Path Planning for Point Robots

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(윤성의)

Course URL:
http://sglab.kaist.ac.kr/~sungeui/MPA

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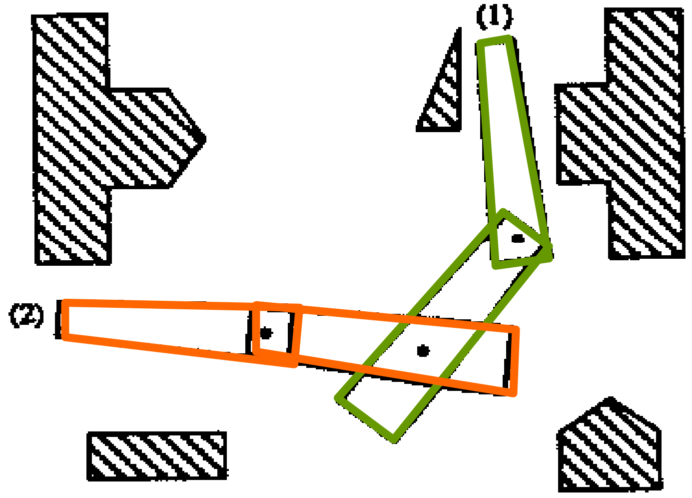
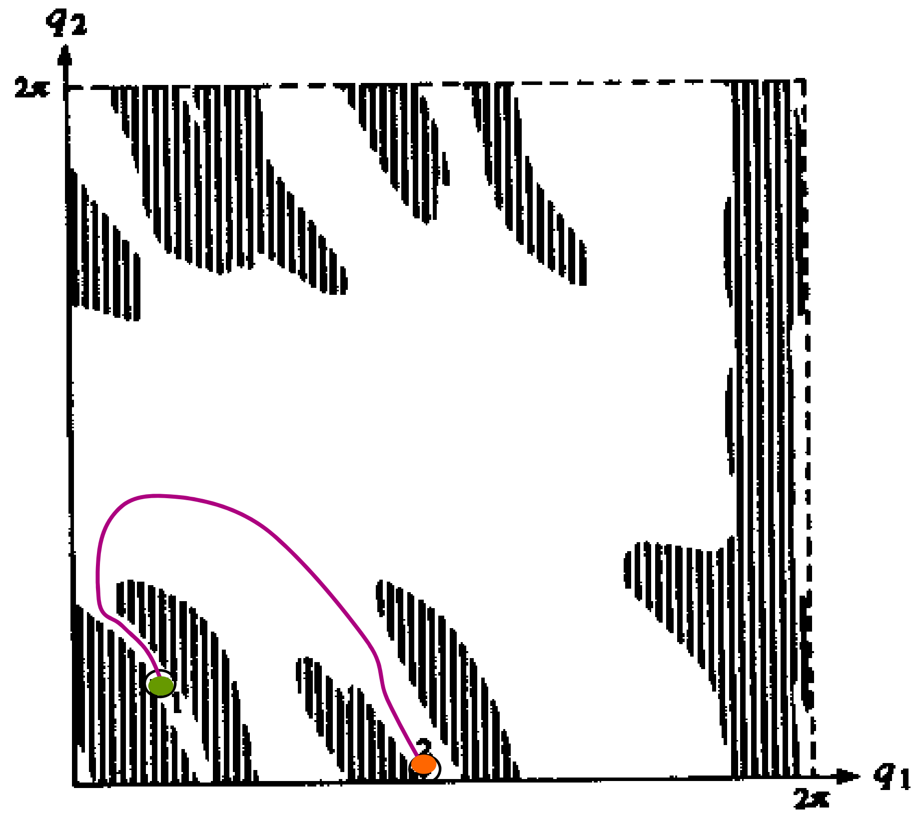
Class Objectives

- **Motion planning framework**
- **Classic motion planning approaches**

Configuration Space: Tool to Map a Robot to a Point

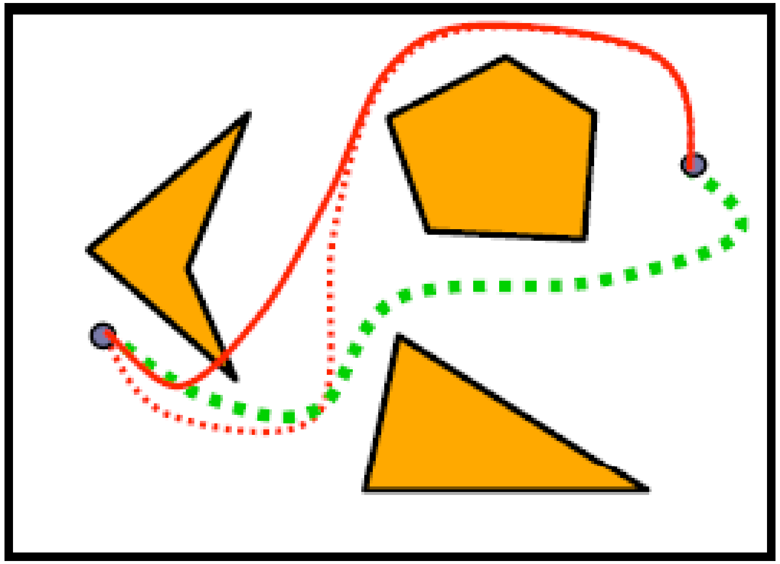




Problem

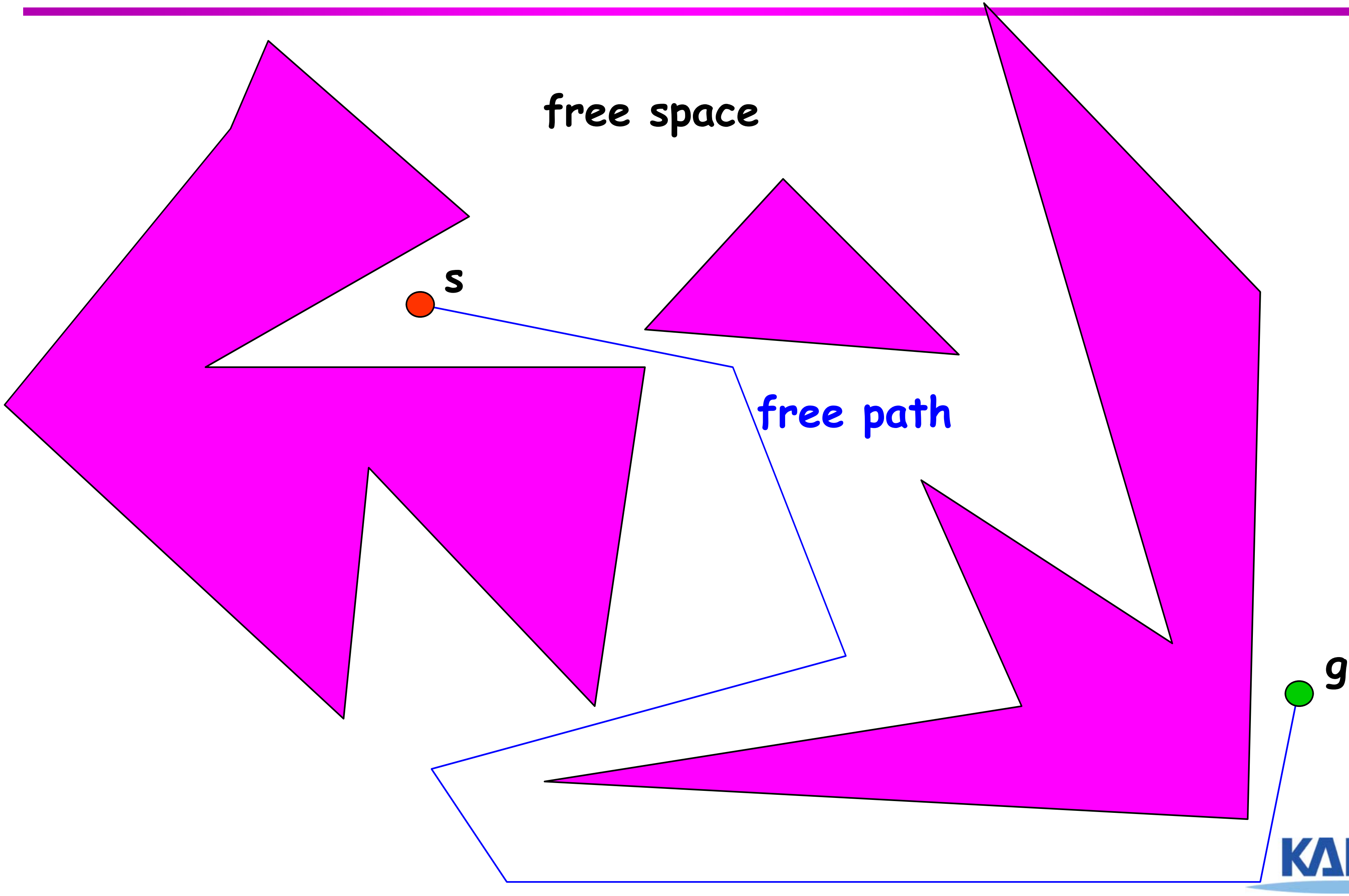
- Input
  - Robot represented as a **point** in the **plane**
  - Obstacles represented as polygons
  - Initial and goal positions
- Output

