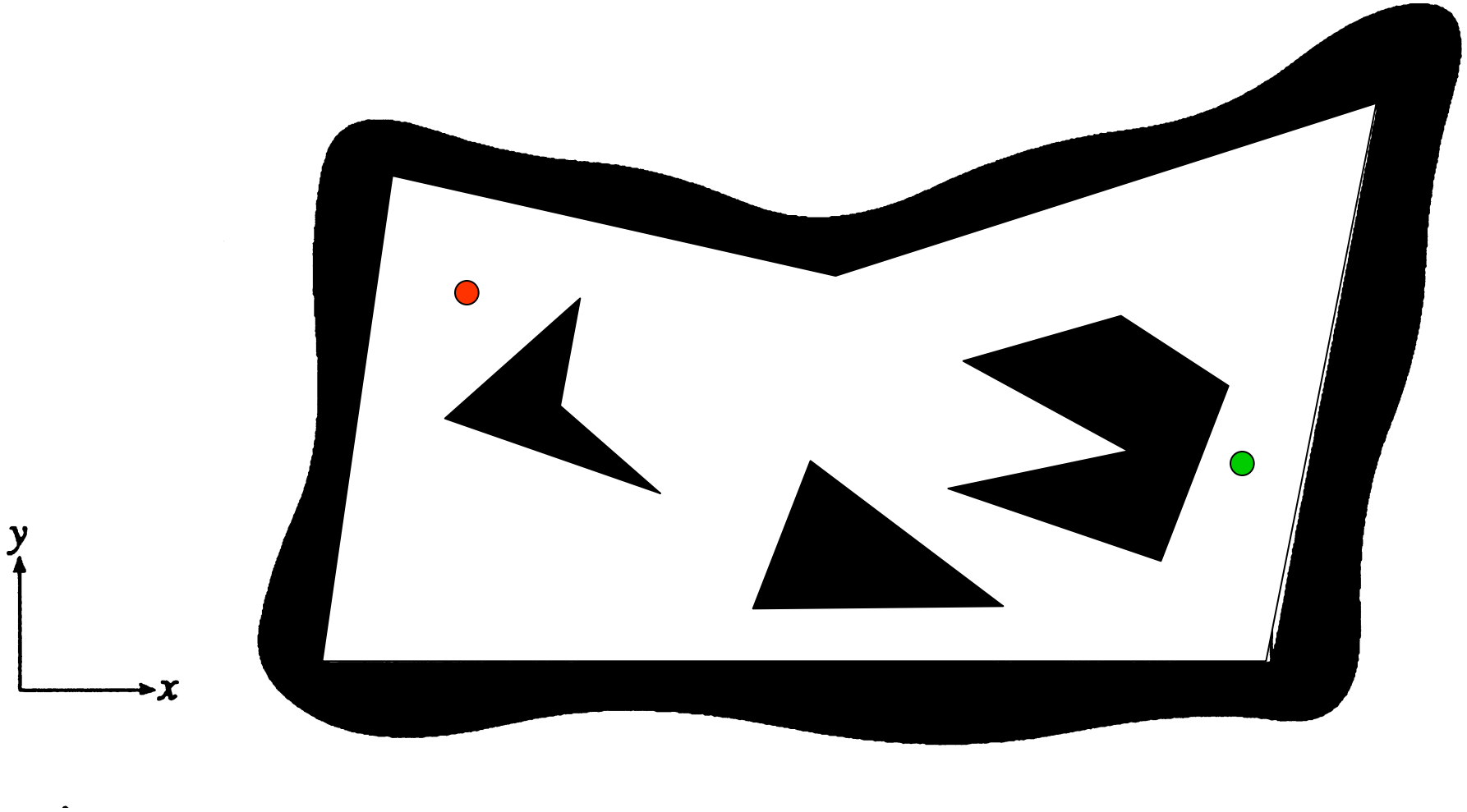
Cell-Decomposition Methods

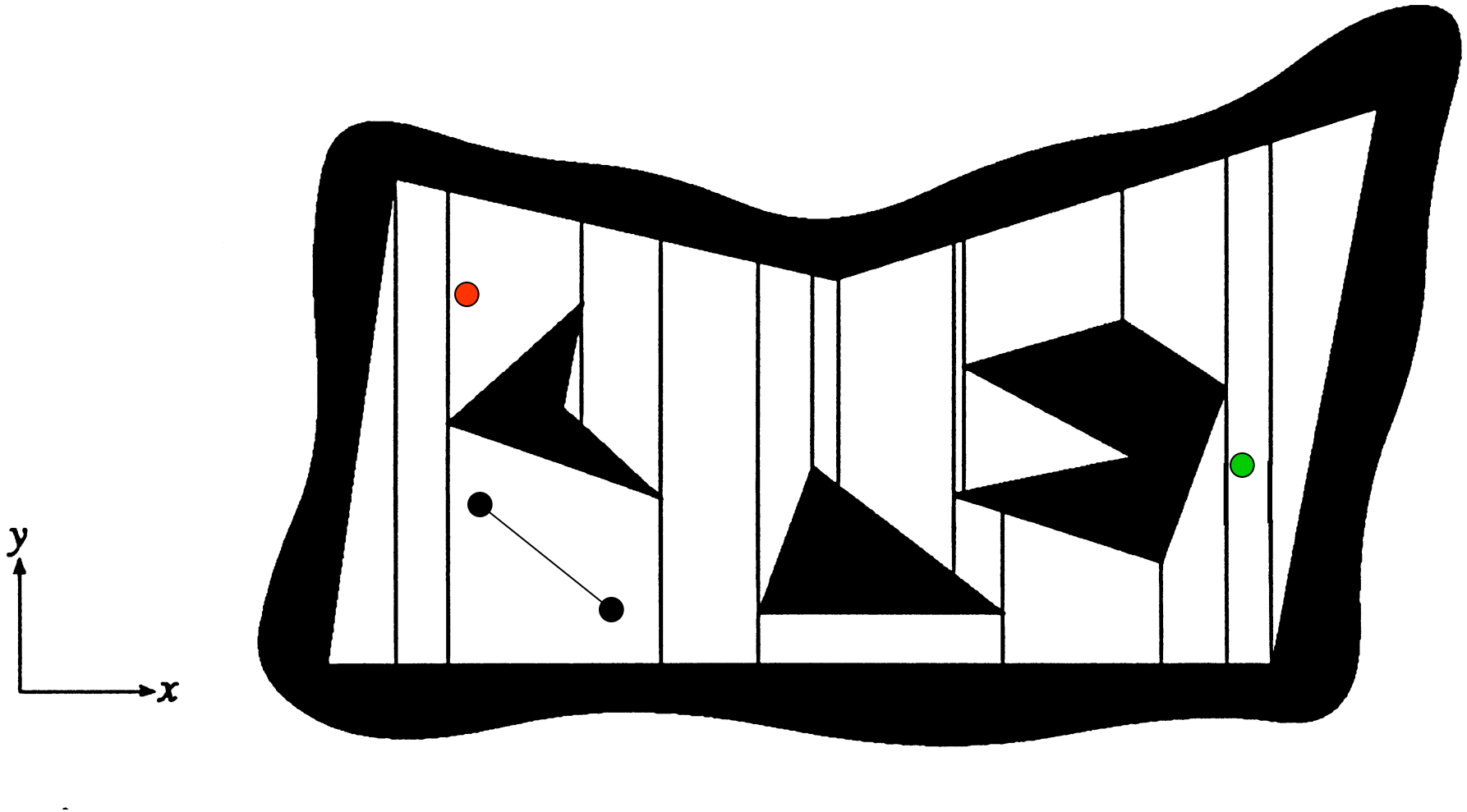
- **Two classes of methods:**
  - **Exact and approximate cell decompositions**

- **Exact cell decomposition**
  - **The free space F is represented by a collection of non-overlapping cells whose union is exactly F**
  - **Example: trapezoidal decomposition**

