Configuration Space II

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Class Objectives

- **Configuration space**
  - **Definitions and examples**
  - **Obstacles**
  - **Paths**
  - **Metrics**

Configuration Space

- Definitions and examples
- Obstacles
- Paths
- Metrics

Obstacles in the Configuration Space

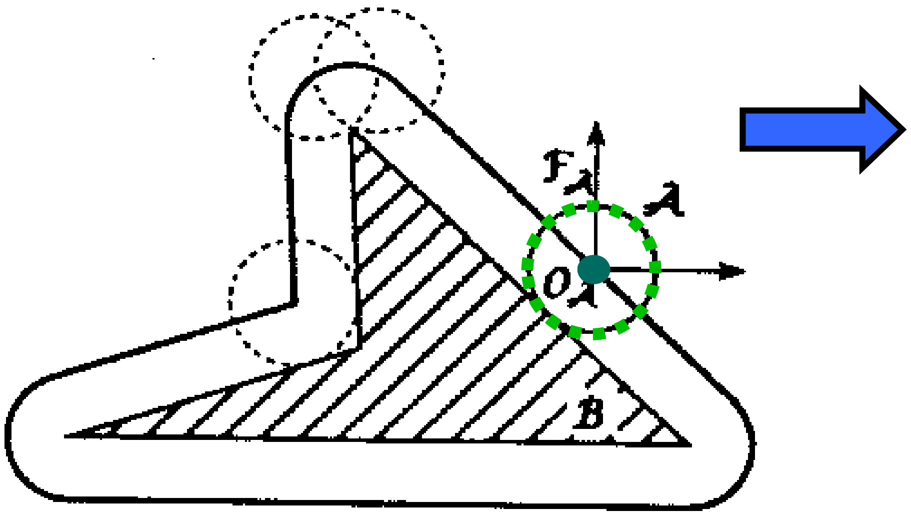
- **A configuration q is collision-free, or free, if a moving object placed at q does not intersect any obstacles in the workspace**

- **The free space F is the set of free configurations**

- **A configuration space obstacle (C-obstacle) is the set of configurations where the moving object collides with workspace obstacles**