  A collision-free path between the initial and goal positions



Courtesy of Prof. David Hsu

Problem



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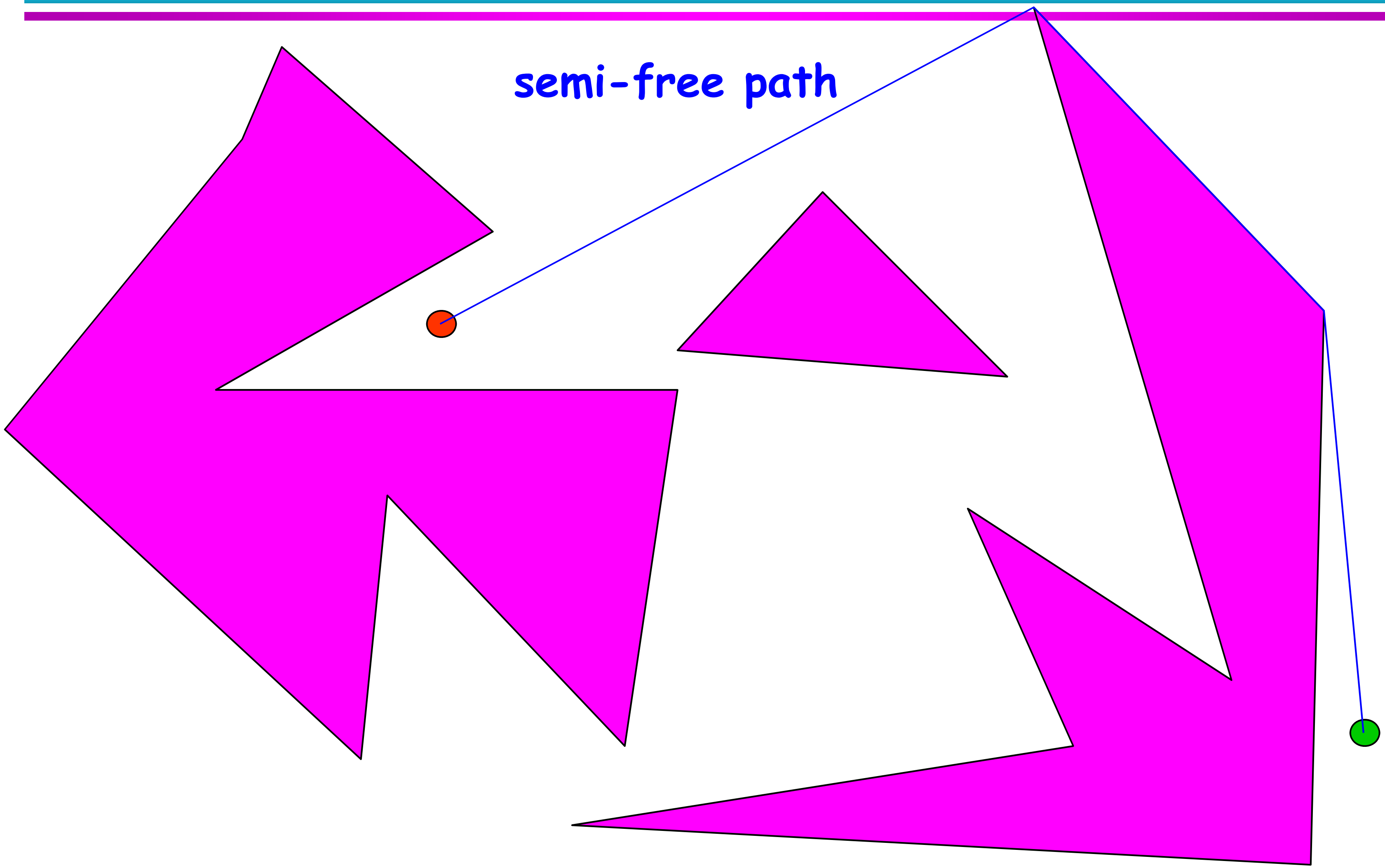
Problem



Types of Path Constraints

- **Local** constraints:
  lie in free space
- **Differential** constraints:
  have bounded curvature
- **Global** constraints:
  have minimal length

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Motion-Planning Framework

Continuous representation

(configuration space formulation)

↓

Discretization

(random sampling, processing critical geometric events)

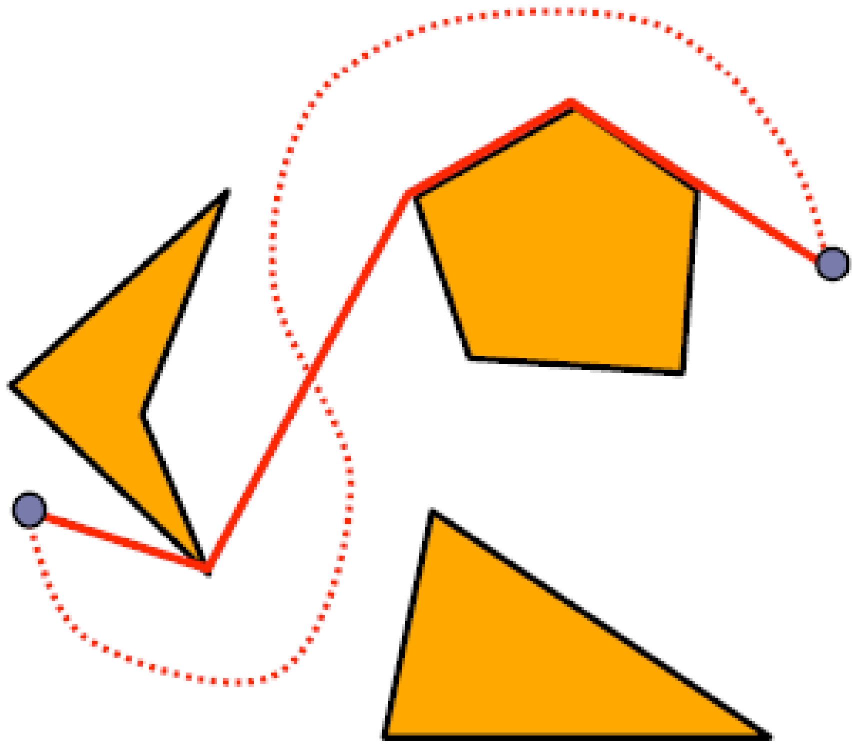
↓

Graph searching

(blind, best-first, A*)

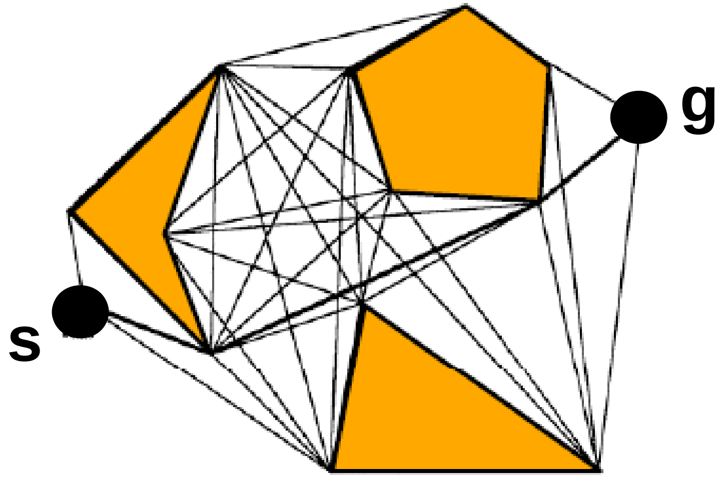
Visibility graph method

- **Observation**: If there is a a collision-free path between two points, then there is a polygonal path that bends only at the obstacles vertices.
- Why?
  Any collision-free path can be transformed into a polygonal path that bends only at the obstacle vertices.



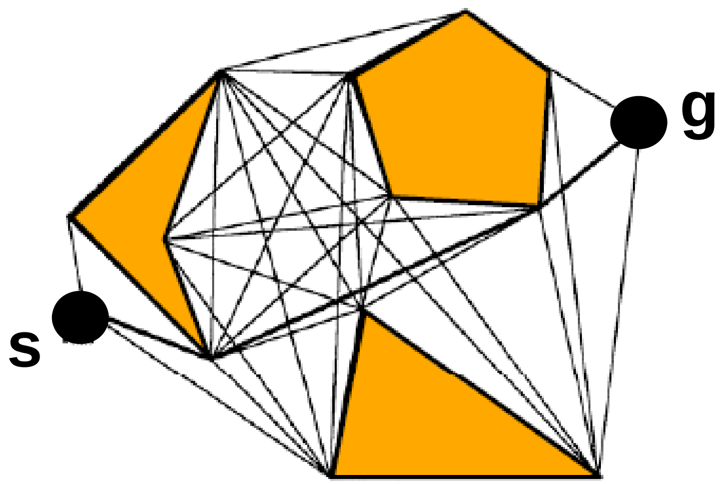
- A polygonal path is a piecewise linear curve.

Visibility Graph



- A **visibility graph** is a graph such that
  - Nodes: s, g, or obstacle vertices
  - Edges: An edge exists between nodes u and v if the line segment between u and v is an obstacle edges or it does not intersect the obstacles

Visibility Graph



- A **visibility graph**
  - Introduced in the late 60s
  - Can produce shortest paths in 2-D configuration spaces

Simple Algorithm

- **Input: s, q, polygonal obstacles**
- **Output: visibility graph G**

```
1: for every pair of nodes u, v
2:   if segment (u, v) is an obstacle edge then
3:     insert edge (u, v) into G;
4:   else
5:     for every obstacle edge e
6:       if segment (u, v) intersects e
7:         go to (1);
8:     insert edge (u, v) into G;
9: Search a path with G using A*
```

Computation Efficiency

```
1: for every pair of nodes u, v                        O(n^2)
2:   if segment (u, v) is an obstacle edge then        O(n)
3:     insert edge (u, v) into G;
4:   else
5:     for every obstacle edge e                       O(n)
6:       if segment (u, v) intersects e
7:         go to (1);
8:     insert edge (u, v) into G;
```

- **Simple algorithm: $O(n^3)$ time**
- **More efficient algorithms**
  - **Rotational sweep $O(n^2 \log n)$ time, etc.**
- **$O(n^2)$ space**

Motion-Planning Framework

Continuous representation

(configuration space formulation)

↓

Discretization

(random sampling, processing critical geometric events)

↓

Graph searching

(blind, best-first, A*)