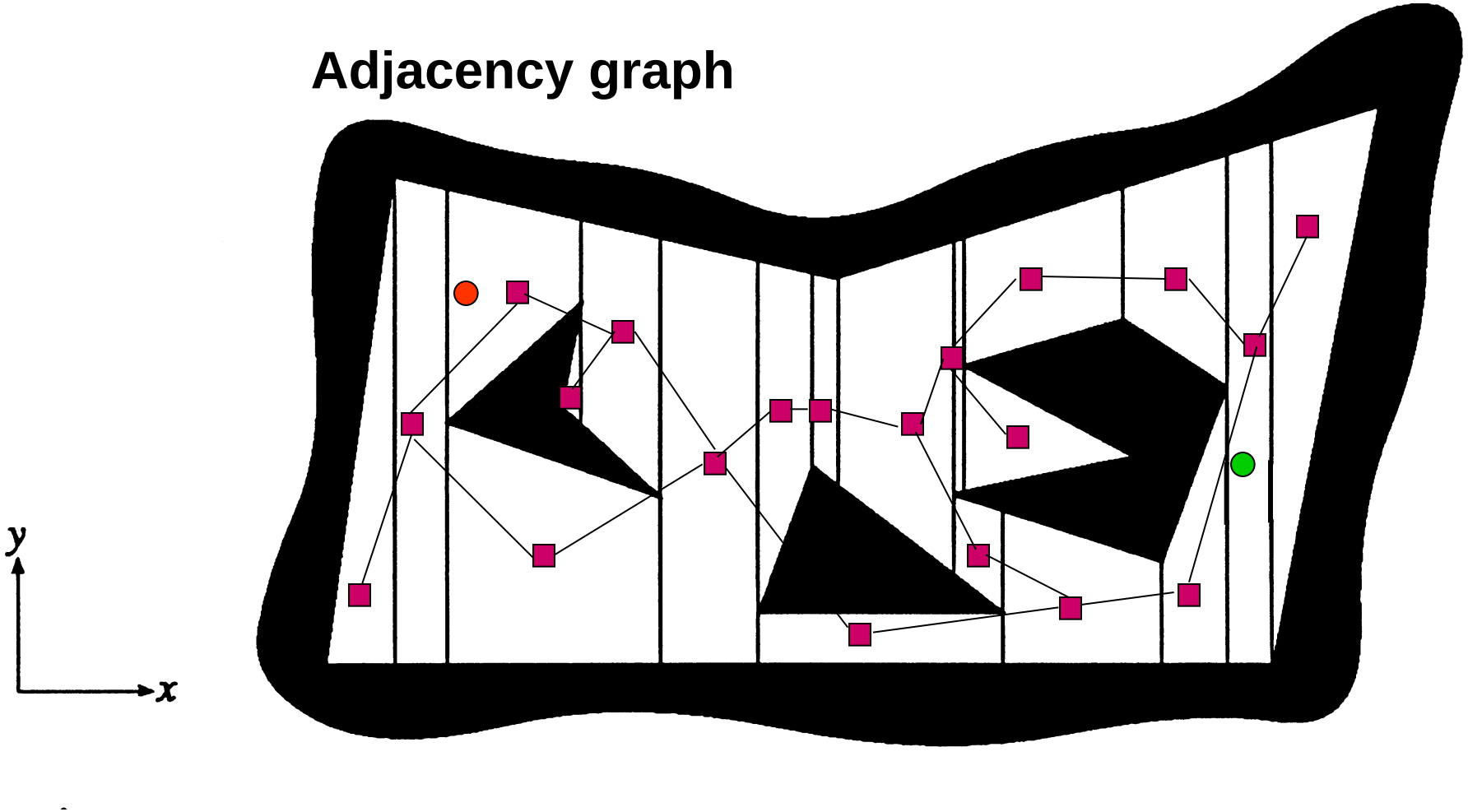
Trapezoidal Decomposition



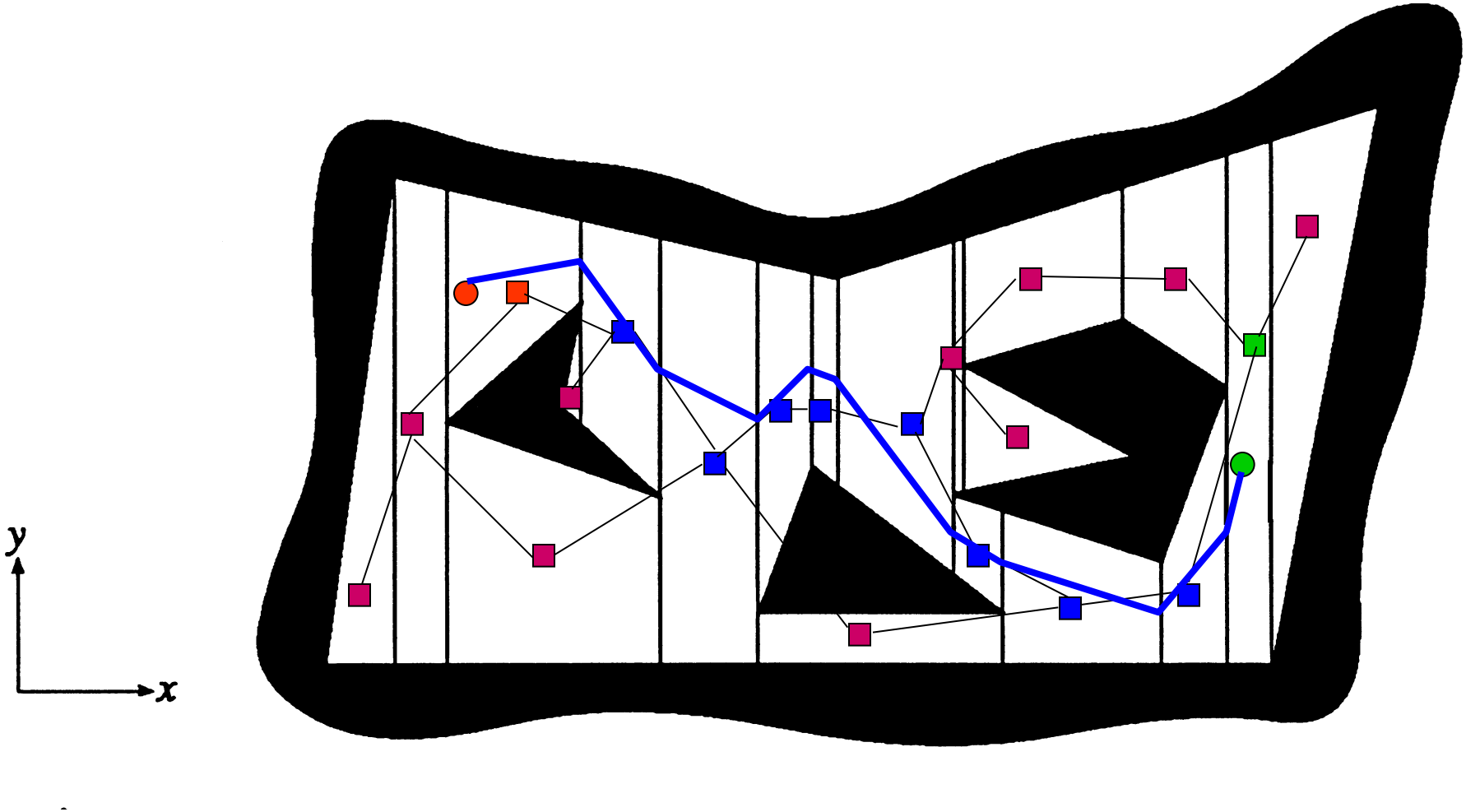
Trapezoidal Decomposition



Trapezoidal Decomposition