Disc in 2-D Workspace

workspace

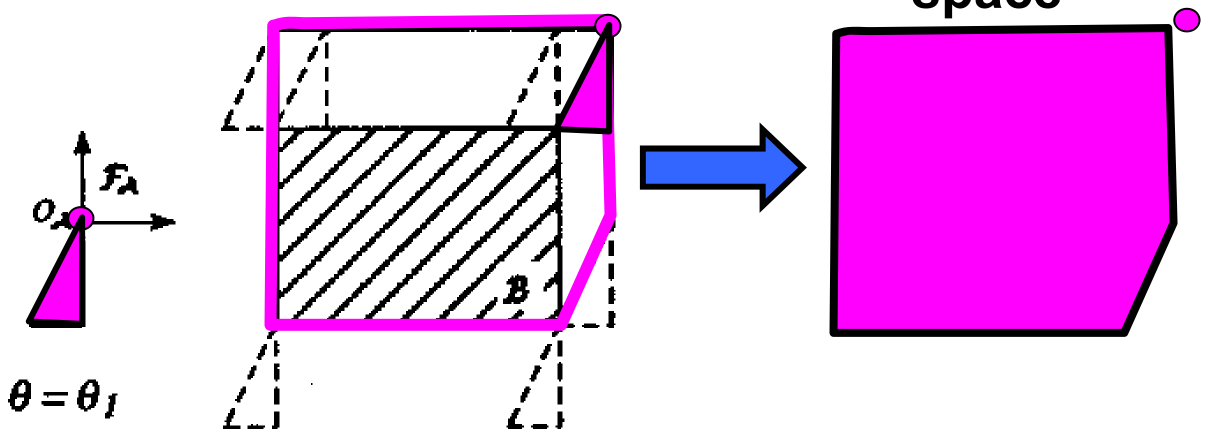


configuration space

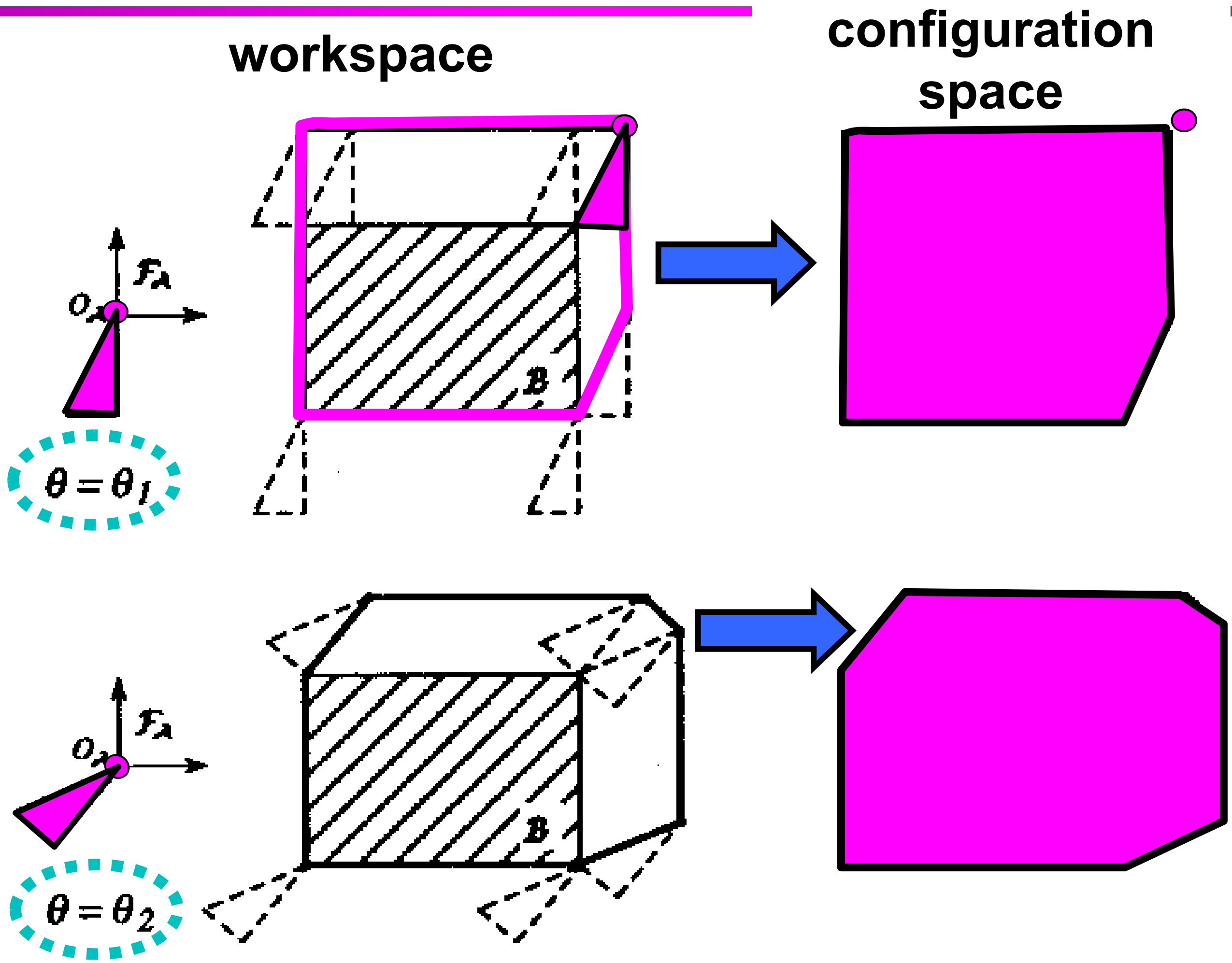
Polygonal Robot Translating in 2-D Workspace