Graph Search Algorithms

- Breadth, depth-first, best-first
- Dijkstra's algorithm
- A*

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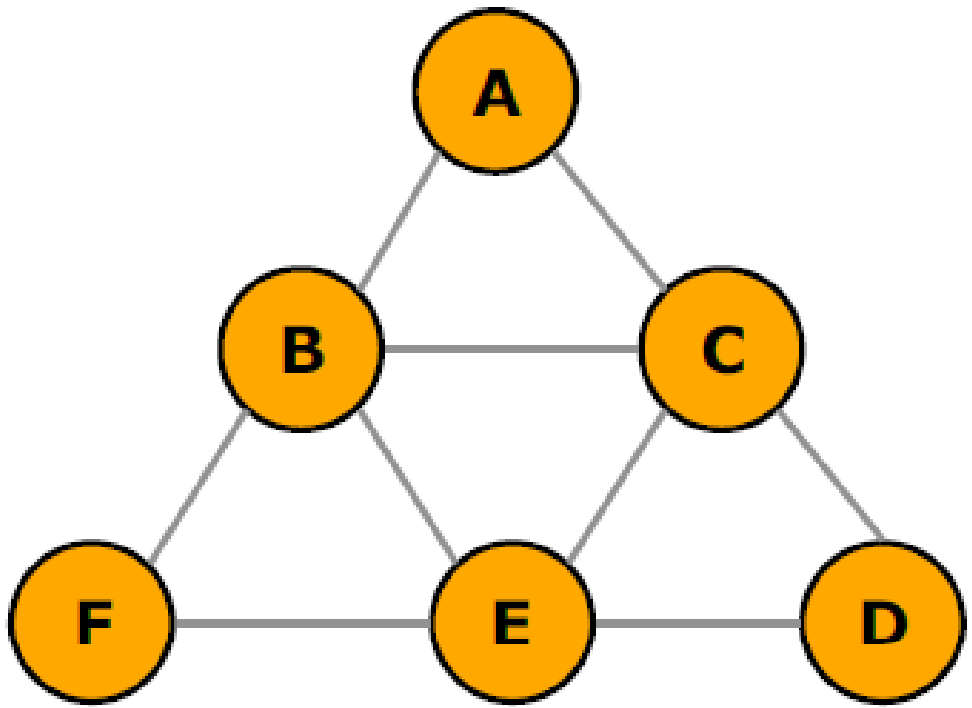
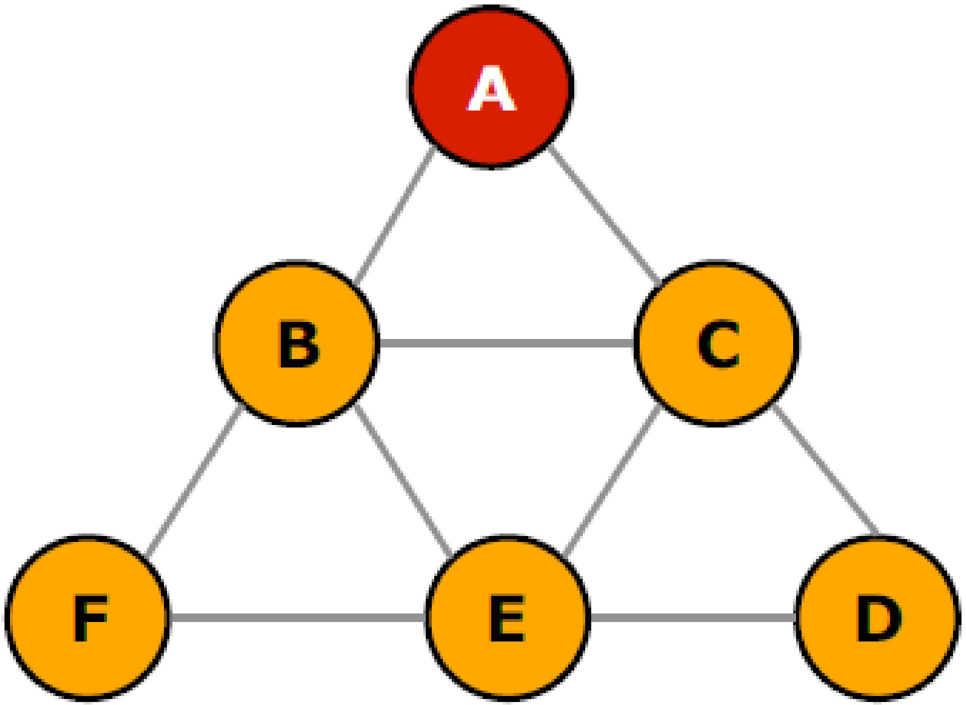
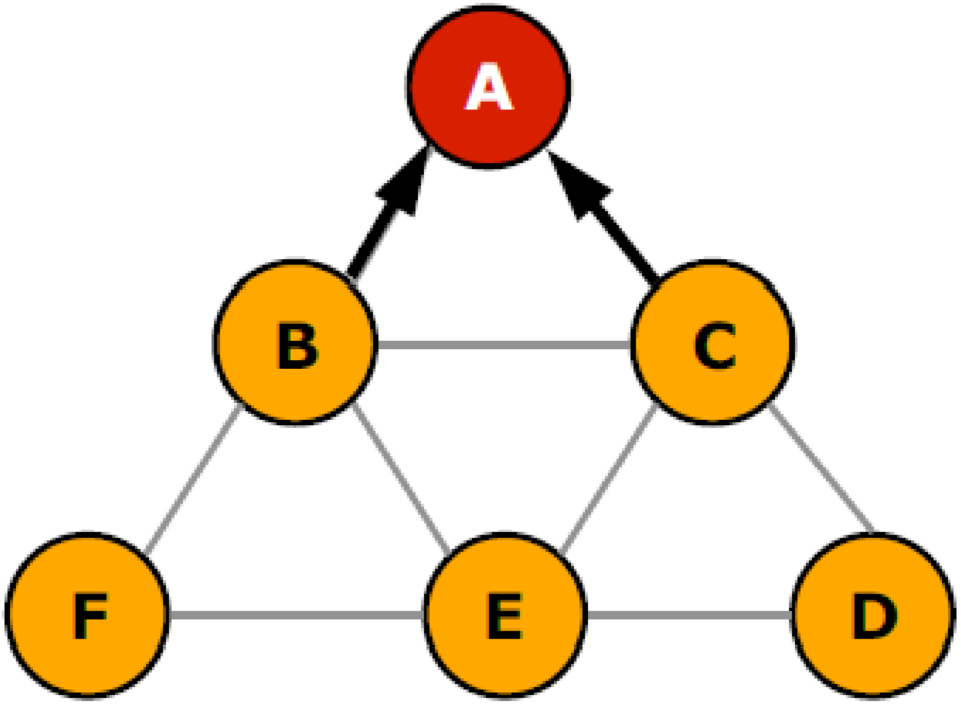
Breadth-first search



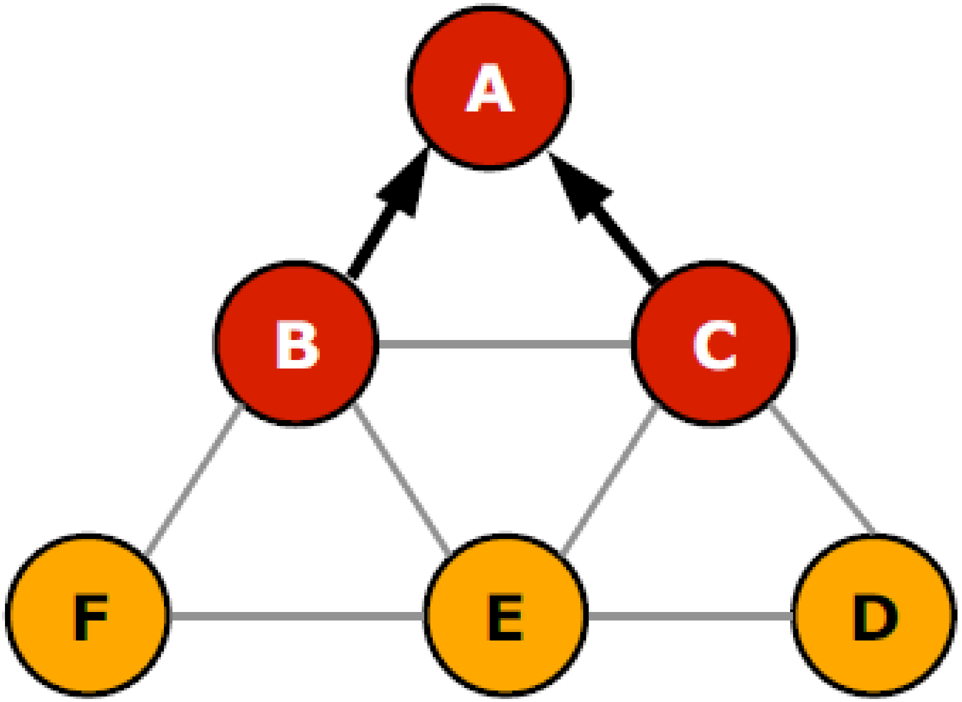
Breadth-first search



Breadth-first search



Breadth-first search



Dijkstra's Shortest Path Algorithm

- Given a (non-negative) weighted graph, two vertices, s and g:
  - Find a path of minimum total weight between them
  - Also, find minimum paths to other vertices
  - Has O (|V| lg|V| + |E|)

Dijkstra's Shortest Path Algorithm

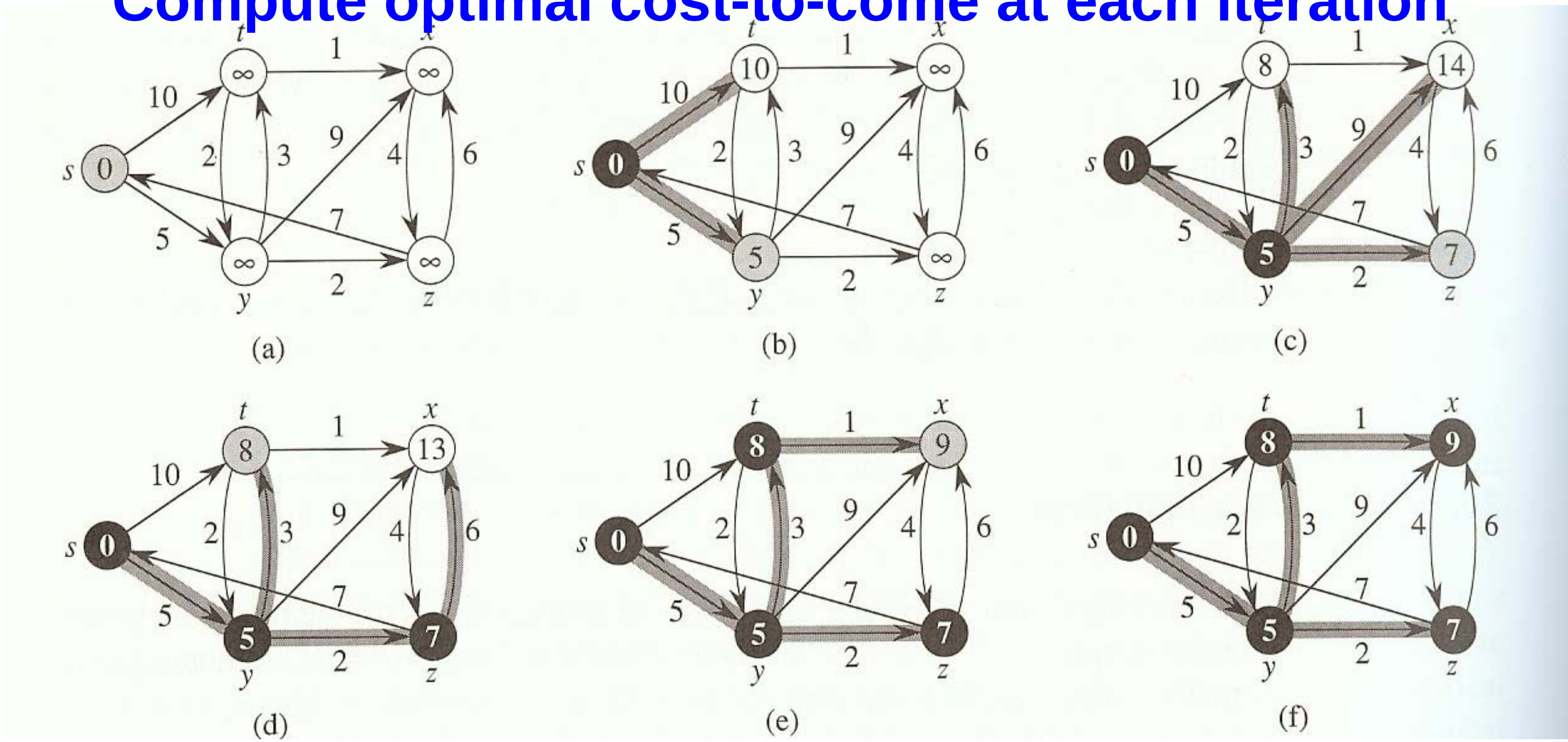
- **A set S**
  - **Contains vertices whose final shortest-path cost has been determined**