Trapezoidal Decomposition



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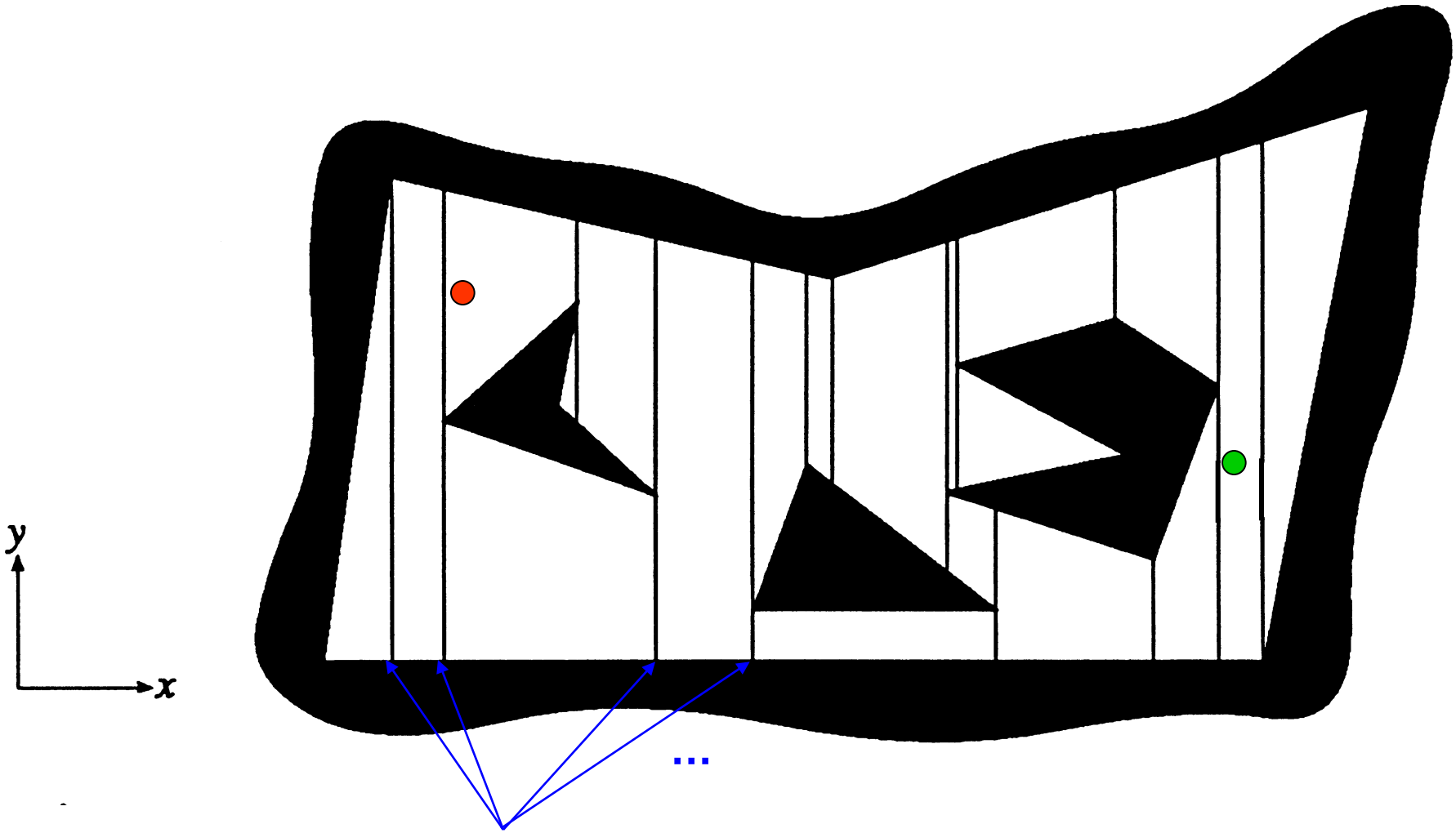
Trapezoidal Decomposition