workspace

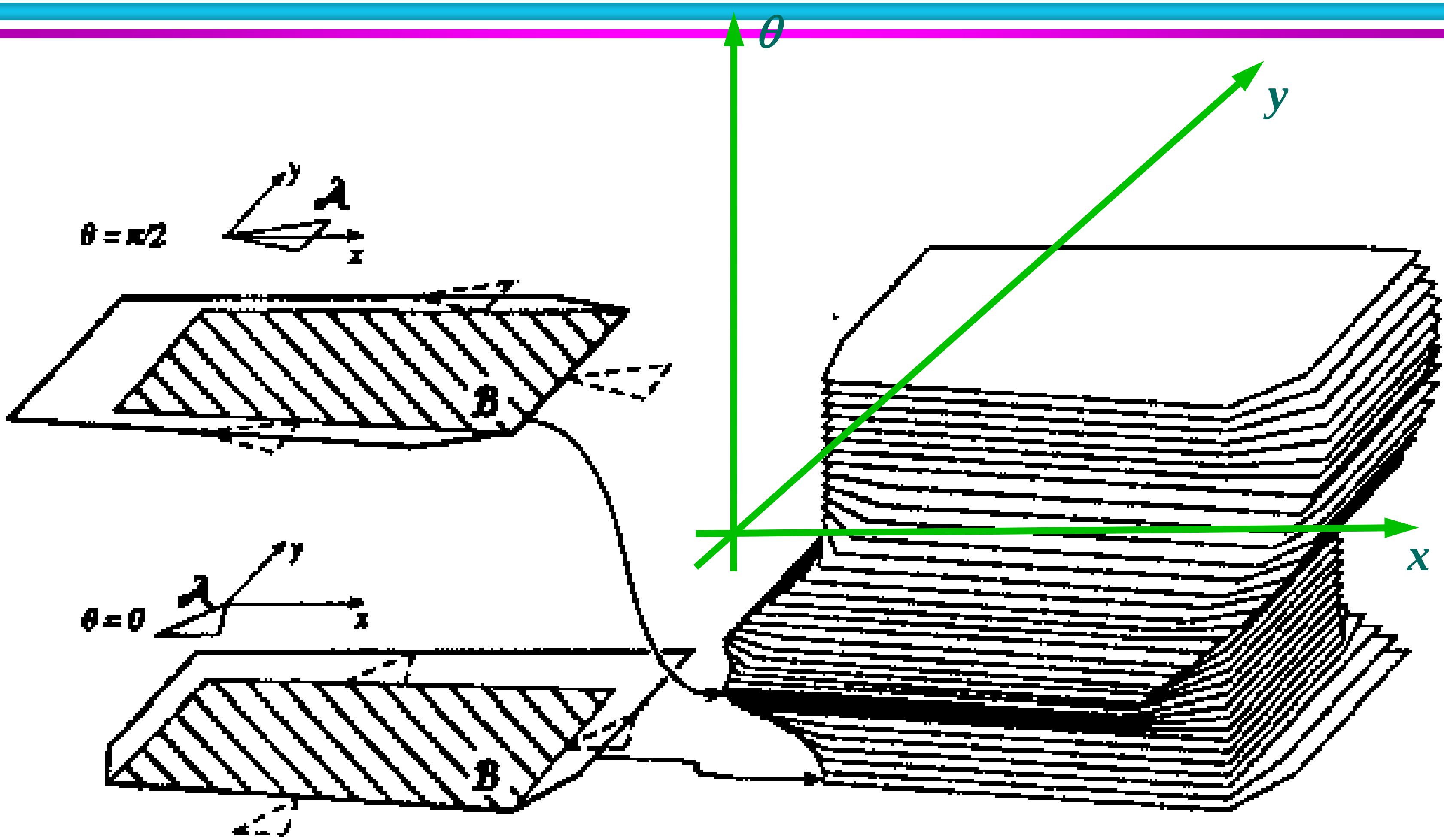
configuration space



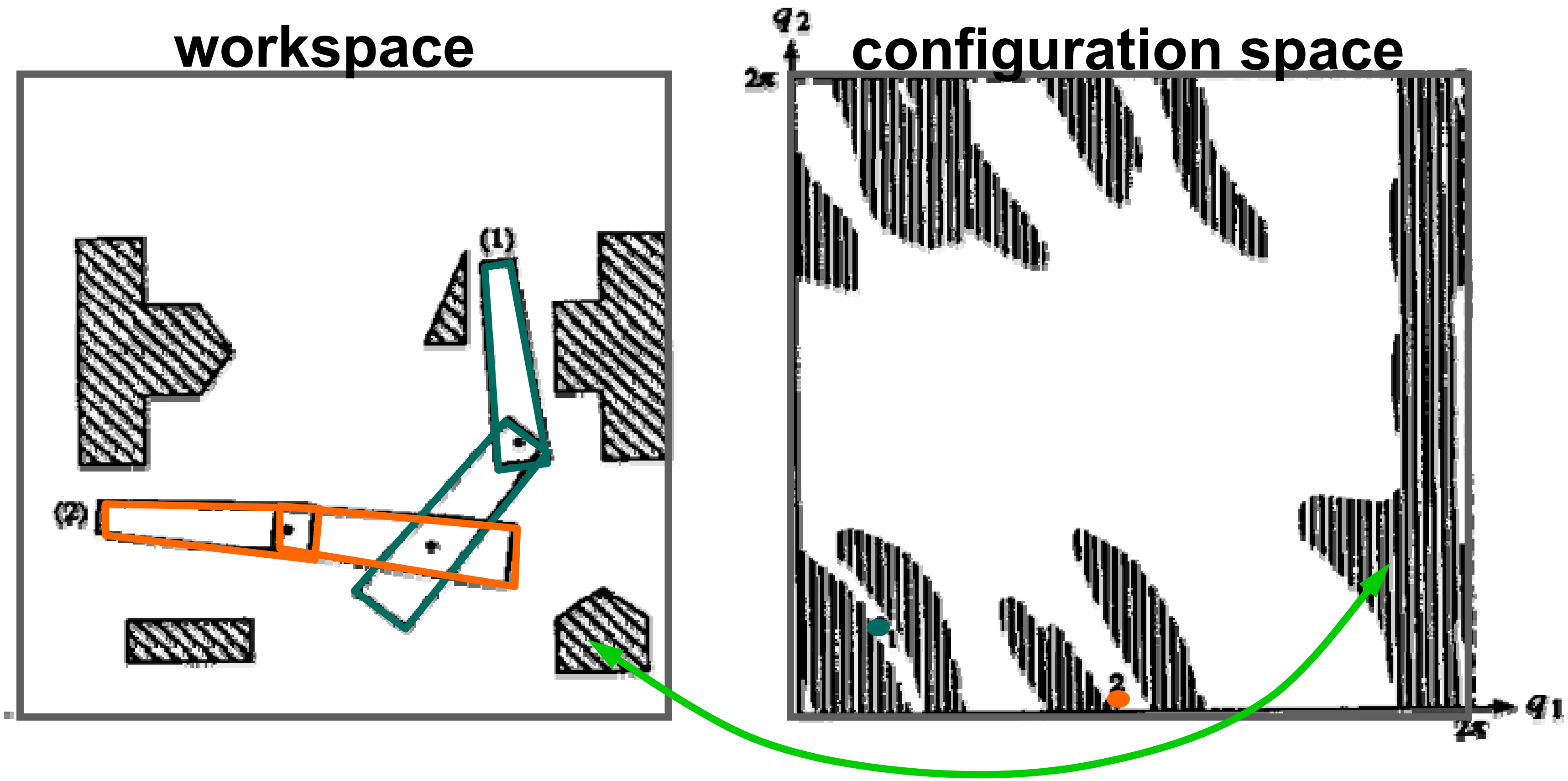
Polygonal Robot Translating & Rotating in 2-D Workspace



Polygonal Robot Translating & Rotating in 2-D Workspace



Articulated Robot in 2-D Workspace



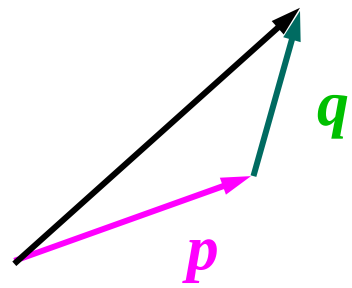
C-Obstacle Construction

- Input:
  - Polygonal moving object translating in 2-D workspace
  - Polygonal obstacles
- Output: configuration space obstacles represented as polygons

Minkowski Sum

- The **Minkowski sum** of two sets P and Q, denoted by $P \oplus Q$, is defined as