- **DIJKSTRA (G, s)**

```
Initialize-Single-Source (G, s)
S ← empty
Queue ← Vertices of G
While Queue is not empty
    Do u ← min-cost from Queue
        S ← union of S  and {u}
        for each vertex v in Adj [u]
            do RELAX (u, v)
```

Dijkstra's Shortest Path Algorithm

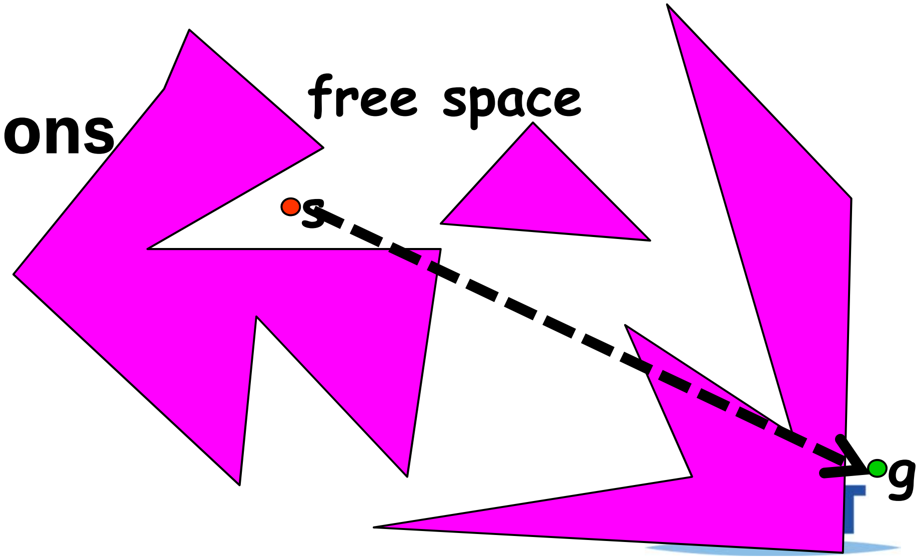
Compute optimal cost-to-come at each iteration



Black vertices are in the set.
White vertices are in the queue.
Shaded one is chosen for relaxation.

A* Search Algorithm

- An extension of Dijkstra's algorithm based on a heuristic estimate
  - Conservatively estimate the cost-to-go from a vertex to the goal
  - The estimate should not be greater than the optimal cost-to-go
  - Sort vertices based on "cost-to-come + the estimated cost-to-go"
  - Can find optimal solutions with fewer steps



Best-First Search

- **Pick a next node based on an estimate of the optimal cost-to-go cost**
  - **Greedily finds solutions that look good**
  - **Solutions may not be optimal**
  - **Can find solutions quite fast, but can be also very slow**

Framework

continuous representation

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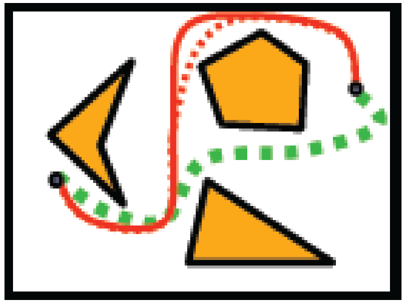
discretization

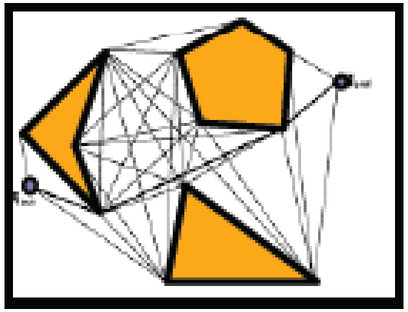
construct visibility graph

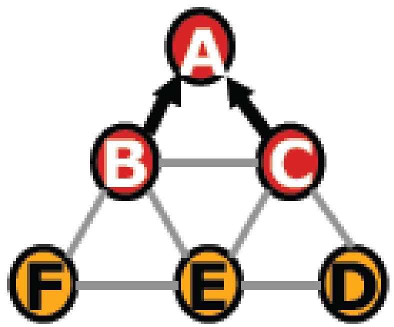
↓

graph searching

breadth-first search







Computational Efficiency

- Running time $O(n^3)$
  - Compute the visibility graph
  - Search the graph
- Space $O(n^2)$

- Can we do better?

Classic Path Planning Approaches

- Roadmap
  - Represent the connectivity of the free space by a network of 1-D curves
- Cell decomposition
  - Decompose the free space into simple cells and represent the connectivity of the free space by the adjacency graph of these cells
- Potential field
  - Define a function over the free space that has a global minimum at the goal configuration and follow its steepest descent

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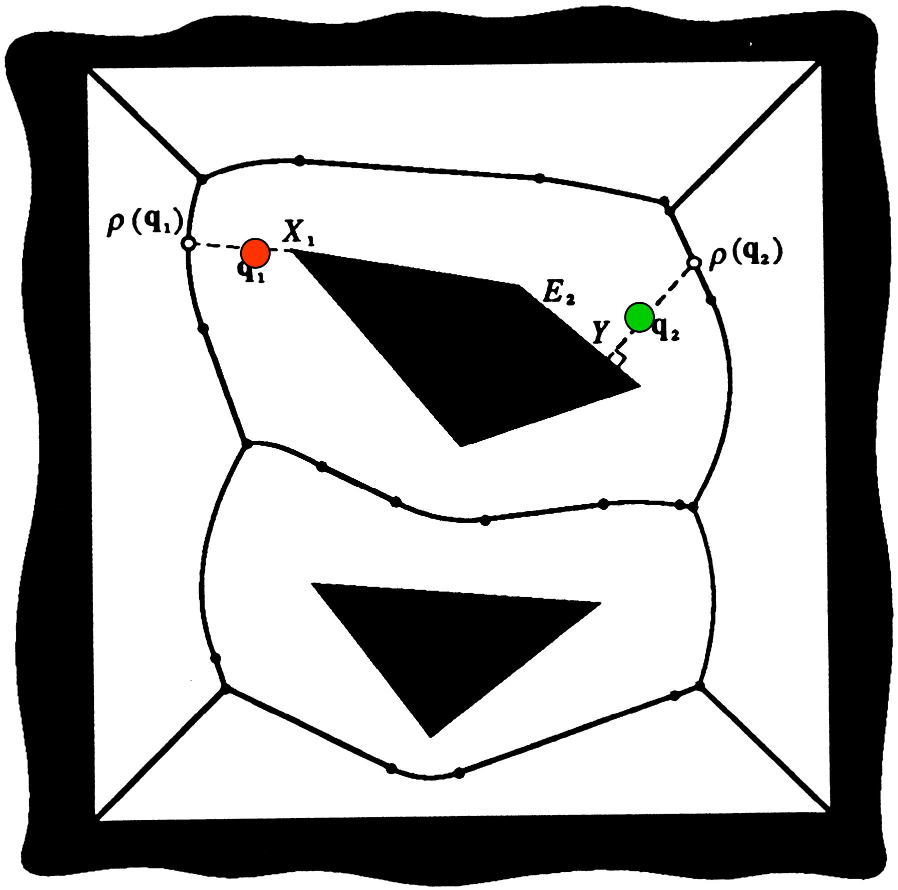
Roadmap Methods

- Visibility Graph
  - Shakey project, SRI [Nilsson 69]
- **Voronoi diagram**
  - **Introduced by computational geometry researchers**
  - **Generate paths that maximize clearance**
  - **O(n log n) time and O(n) space**



Other Roadmap Methods

- Visibility graph
- Voronoi diagram
- Silhouette
  - First complete general method that applies to spaces of any dimension and is singly exponential in # of dimensions [Canny, 87]
- Probabilistic roadmaps