critical events → criticality-based decomposition

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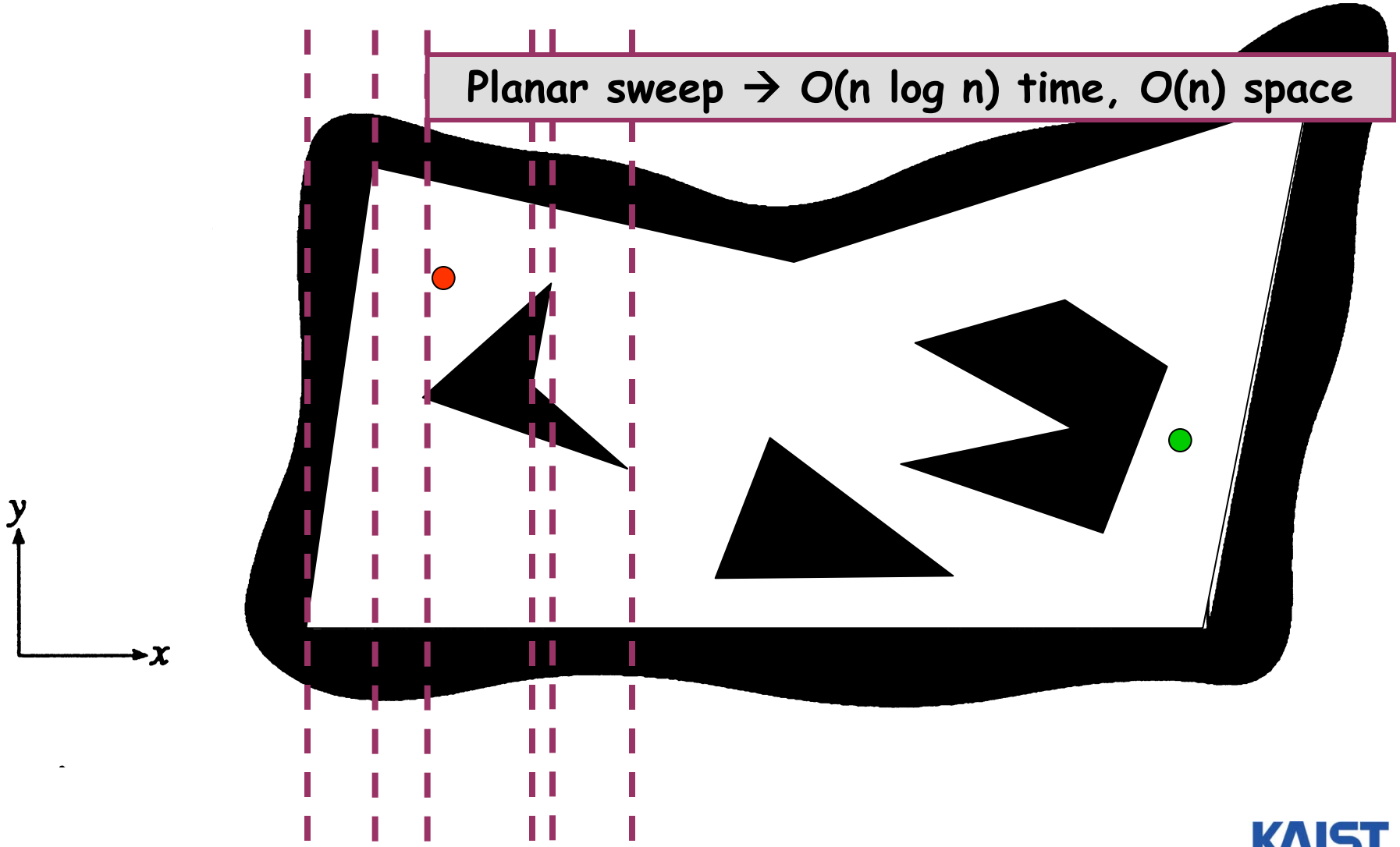
Trapezoidal Decomposition

