
CS686: Probabilistic Roadmaps

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Course URL:
<http://sgvr.kaist.ac.kr/~sungeui/MPA>

KAIST

The KAIST logo consists of the letters "KAIST" in a bold, blue, sans-serif font. Below the text is a light blue, horizontal oval shape that serves as a shadow or base for the letters.

Announcements

- **Mid-term exam**
 - **Closed book: simple reviews on lecture materials)**
 - **4:00pm on Oct-22 at the class room**

Reminder

- **Declare your chosen 2 papers at the KLMS by Oct-14 (Mon.)**
 - **First come, first served**
 - **Paper title, conf. name, publication year**
- **Student presentations will start right after the mid-term exam**
 - **2 talks per each class; 20 min for each talk**
 - **Each presenter needs two short quiz**

Project Guidelines: Project Topics

- Any topics related to the course theme are okay
 - You can find topics by browsing recent papers
- You can bring your own research to the class, only if it is related to the course theme
 - You need to get a permission from me for this

Expectations

- **Mid-term project presentation**
 - **Introduce problems and explain why it is important**
 - **Give an overall idea on the related work**
 - **Explain what problems those existing techniques have**
 - **(Optional) explain how you can address those problems**
 - **Explain roles of each member**

Expectations

- **Final-term project presentation**
 - Review materials that you talked for your mid-term project
 - Present your ideas that can address problems of those state-of-the-art techniques
 - Give your qualitatively (or intuitive) reasons how your ideas address them
 - Also, explain expected benefits and drawbacks of your approach
 - (Optional) backup your claims with quantitative results collected by some implementations
 - Explain roles of each members

A few more comments

- **Start to implement a paper, if you don't have any clear ideas**
 - **While you implement it, you may get ideas about improving it**

Final-project evaluation sheet

You name:

ID:

Score table: higher score is better.

Speaker	Novelty of the project and idea (1 ~ 5)	Practical benefits of the method (1 ~ 5)	Completeness level of the project (1 ~ 5)	Total score (3 ~ 15)	Role of each student is clear and well balanced? (Yes or No)
XXX					
YYY					

Class Objectives

- **Understand probabilistic roadmap (PRM) approaches**
 - **Multi-query PRMs**

Difficulty with Classic Approaches

- Running time increases exponentially with the dimension of the configuration space
 - For a d -dimension grid with 10 grid points on each dimension, how many grid cells are there?