$$P \oplus Q = \{ p+q \mid p \in P, q \in Q \}$$



- Similarly, the **Minkowski difference** is defined as

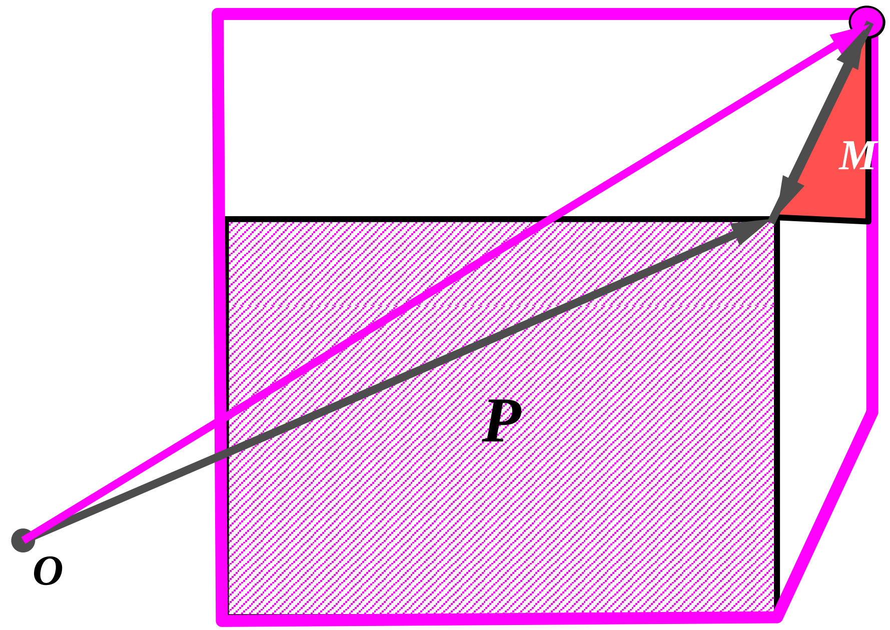
$$P \ominus Q = \{ p-q \mid p \in P, q \in Q \}$$
$$= P \oplus -Q$$

Minkowski Sum of Convex polygons

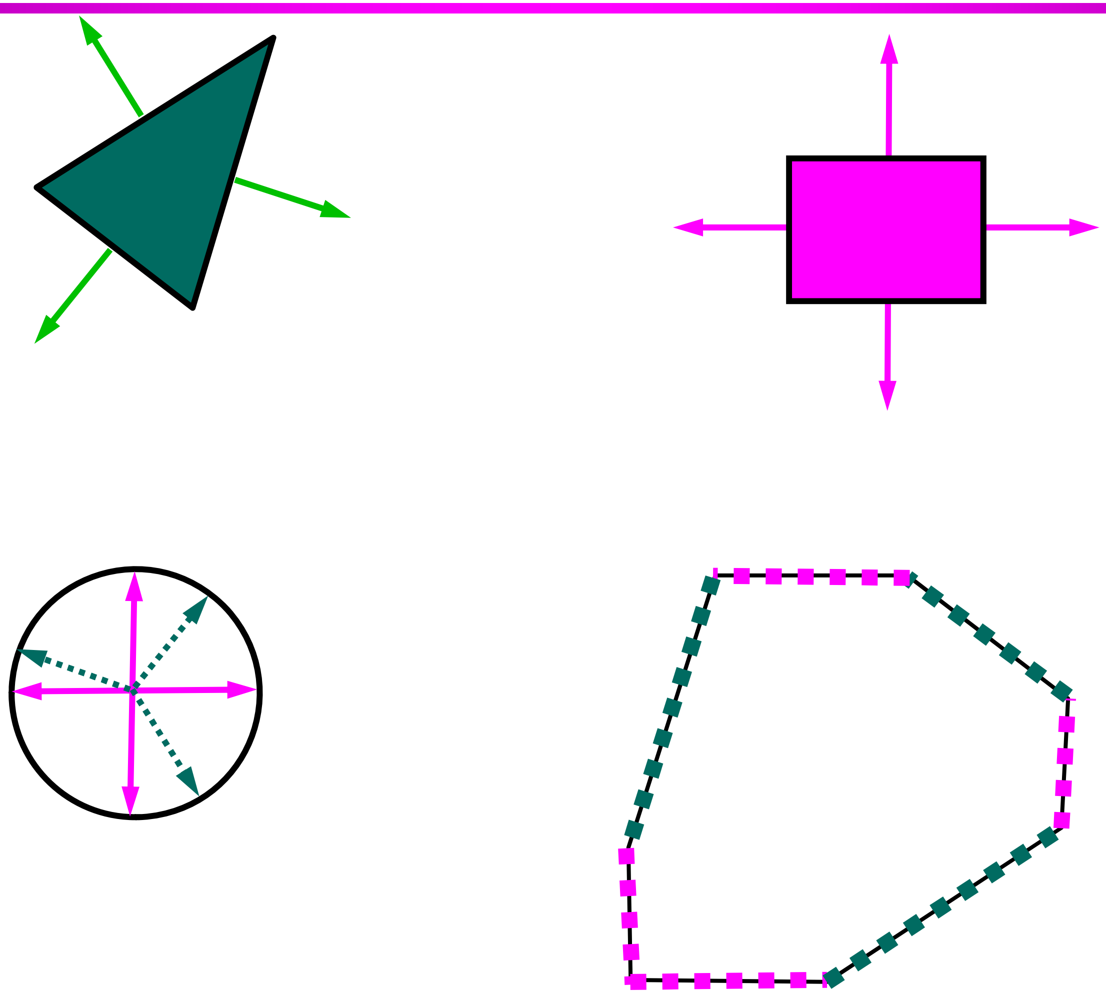
- The Minkowski sum of two convex polygons P and Q of m and n vertices respectively is a convex polygon $P \oplus Q$ of $m + n$ vertices.
  - The vertices of $P \oplus Q$ are the “sums” of vertices of P and Q.

Observation

- If P is an obstacle in the workspace and M is a moving object. Then the C-space obstacle corresponding to P is $P \ominus M$.