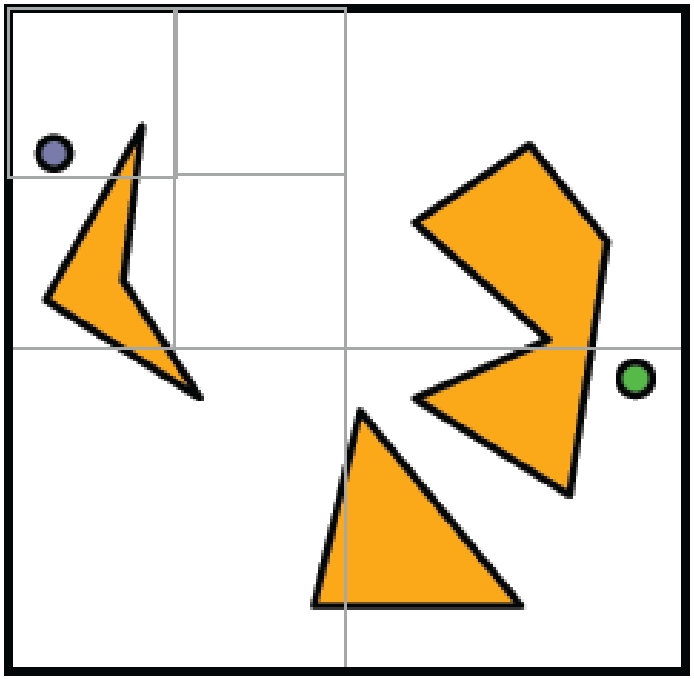


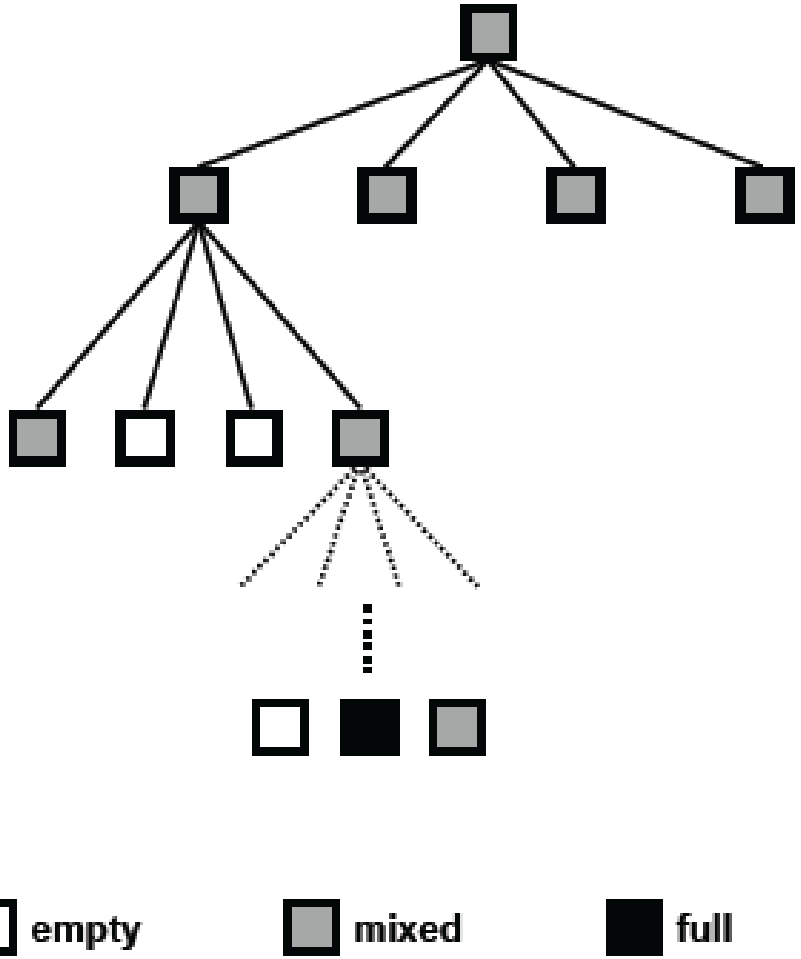
Cell-Decomposition Methods

- **Two classes of methods:**
  - **Exact and approximate cell decompositions**

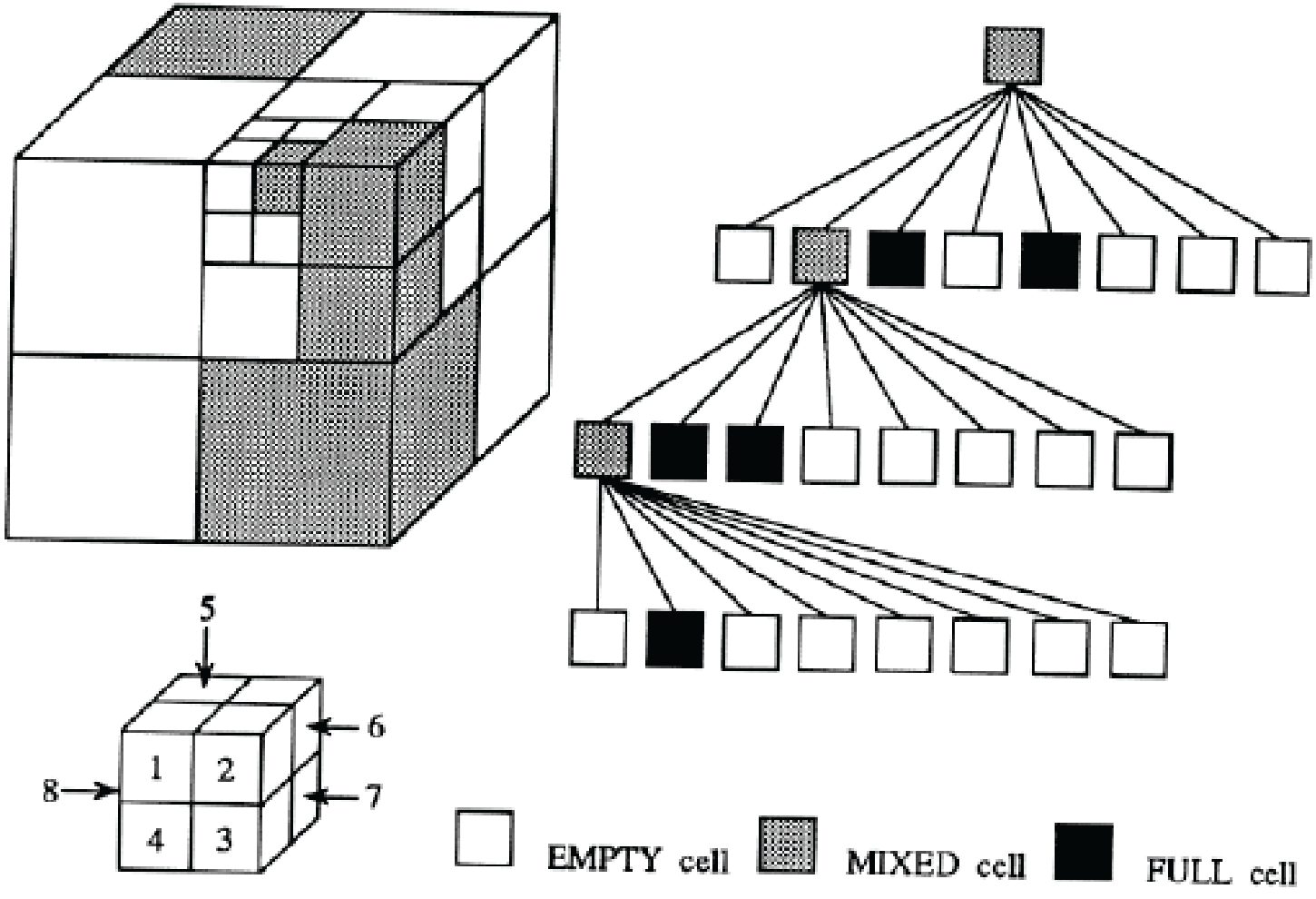
- **Approximate cell decomposition**
  - **The free space F is represented by a collection of non-overlapping cells whose union is contained in F**
  - **Cells usually have simple, regular shapes (e.g., rectangles and squares)**
  - **Facilitates hierarchical space decomposition**

Quadtree decomposition





Octree decomposition



Sketch of Algorithm

1. Decompose the free space F into cells
2. Search for a sequence of **mixed** or **free** cells that connect that initial and goal positions
3. Further decompose the mixed
4. Repeat 2 and 3 until a sequence of **free** cells is found