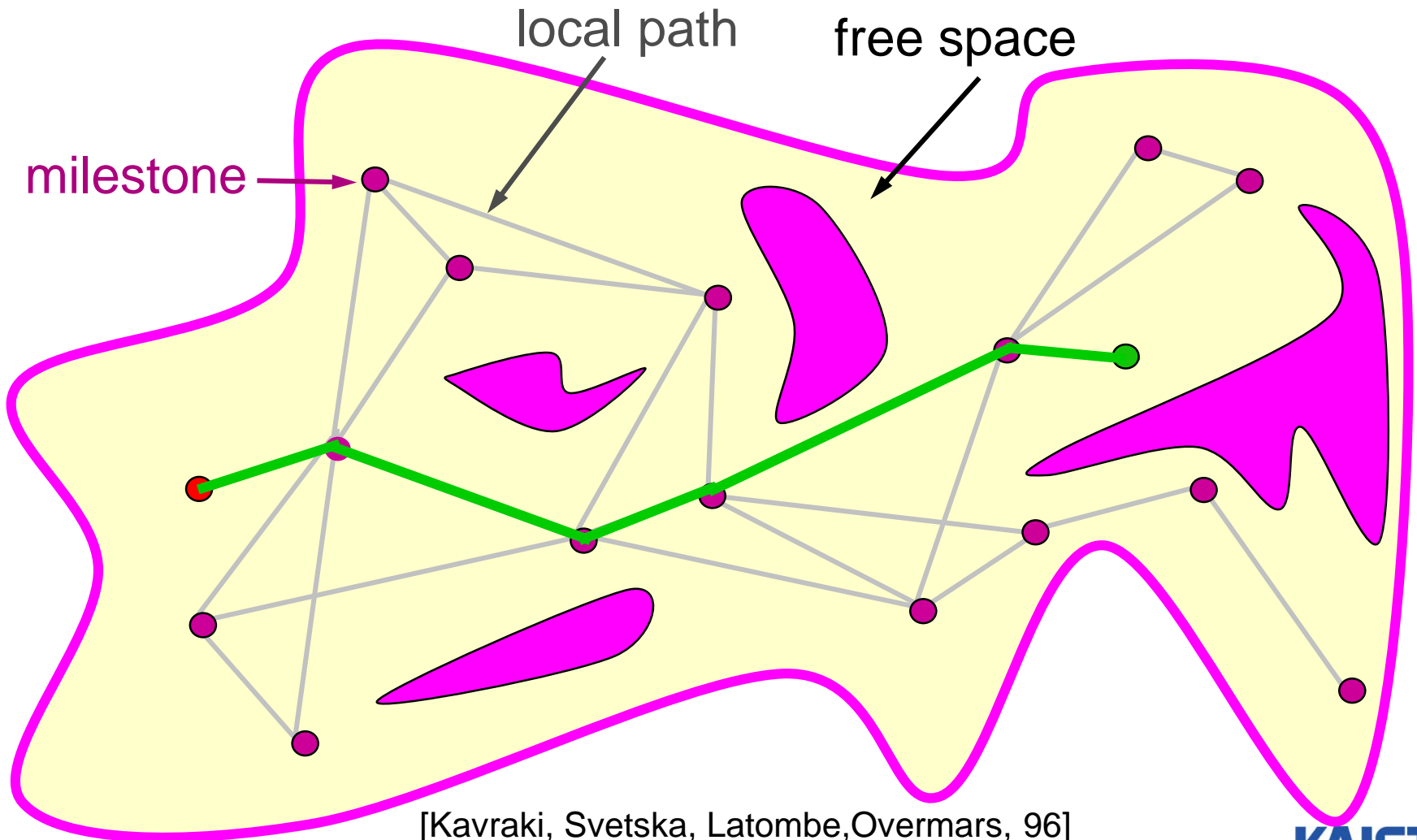
$$10^d$$

- Several variants of the path planning problem have been proven to be PSPACE-hard

Completeness

- Complete algorithm → Slow
 - A **complete** algorithm finds a path if one exists and reports no otherwise
 - Example: Canny's roadmap method
- Heuristic algorithm → Unreliable
 - Example: potential field
- **Probabilistic completeness**
 - Intuition: If there is a solution path, the algorithm will find it with high probability

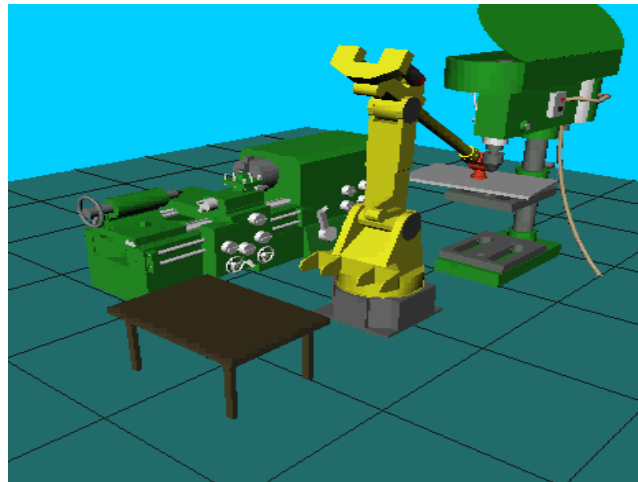
Probabilistic Roadmap (PRM): multiple queries



[Kavraki, Svetska, Latombe, Overmars, 96]

Assumptions

- **Static obstacles**
- **Many queries to be processed in the same environment**
- **Examples**
 - **Navigation in static virtual environments**
 - **Robot manipulator arm in a workcell**



Overview

- **Precomputation: roadmap construction**
 - Uniform sampling
 - Resampling (expansion)
- **Query processing**