Computing C-obstacles



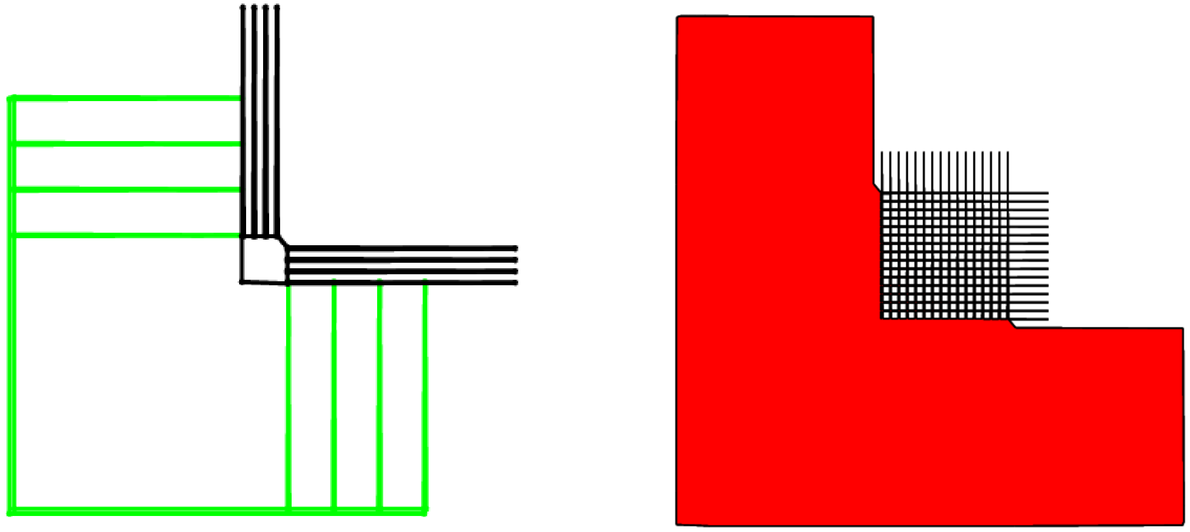
Computational efficiency

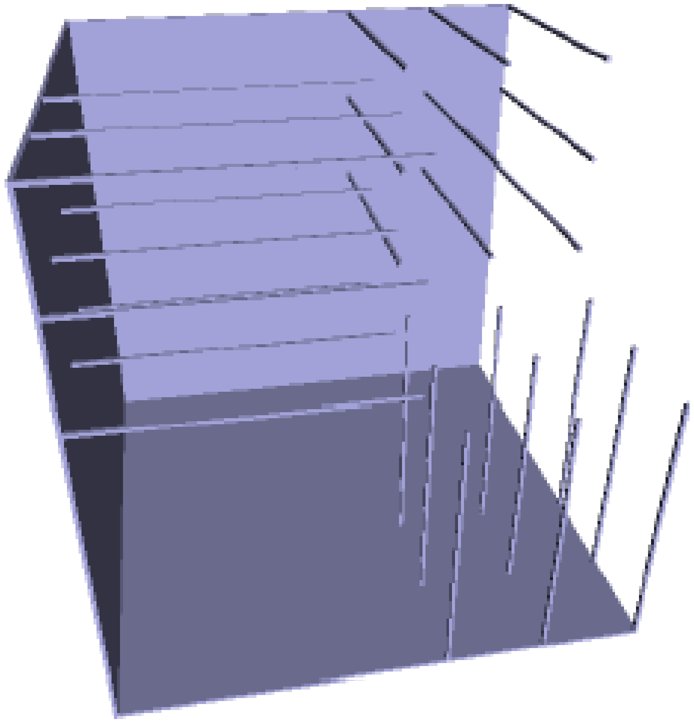
- **Running time** $O(n+m)$
- **Space** $O(n+m)$
- **Non-convex obstacles**
  - **Decompose into convex polygons (*e.g.,* triangles or trapezoids), compute the Minkowski sums, and take the union**
  - **Complexity of Minkowksi sum** $O(n^2m^2)$
- **3-D workspace**
  - **Convex case:** $O(nm)$
  - **Non-convex case:** $O(n^3m^3)$