Classic Path Planning Approaches

- Roadmap
  - Represent the connectivity of the free space by a network of 1-D curves
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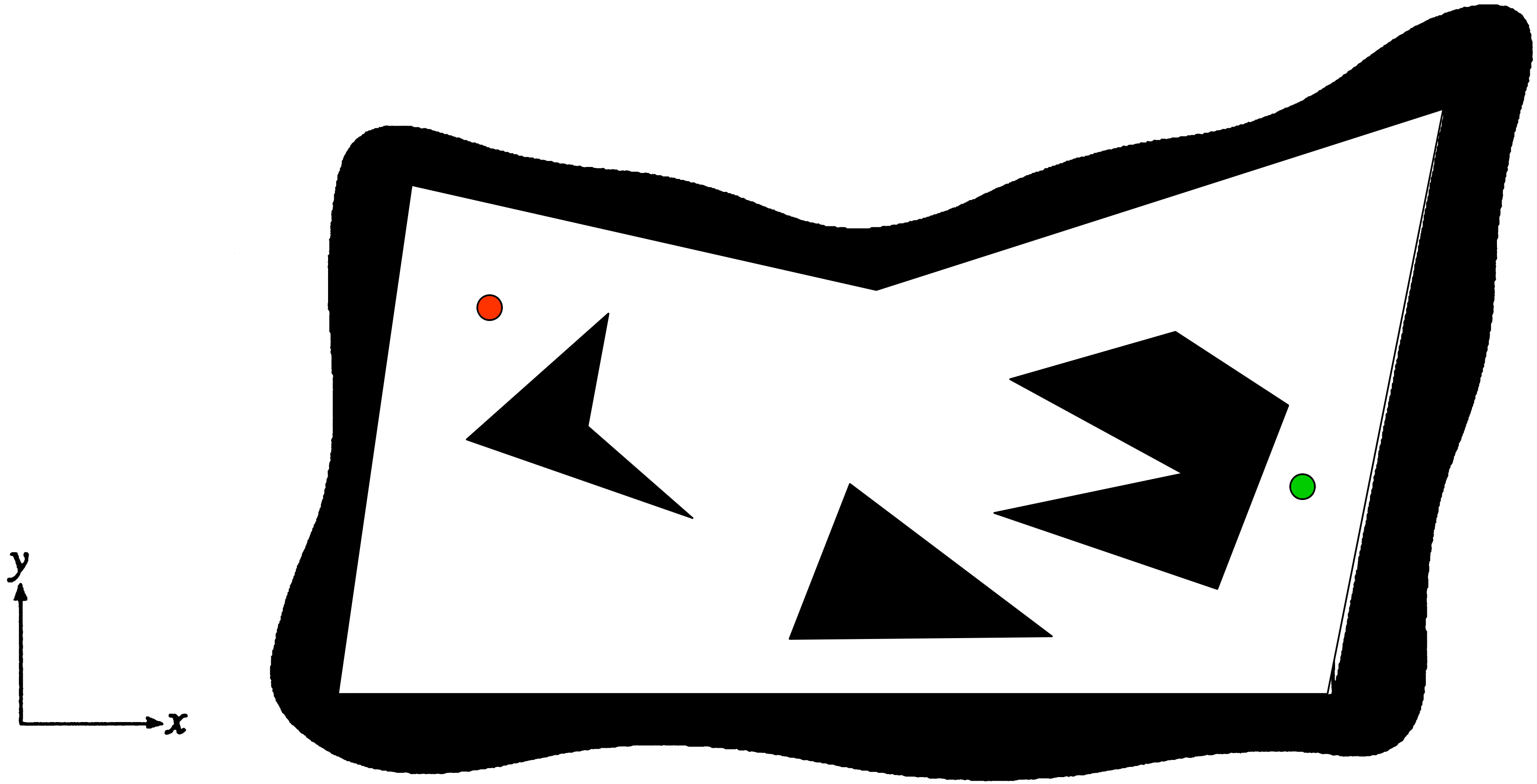
Cell-Decomposition Methods

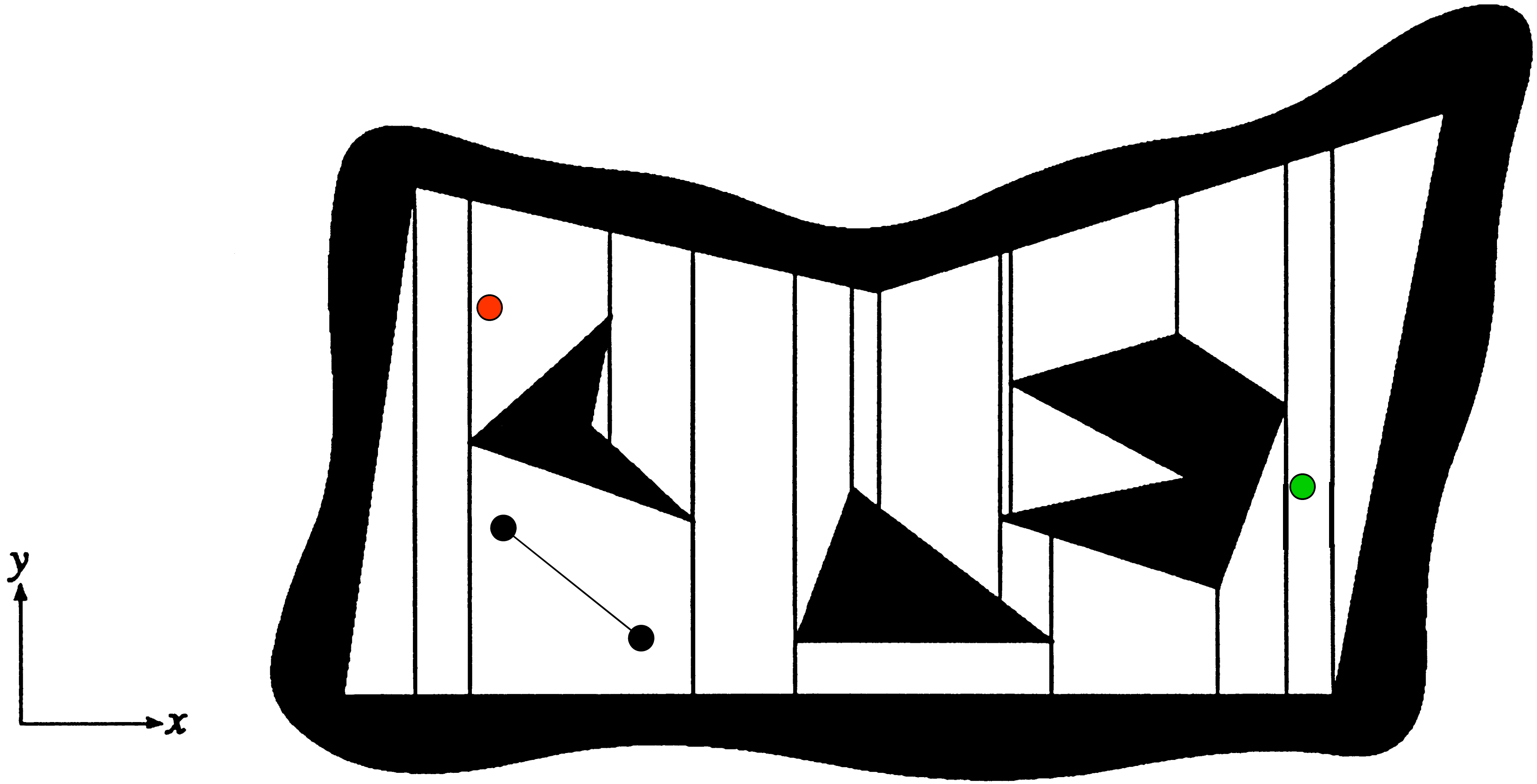
- Two classes of methods:
  - Exact and approximate cell decompositions

- Exact cell decomposition
  - The free space F is represented by a collection of non-overlapping cells whose union is exactly F
  - Example: trapezoidal decomposition