Uniform sampling

Input: geometry of the moving object & obstacles

Output: roadmap $G = (V, E)$

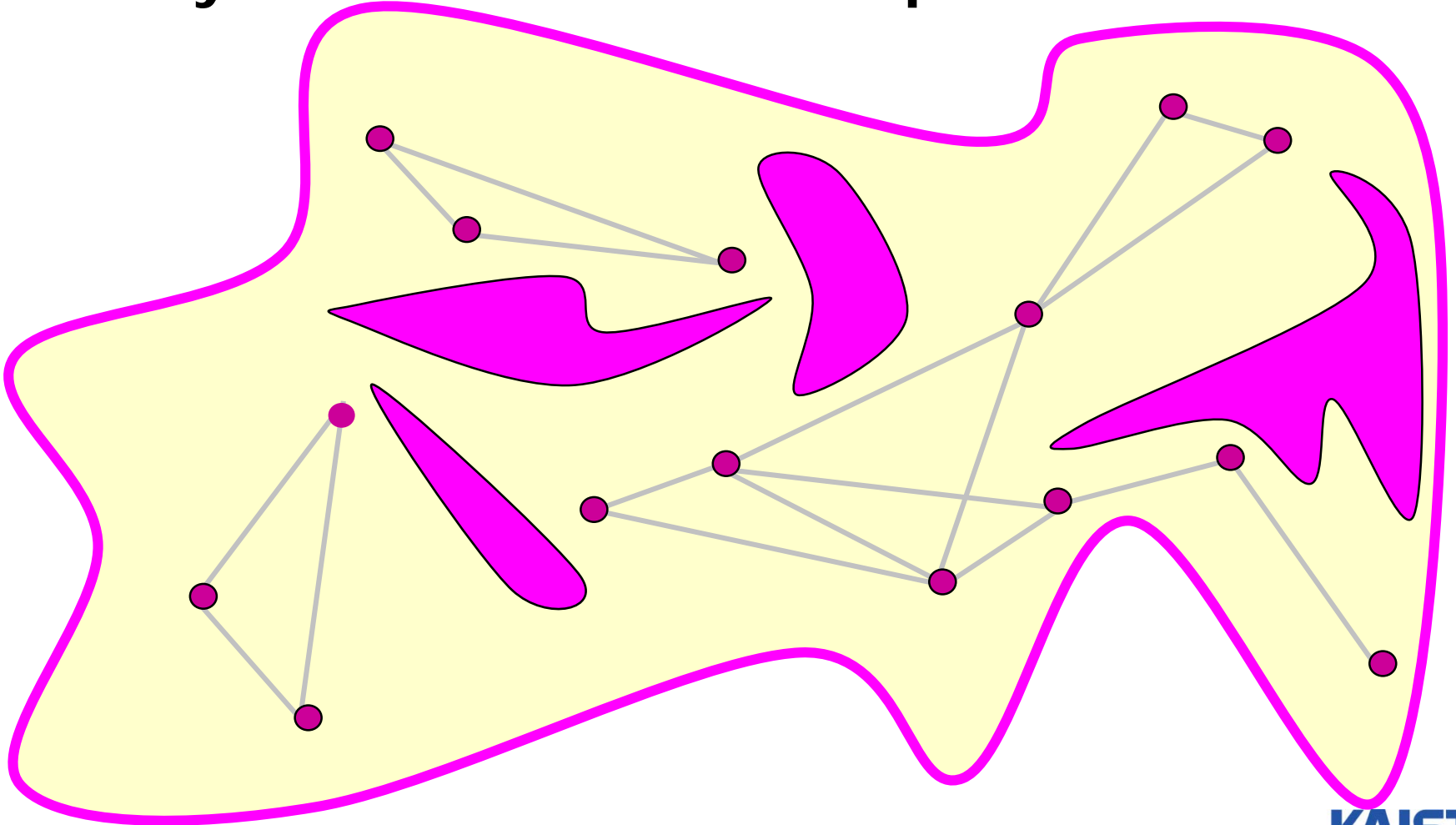
```
1:  $V \leftarrow \emptyset$  and  $E \leftarrow \emptyset$ .
2: repeat
3:    $q \leftarrow$  a configuration sampled uniformly at random from  $C$ 
4:   if CLEAR( $q$ )then
5:     Add  $q$  to  $V$ .
6:      $N_q \leftarrow$  a set of nodes in  $V$  that are close to  $q$ .
6:     for each  $q' \in N_q$ , in order of increasing  $d(q, q')$ 
7:       if LINK( $q', q$ )then
8:         Add an edge between  $q$  and  $q'$  to  $E$ .
```

The graph G is called a **probabilistic roadmap**

The nodes in G are called **milestones**

Difficulty

- Many small connected components



Resampling (expansion)