Worst case example

- $O(n^2m^2)$ **complexity**

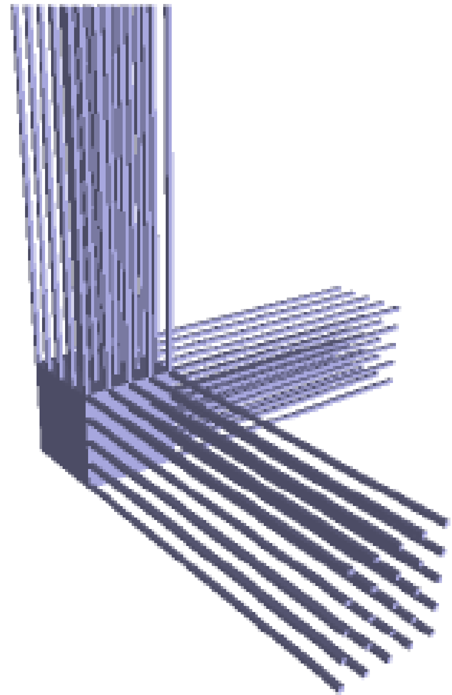
2D example
Agarwal et al. 02



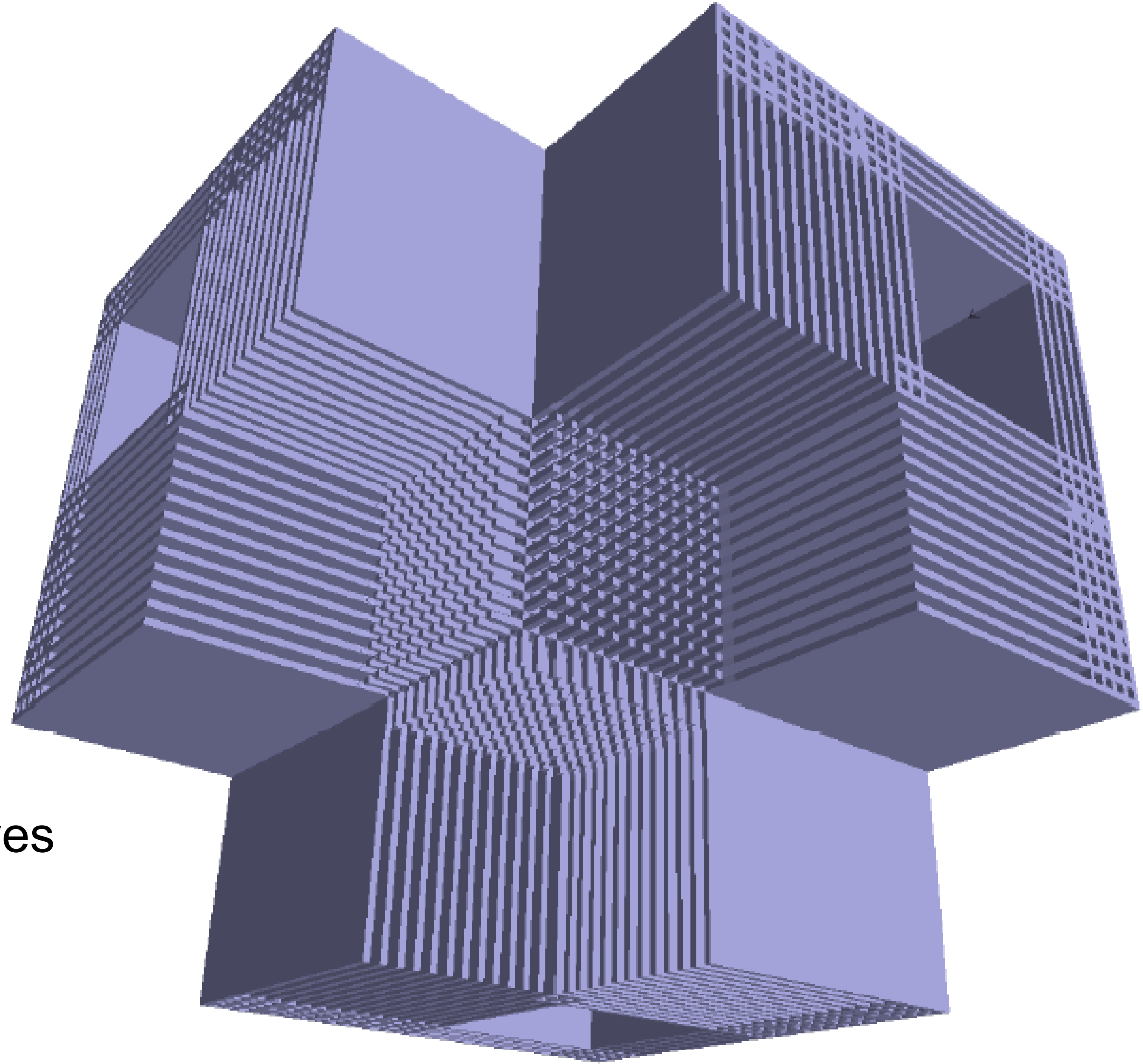


444 tris

⊕



1,134 tris



Union of
66,667 primitives

Configuration space

- Definitions and examples
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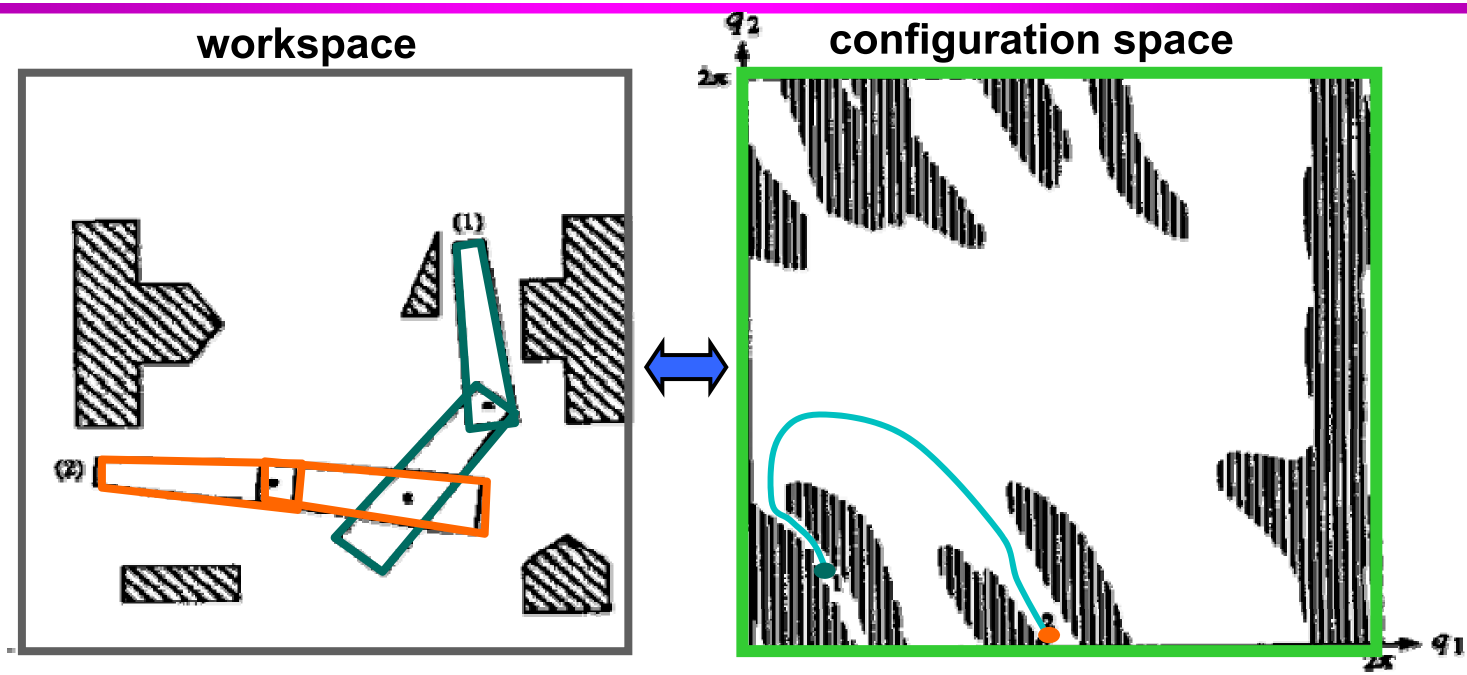
Paths in the configuration space



- A **path** in C is a continuous curve connecting two configurations q and q':

$$\tau : s \in [0,1] \rightarrow \tau(s) \in C$$

such that $\tau(0) = q$ and $\tau(1) = q'$.