Trapezoidal Decomposition

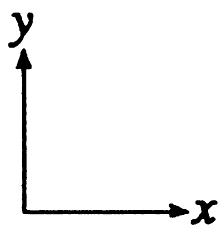


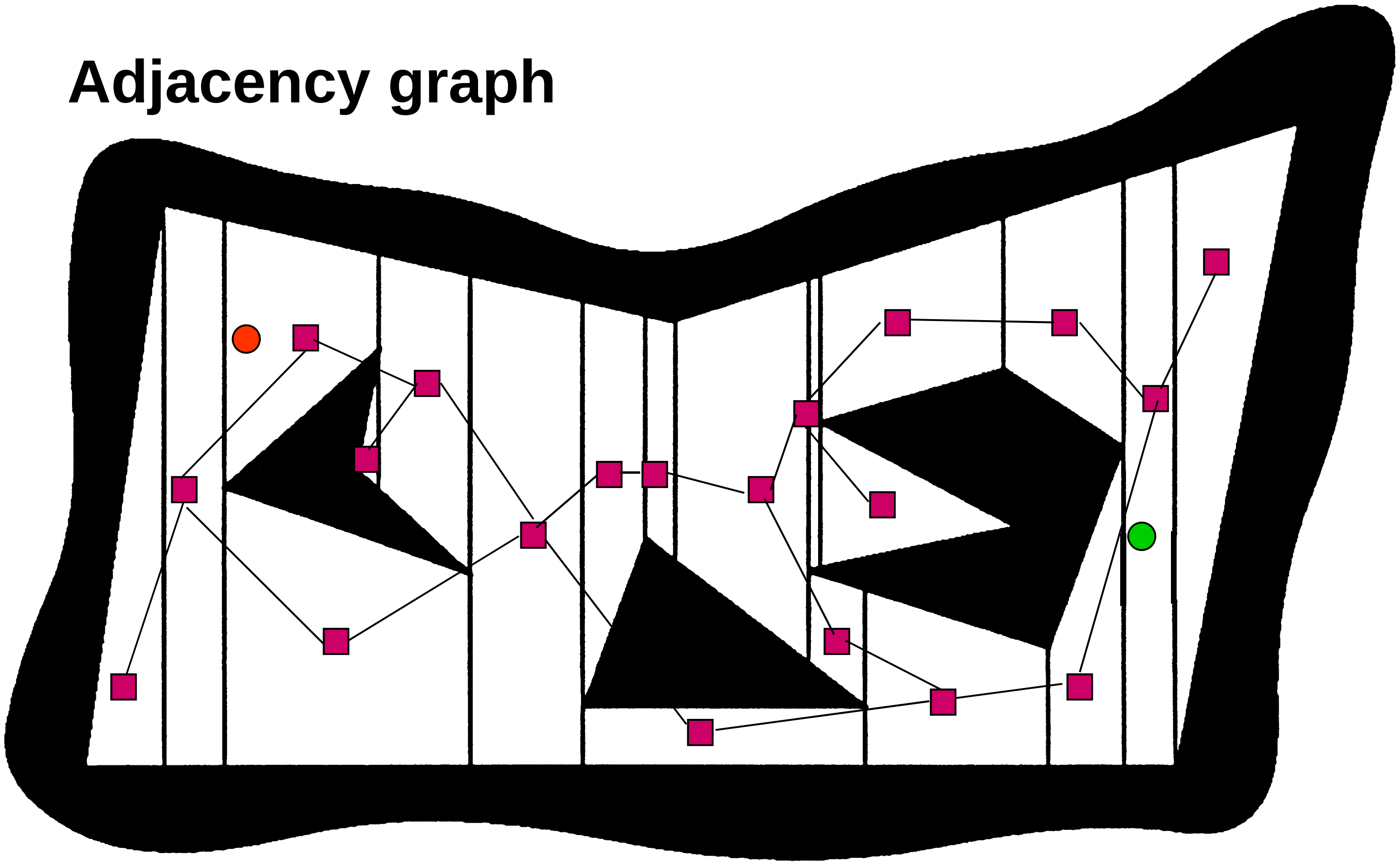
Trapezoidal Decomposition



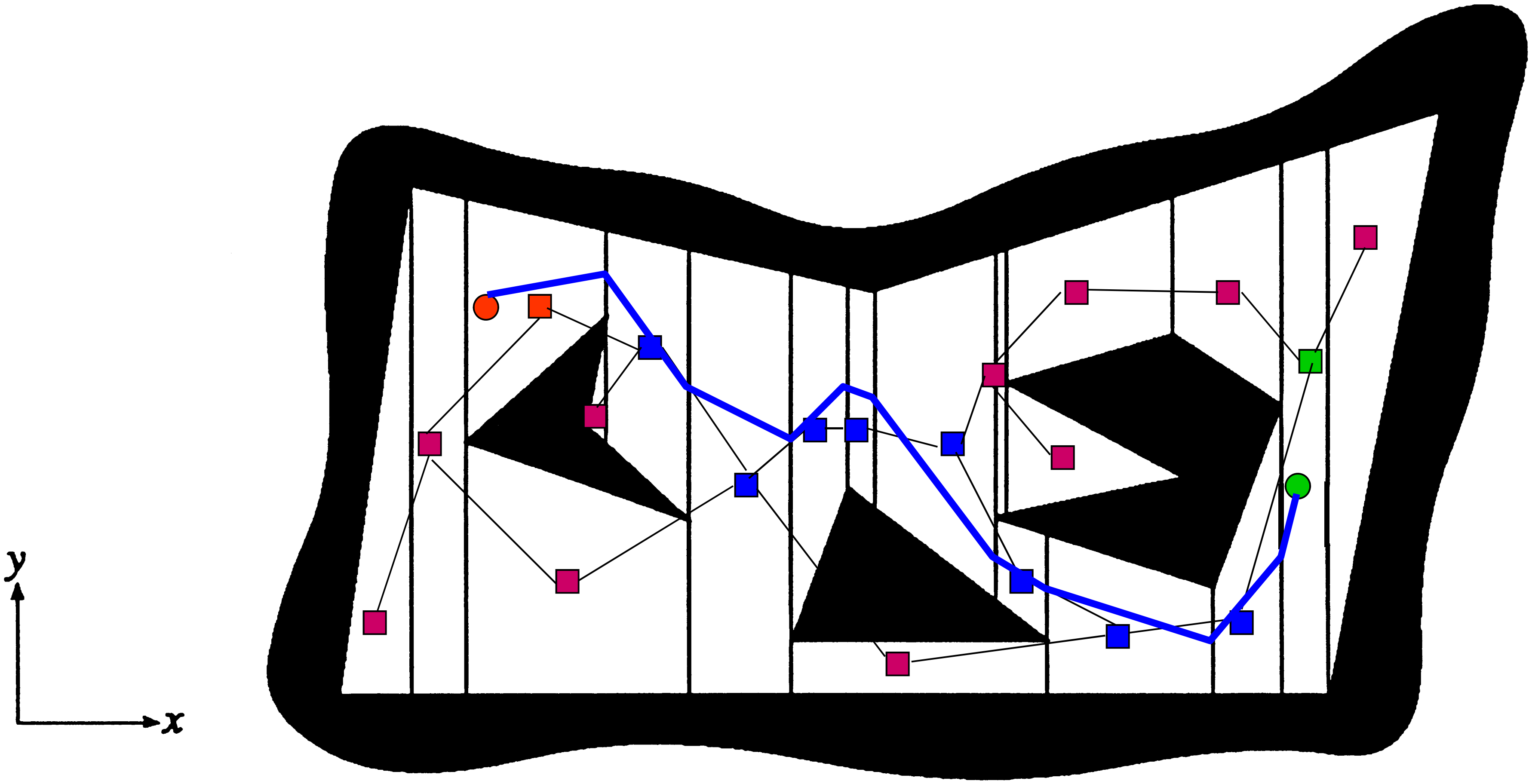
Trapezoidal Decomposition

Adjacency graph

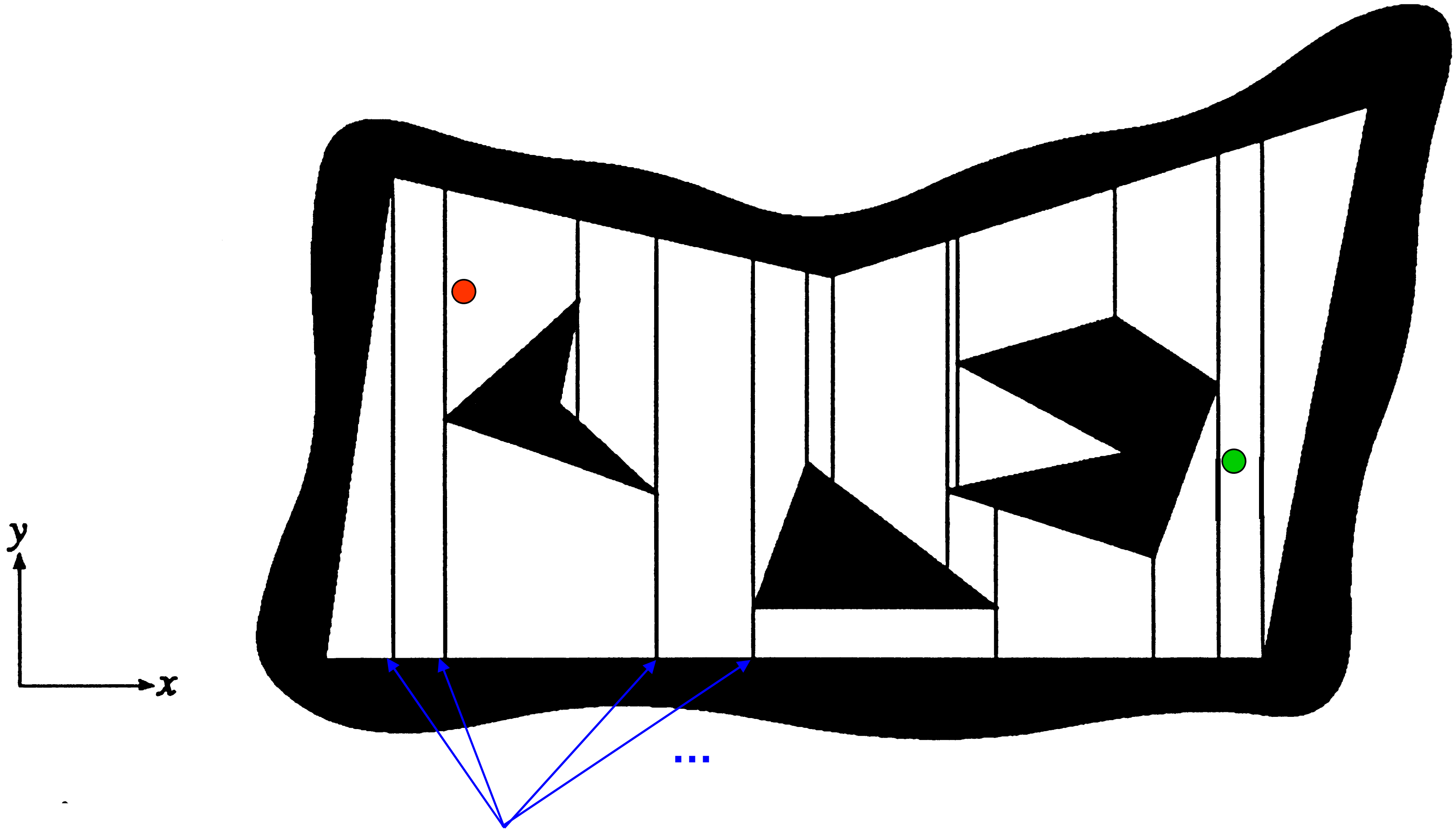




Trapezoidal Decomposition

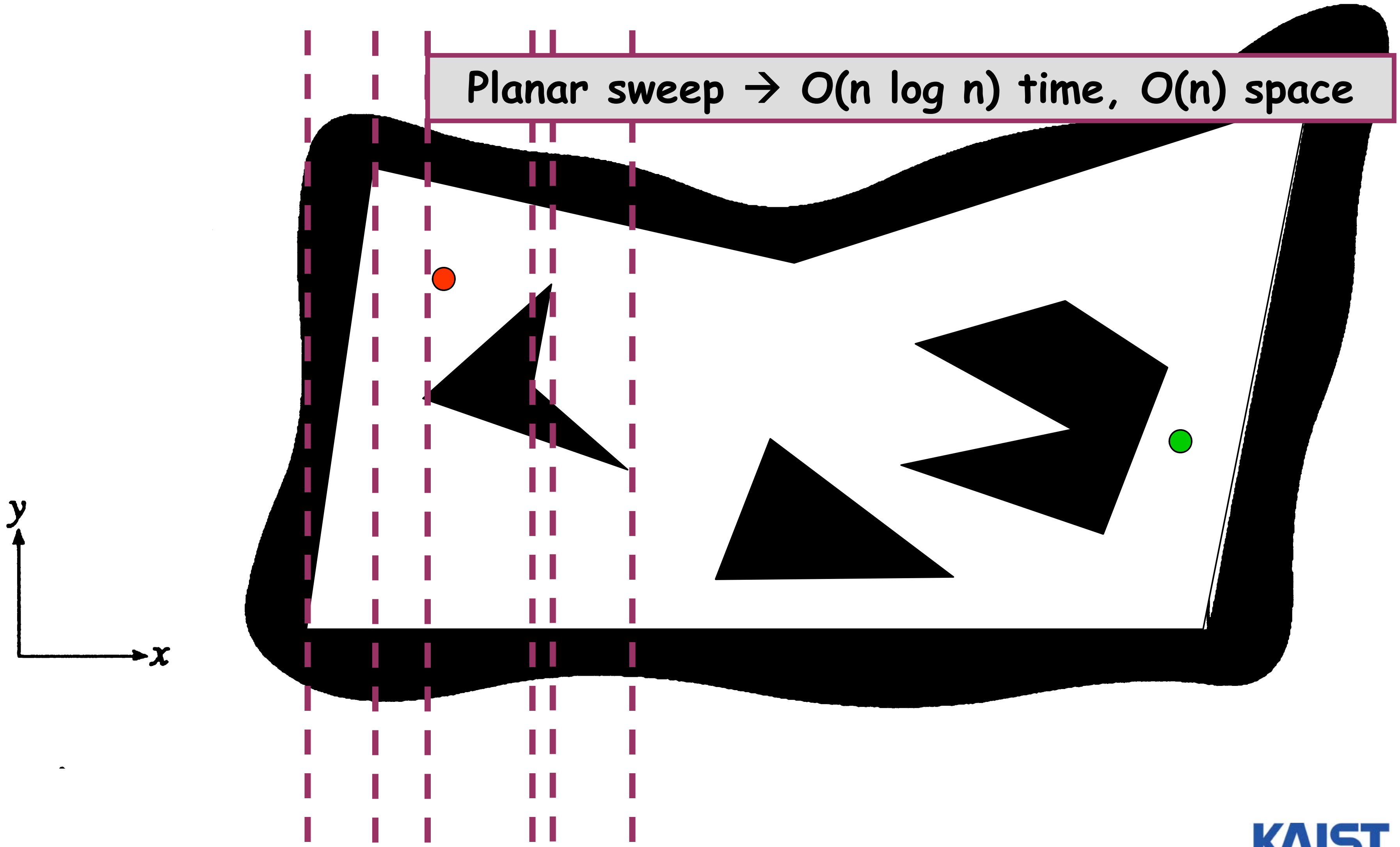


Trapezoidal Decomposition



critical events → criticality-based decomposition

Trapezoidal Decomposition

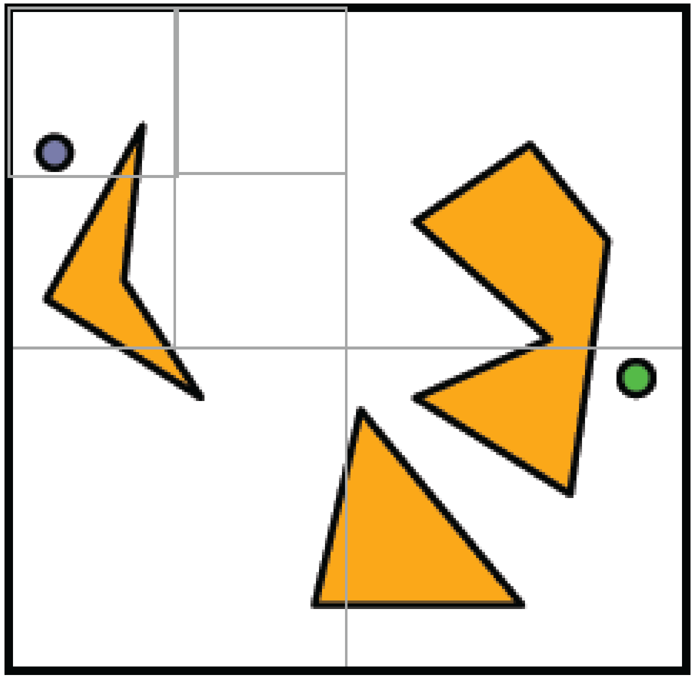


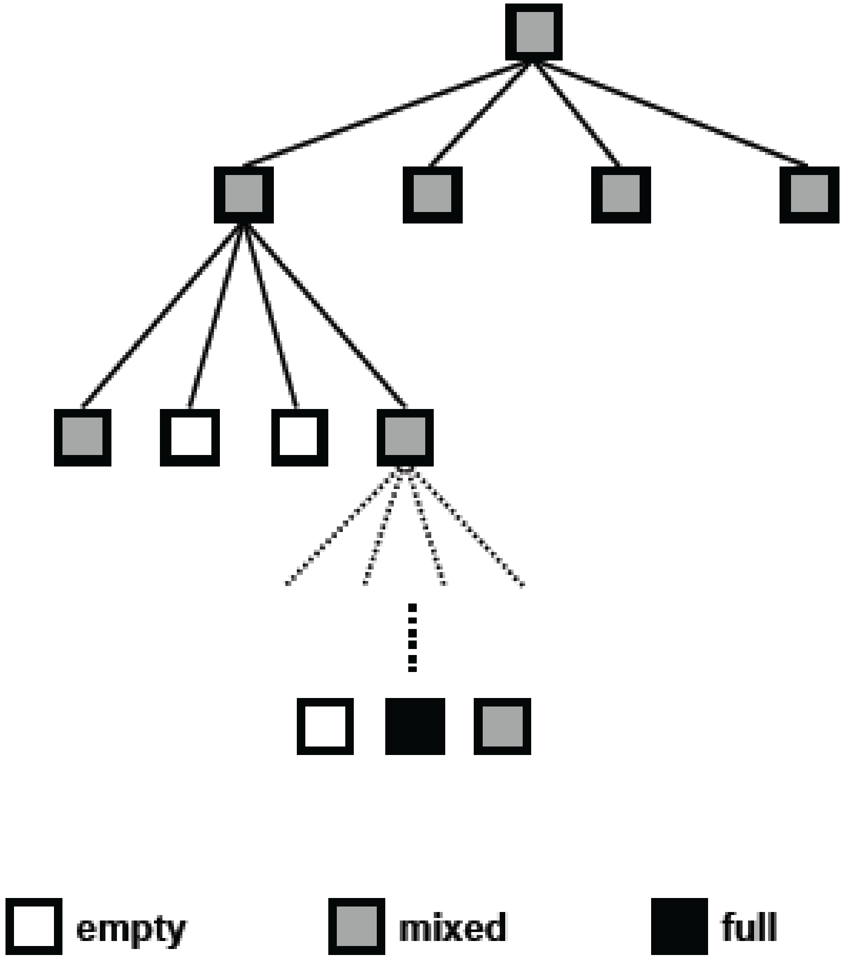
Cell-Decomposition Methods

- **Two classes of methods:**
  - **Exact and approximate cell decompositions**