- Failure rate

$$r(q) = \frac{\text{\#. failed LINK}}{\text{\#. LINK}}$$

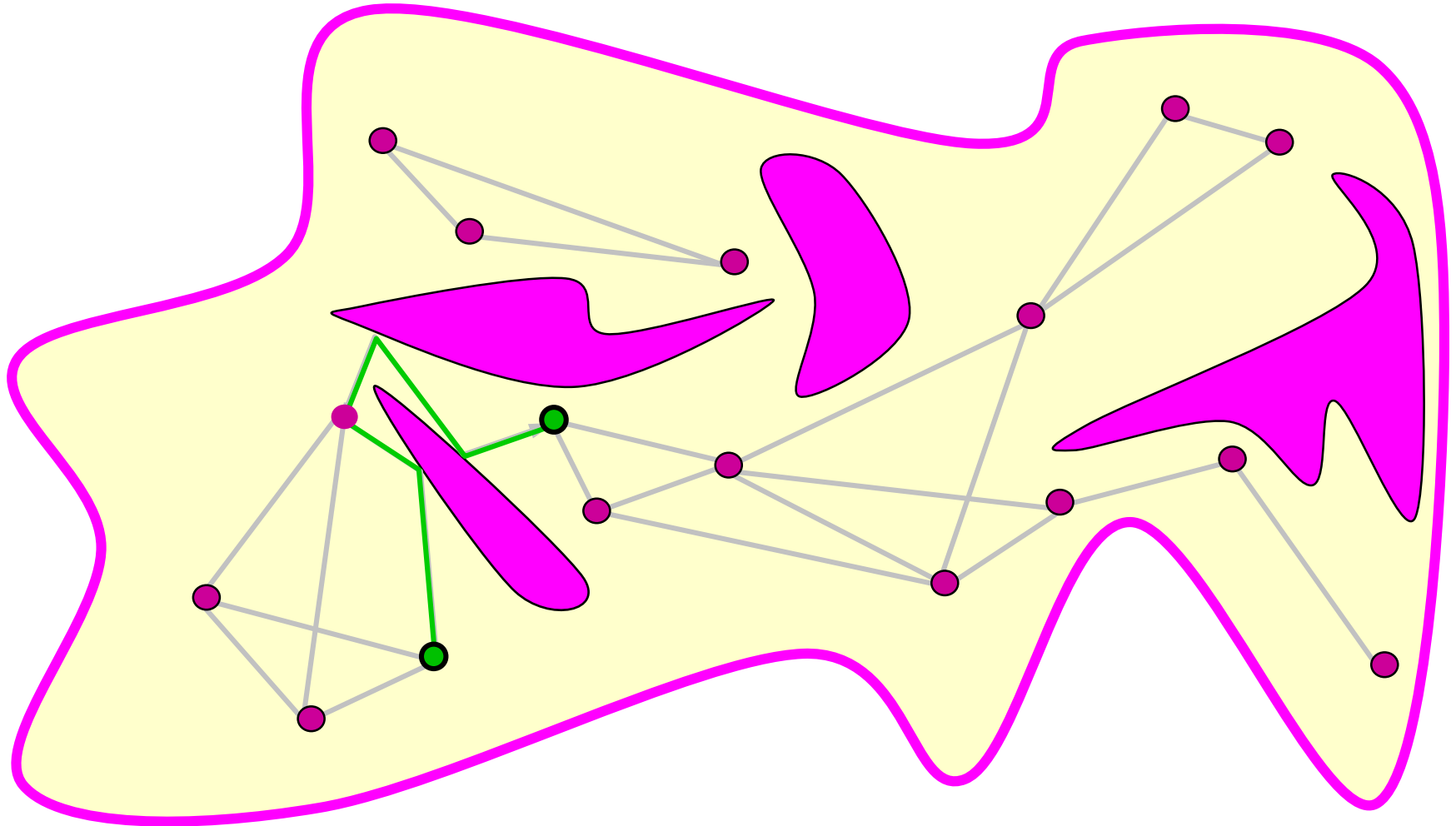
- Normalized weight

$$w(q) = \frac{r(q)}{\sum_p r(p)}$$

- Resampling probability

$$\Pr(q) = w(q)$$

Resampling (expansion)



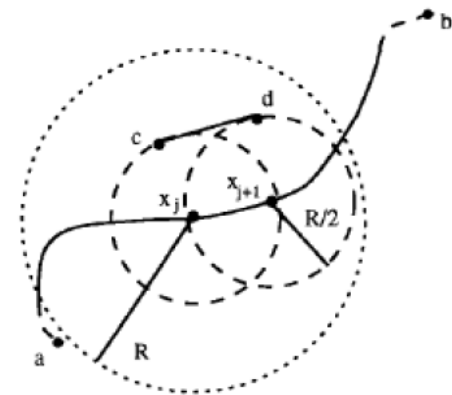
Query processing

- Connect q_{init} and q_{goal} to the roadmap
- Start at q_{init} and q_{goal} , perform a random walk, and try to connect with one of the milestones nearby
- Try multiple times