Constraints on paths

- A trajectory is a path parameterized by time:

$$\tau : t \in [0, T] \rightarrow \tau(t) \in C$$

- Constraints
  - Finite length
  - Bounded curvature
  - Smoothness
  - Minimum length
  - Minimum time
  - Minimum energy
  - ...

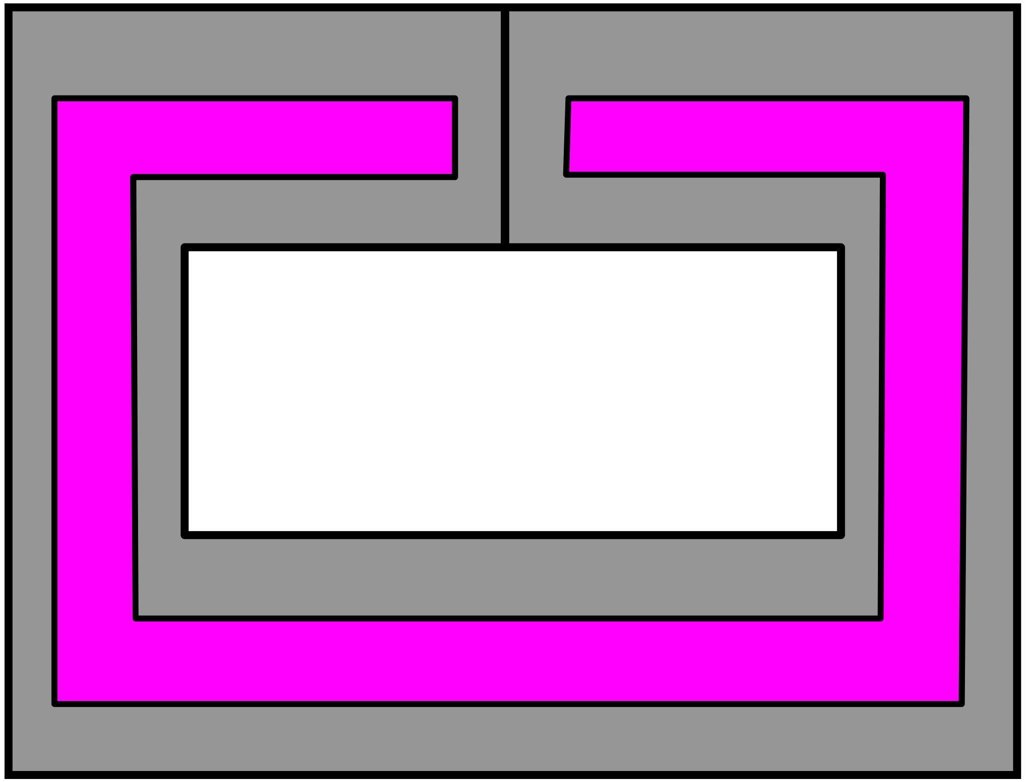
Free Space Topology

- A free path lies entirely in the free space F.
- The moving object and the obstacles are modeled as closed subsets, meaning that they contain their boundaries.
- One can show that the C-obstacles are closed subsets of the configuration space C as well.
- Consequently, the free space F is an open subset of C.

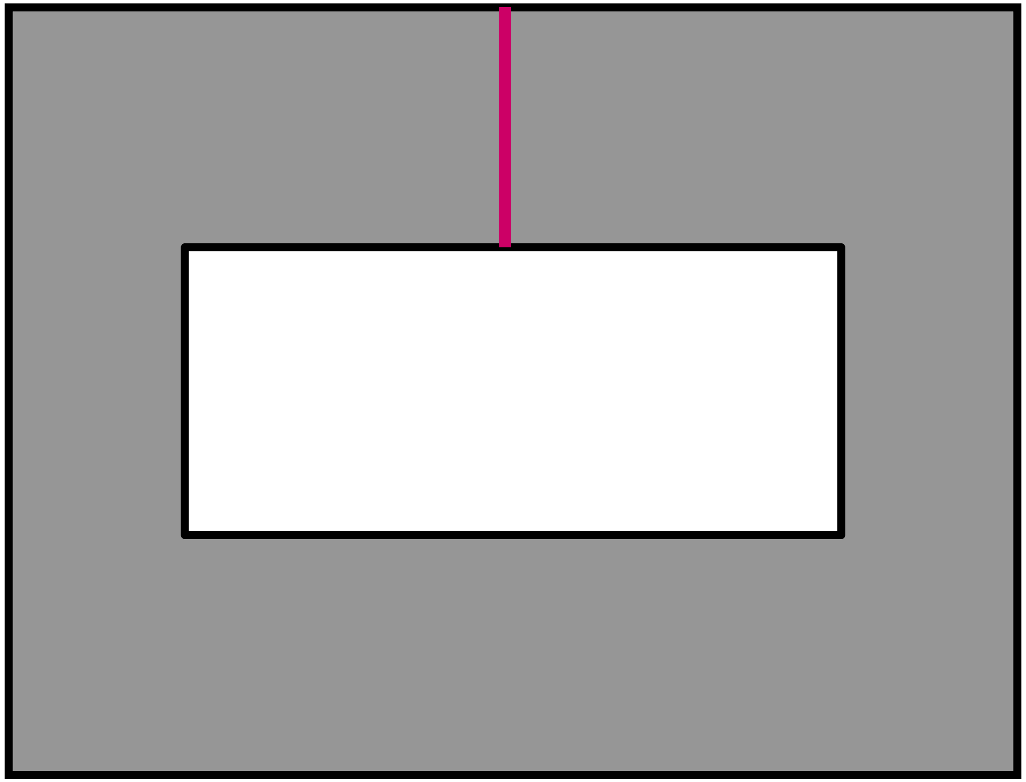
Semi-Free Space

- A configuration q is semi-free if the moving object placed q touches the boundary, but not the interior of obstacles.
  - Free, or
  - In contact
- The semi-free space is a closed subset of C.