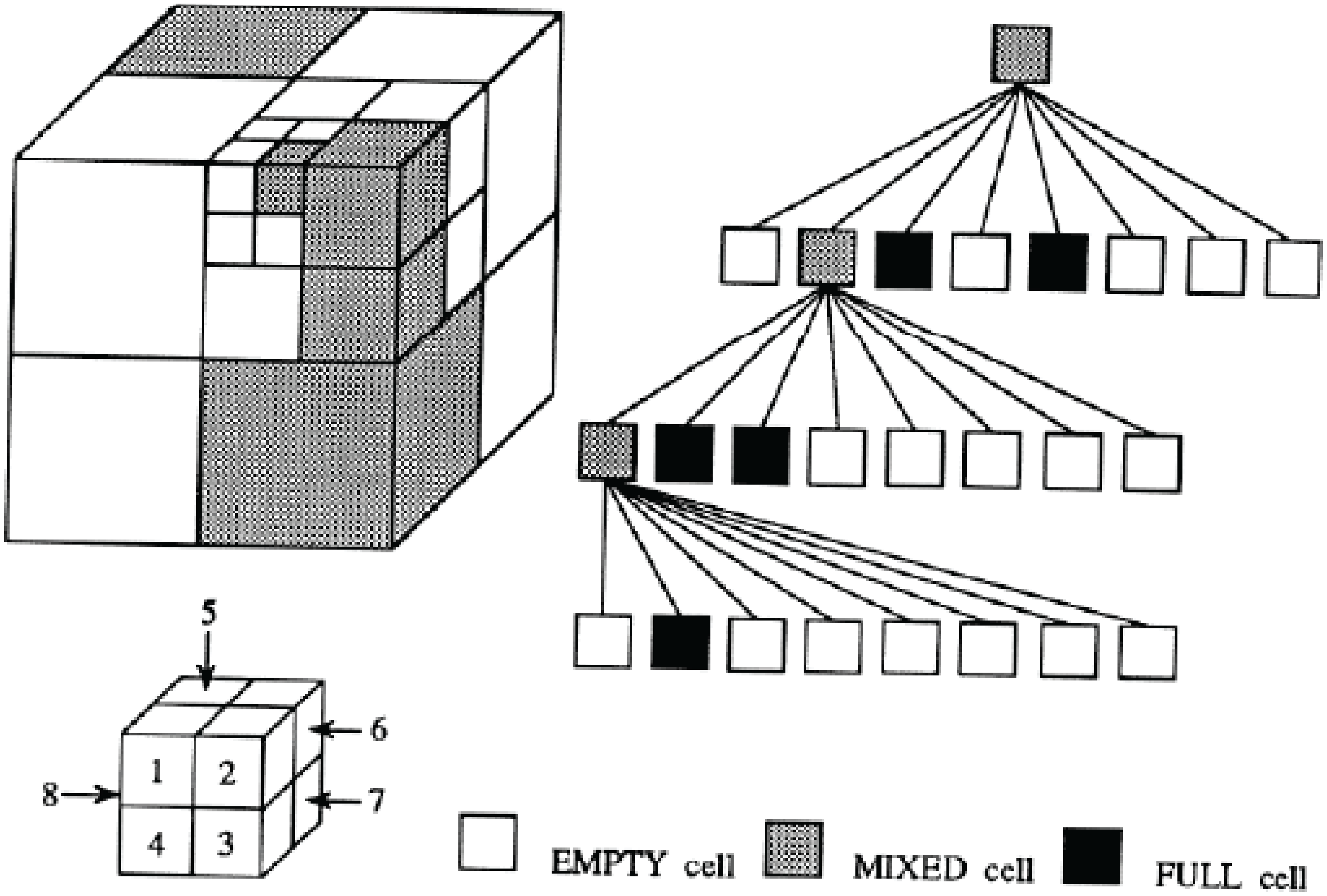
- **Approximate cell decomposition**
  - **The free space F is represented by a collection of non-overlapping cells whose union is contained in F**
  - **Cells usually have simple, regular shapes (e.g., rectangles and squares)**
  - **Facilitates hierarchical space decomposition**

Quadtree decomposition





Octree decomposition



Sketch of Algorithm

1. Decompose the free space F into cells
2. Search for a sequence of mixed or free cells that connect that initial and goal positions
3. Further decompose the mixed
4. Repeat 2 and 3 until a sequence of free cells is found