Error

- If a path is returned, the answer is always correct
- If no path is found, the answer may or may not be correct. We hope it is correct with high probability.
 - Refer to Theoretical Analysis of my draft

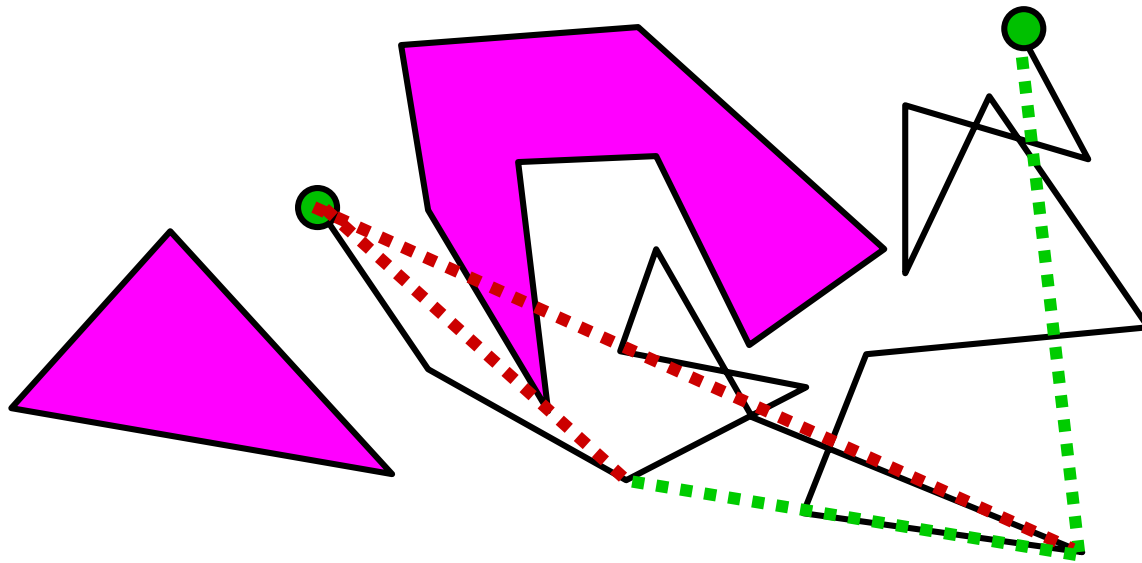
$$P(\text{Fail}) \leq \frac{2L}{R} \exp(-\alpha_D R^D N).$$



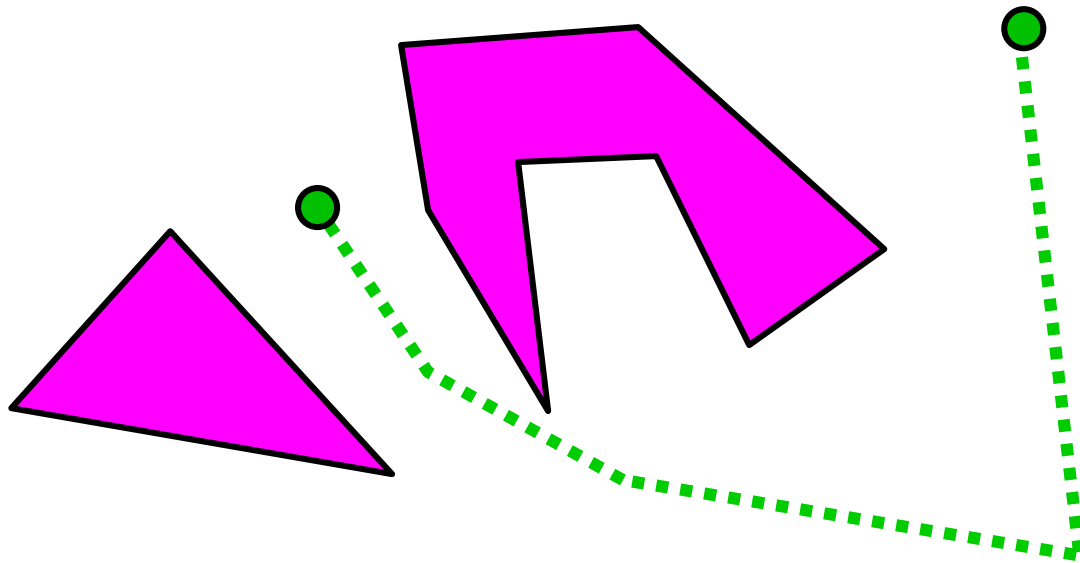
L: path lengths, **N:** # of samples, **D** is dimension
R: the clearance between the robot and obstacles