Classic Path Planning Approaches

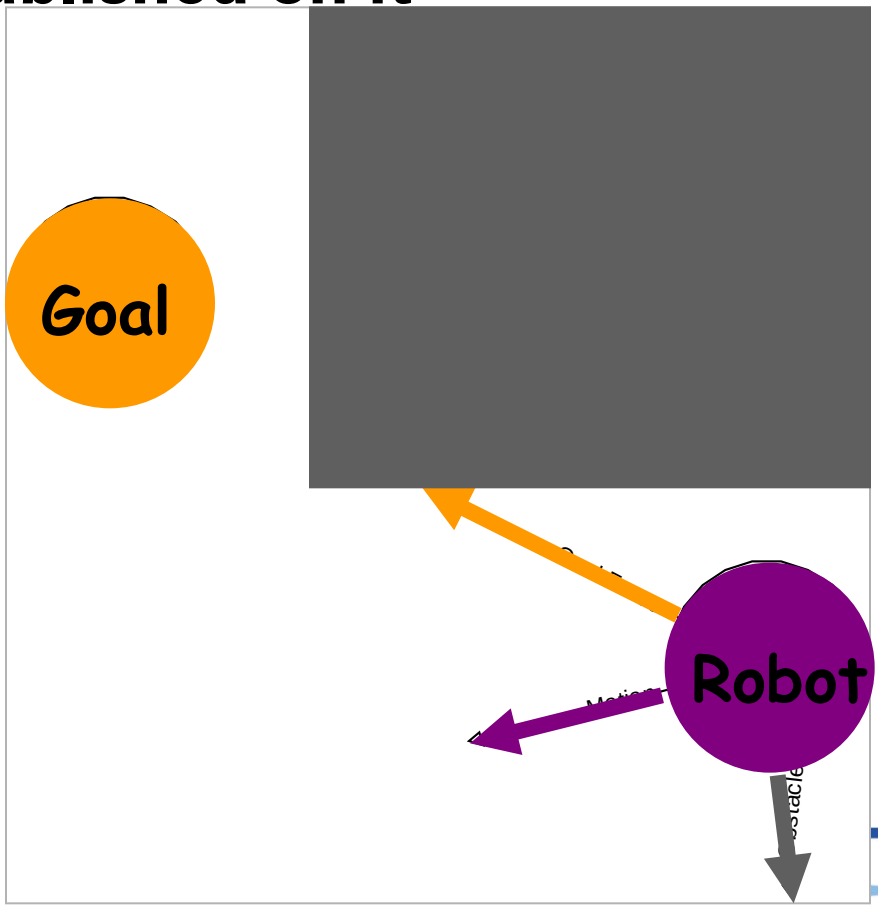
- **Roadmap**
  - **Represent the connectivity of the free space by a network of 1-D curves**
- **Cell decomposition**
  - **Decompose the free space into simple cells and represent the connectivity of the free space by the adjacency graph of these cells**
- **Potential field**
  - **Define a function over the free space that has a global minimum at the goal configuration and follow its steepest descent**

Potential Field Methods

- **Initially proposed for real-time collision avoidance [Khatib, 86]**
  - **Hundreds of papers published on it**

$$F_{Goal} = -k_p(x - x_{Goal})$$

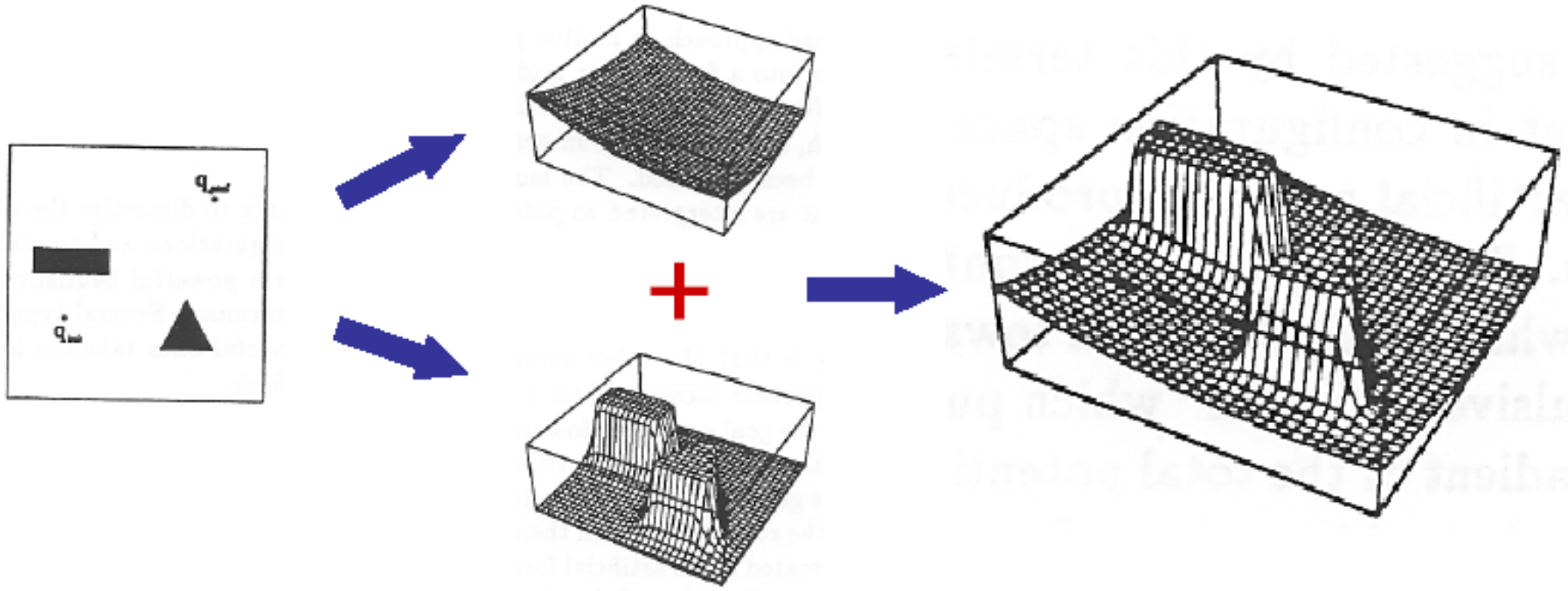
$$F_{Obstacle} = \begin{cases} \eta\left(\frac{1}{\rho} - \frac{1}{\rho_0}\right)\frac{1}{\rho^2}\frac{\partial \rho}{\partial x} & \text{if } \rho \le \rho_0, \\ 0 & \text{if } \rho > \rho_0 \end{cases}$$