Classic Path Planning Approaches

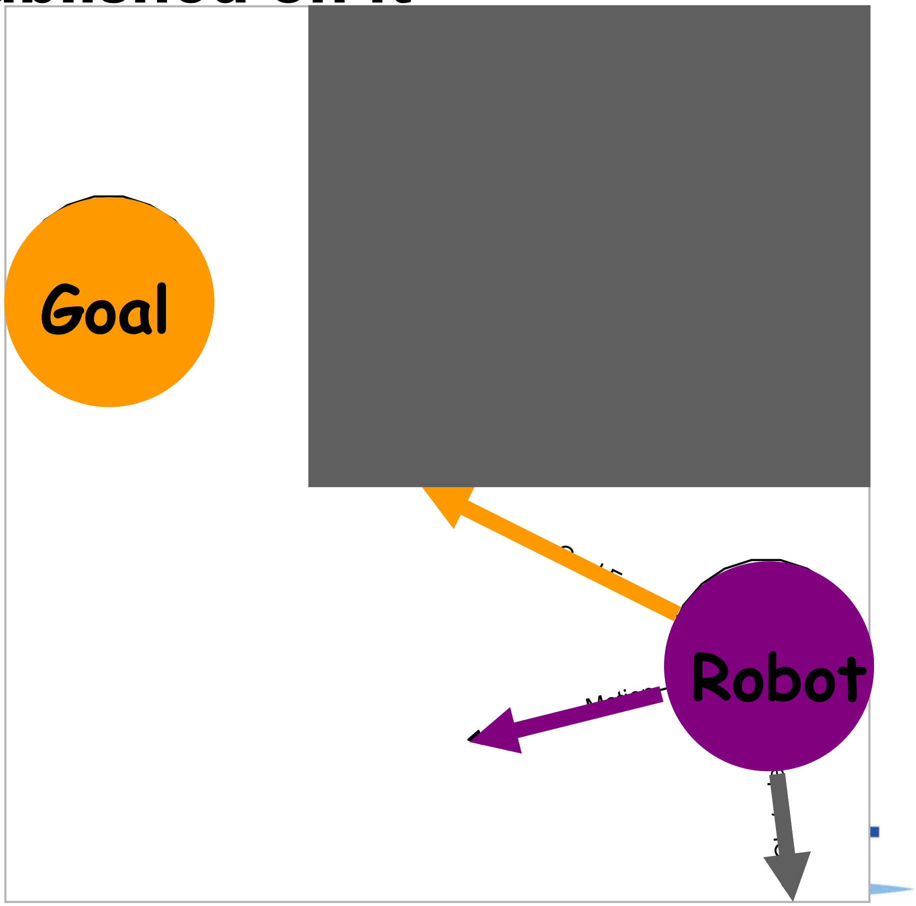
- Roadmap
  - Represent the connectivity of the free space by a network of 1-D curves
- Cell decomposition
  - Decompose the free space into simple cells and represent the connectivity of the free space by the adjacency graph of these cells
- Potential field
  - Define a function over the free space that has a global minimum at the goal configuration and follow its steepest descent

Potential Field Methods

- **Initially proposed for real-time collision avoidance [Khatib, 86]**
  - **Hundreds of papers published on it**

$$F_{Goal} = -k_p (x - x_{Goal})$$

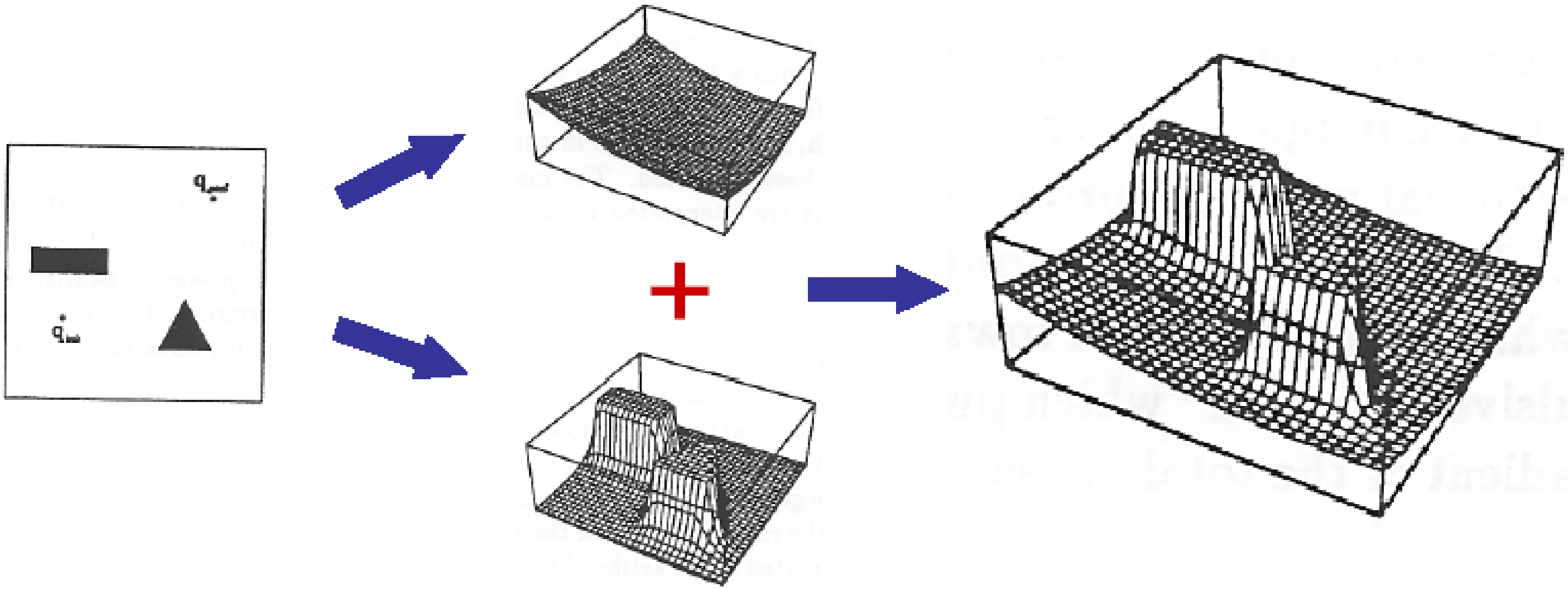
$$F_{Obstacle} = \begin{cases} \eta \left(\frac{1}{\rho} - \frac{1}{\rho_0} \right) \frac{1}{\rho^2} \frac{\partial \rho}{\partial x} & \text{if } \rho \le \rho_0, \\ 0 & \text{if } \rho > \rho_0 \end{cases}$$



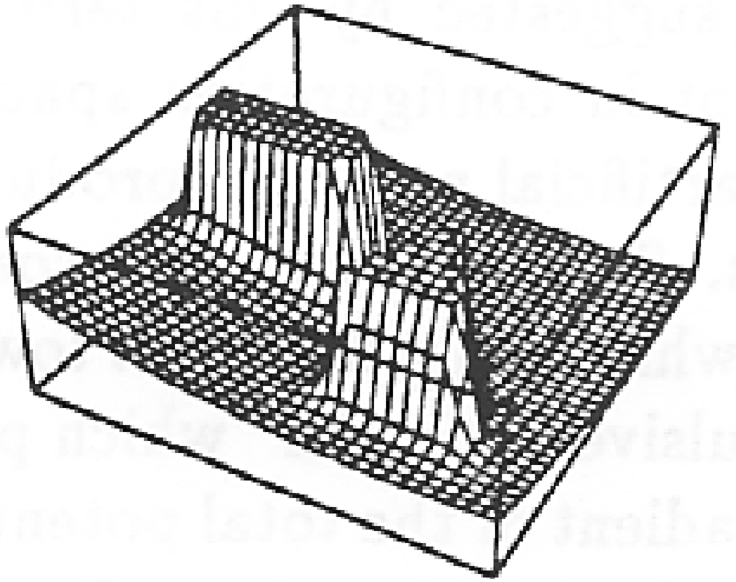
Potential Field

- A scalar function over the free space
- To navigate the robot applies a force proportional to the negated gradient of the potential field
- A navigation function is an ideal potential field that
  - Has global minimum at the goal
  - Has no local minima
  - Grows to infinity near obstacles
  - Is smooth

Attractive and Repulsive fields



Local Minima



- **What can we do?**
  - **Escape from local minima by taking random walks**
  - **Build an ideal potential field that does not have local minima**

Sketch of Algorithm

- Place a regular grid G over the configuration space
- Compute the potential field over G
- Search G using a best-first algorithm with potential field as the heuristic function

Question

- **Can such an ideal potential field be constructed efficiently in general?**

Completeness

- A complete motion planner always returns a solution when one exists and indicates that no such solution exists otherwise
  - Is the visibility algorithm complete? Yes
  - How about the exact cell decomposition algorithm and the potential field algorithm?

Class Objectives were:

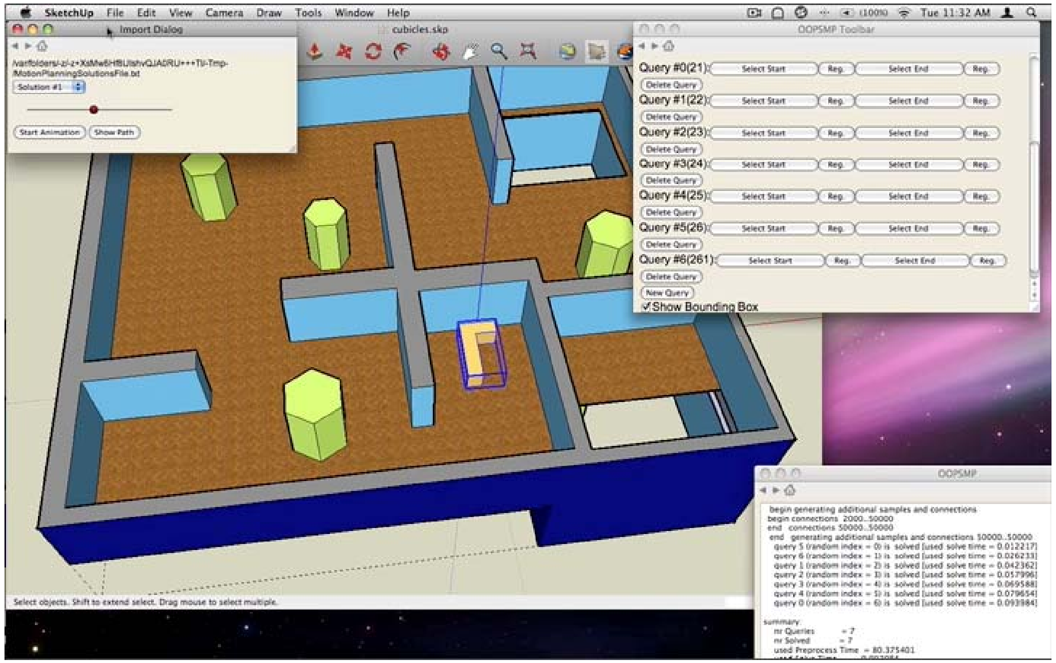
- Motion planning framework
- Classic motion planning approaches

Homework for Every Class

- Go over the next lecture slides
- Come up with one question on what we have discussed today and submit at the beginning of the next class
    - 0 for no questions
    - 2 for typical questions
    - 3 for questions with thoughts
    - 4 for questions that surprised me

Homework

- Install Open Motion Planning Library (OMPL)
- Create a scene and a robot
- Find a collision-free path and visualize the path



Conf. Deadline

- ICRA
  - Sep., 2011
- IROS
  - Mar., 2012

Next Time….

- Configuration spaces