Example



Example



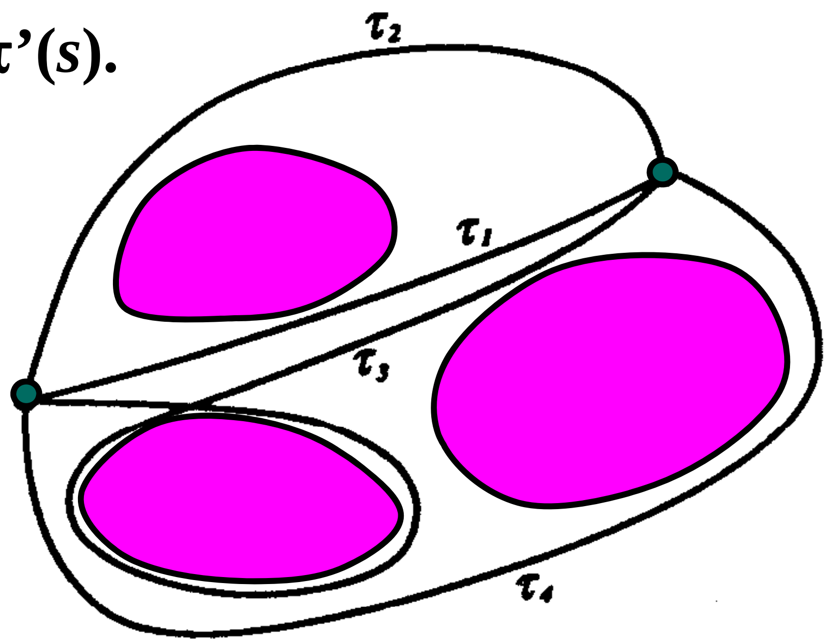
Homotopic Paths

- **Two paths τ and τ' (that map from U to V) with the same endpoints are homotopic if one can be continuously deformed into the other:**

$$h : U \times [0,1] \rightarrow V$$

with $h(s,0) = \tau(s)$ and $h(s,1) = \tau'(s)$.

- **A homotopic class of paths contains all paths that are homotopic to one another.**



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Connectedness of C-Space

- C is **connected** if every two configurations can be connected by a path.
- C is **simply-connected** if any two paths connecting the same endpoints are homotopic.
  Examples: $\mathbf{R}^2$ or $\mathbf{R}^3$
- Otherwise C is multiply-connected.

Connectedness of C-Space

- *C* is **connected** if every two configurations can be connected by a path.

- *C* is **simply-connected** if any two paths connecting the same endpoints are homotopic.
  Examples: R^2 or R^3

- Otherwise *C* is multiply-connected.
  Examples: S^1 and $SO(3)$ are multiply- connected:
  - In S^1, infinite number of homotopy classes
  - In $SO(3)$, only two homotopy classes

Configuration space

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Metric in Configuration Space

- A **metric** or **distance** function d in a configuration space C is a function

$$d : (q, q') \in C^2 \rightarrow d(q, q') \geq 0$$

  such that
  - $d(q, q') = 0$ if and only if $q = q'$,
  - $d(q, q') = d(q', q)$,
  - $d(q, q') \leq d(q, q'') + d(q'', q')$.

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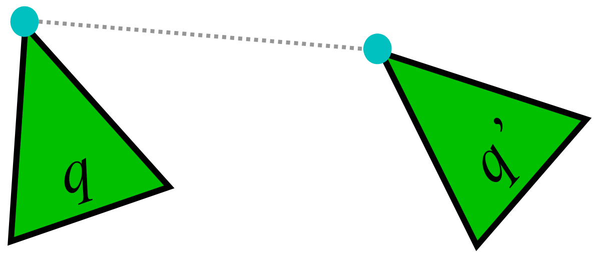
Example

- **Robot A and a point x on A**
- **$x(q)$: position of x in the workspace when A is at configuration q**
- **A distance d in C is defined by**

$$d(q, q') = \max_{x \in A} \| x(q) - x(q') \|$$

, where $\|x - y\|$ denotes the Euclidean distance between points x and y in the workspace.



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L_p Metrics

$$d(x, x') = \left(\sum_{i=1}^{n} | x_i - x_i' |^p \right)^{1/p}$$

- L_2: Euclidean metric
- L_1: Manhattan metric
- L_∞: Max ($| x_i - x_i' |$)

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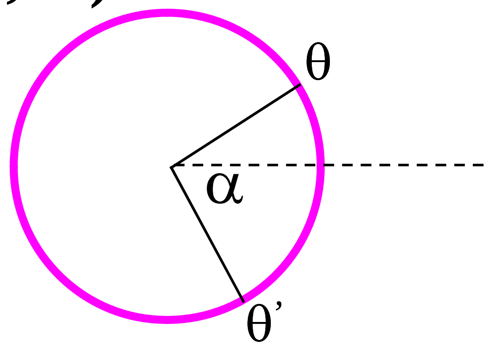
Examples in R^2 x S^1

- **Consider R^2 x S^1**
  - $q = (x, y, \theta)$, $q' = (x', y', \theta')$ **with** $\theta, \theta' \in [0, 2\pi)$
  - $\alpha = \min \{ |\theta - \theta'|, 2\pi - |\theta - \theta'| \}$



- $d(q, q') = \mathrm{sqrt}((x-x')^2 + (y-y')^2 + \alpha^2))$

Class Objectives were:

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Next Time….

- Collision detection and distance computation