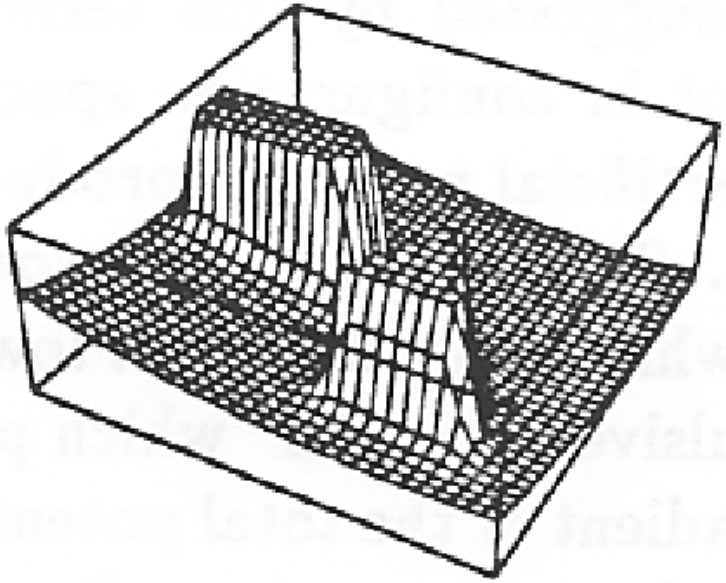
Potential Field

- A scalar function over the free space
- To navigate the robot applies a force proportional to the negated gradient of the potential field
- A navigation function is an ideal potential field that
  - Has global minimum at the goal
  - Has no local minima
  - Grows to infinity near obstacles
  - Is smooth

Attractive and Repulsive fields



Local Minima



- **What can we do?**
  - **Escape from local minima by taking random walks**
  - **Build an ideal potential field that does not have local minima**

Sketch of Algorithm

- Place a regular grid G over the configuration space
- Compute the potential field over G
- Search G using a best-first algorithm with potential field as the heuristic function

Question

- **Can such an ideal potential field be constructed efficiently in general?**

Completeness

- A complete motion planner always returns a solution when one exists and indicates that no such solution exists otherwise
  - Is the visibility algorithm complete? Yes
  - How about the exact cell decomposition algorithm and the potential field algorithm?

Class Objectives were:

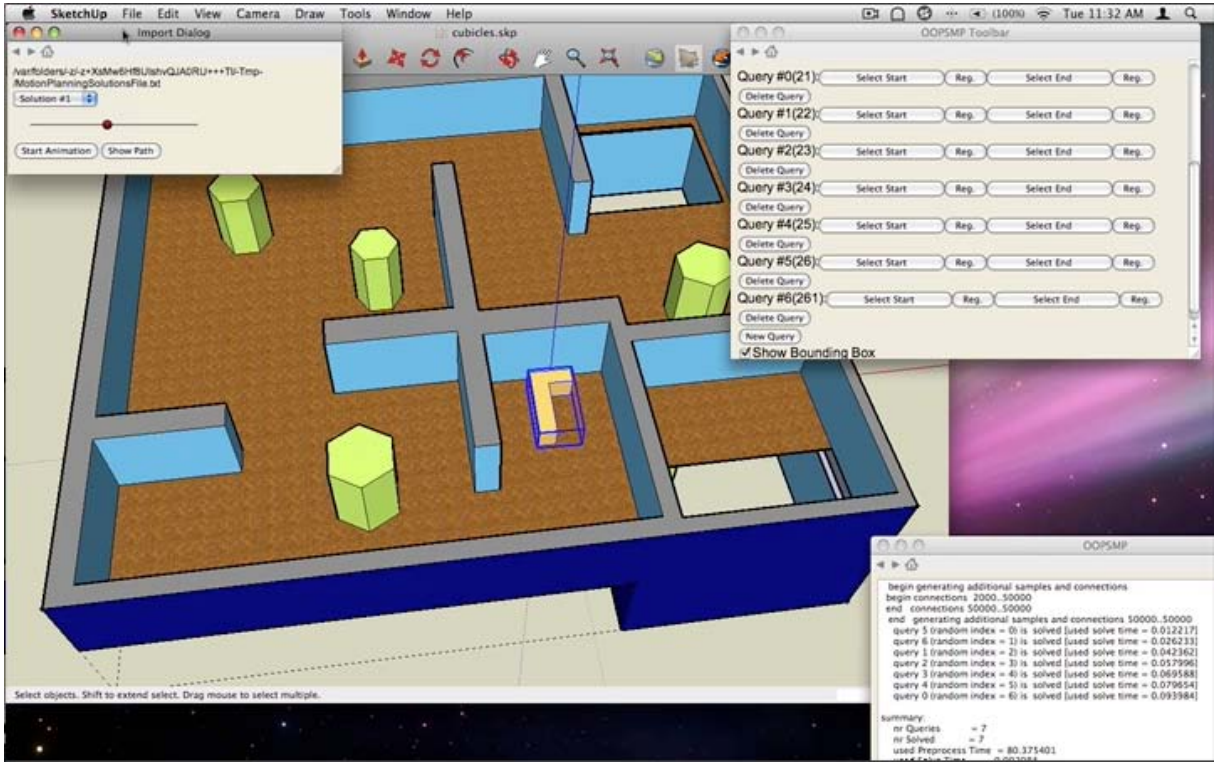
- **Motion planning framework**
- **Classic motion planning approaches**

Homework for Every Class

- **Go over the next lecture slides**
- **Come up with one question on what we have discussed today and submit at the beginning of the next class**
  - **1**
  - **2 for typical questions**
  - **3 for questions with thoughts**
  - **4 for questions that surprised me**

Homework

- Install OOPSMP Motion Planning Library
- Create a scene and a robot
- Find a collision-free path and visualize the path



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Conf. Deadline

- ICRA
  - Sept. 15, 2010
- IROS
  - Feb. 28, 2010

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Next Time….

- **Configuration spaces**