$$\alpha_D = 2^{-D} \frac{\pi^{D/2}}{\Gamma(D/2+1) \text{Vol}(C_{free})}$$

Smoothing the path

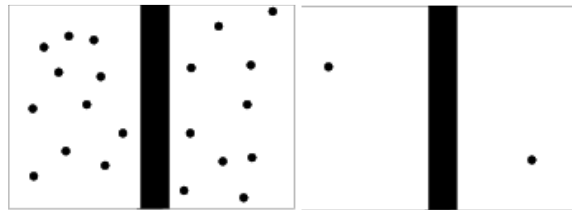


Smoothing the path

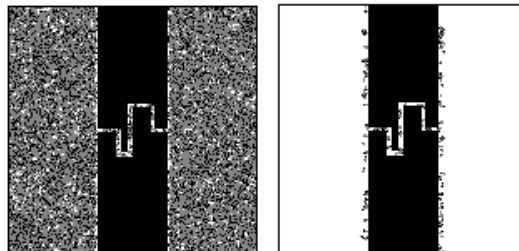


Sampling Strategies

- Visibility-based Probabilistic roadmaps for Motion planning

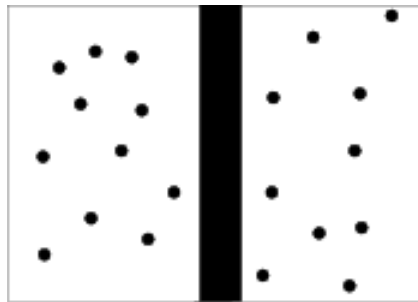


- The Gaussian Sampling Strategy for PRM's
 - *Sample near the boundaries of the C-space obstacles with higher probability*



Visibility-based PRM

- Computes a very compact roadmap



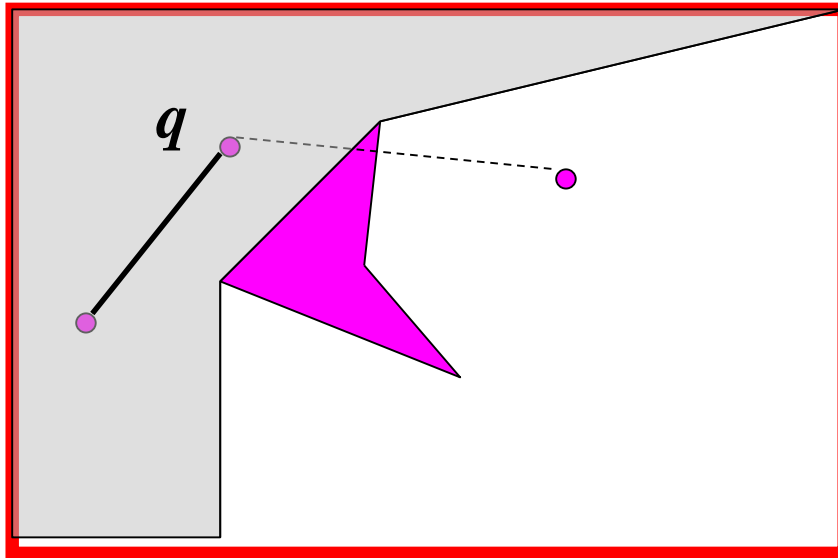
Classical PRM



Visibility roadmap

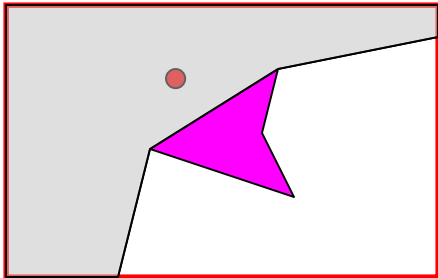
Visibility Domain

- Visibility domain of a free configuration q :
 - The grey region



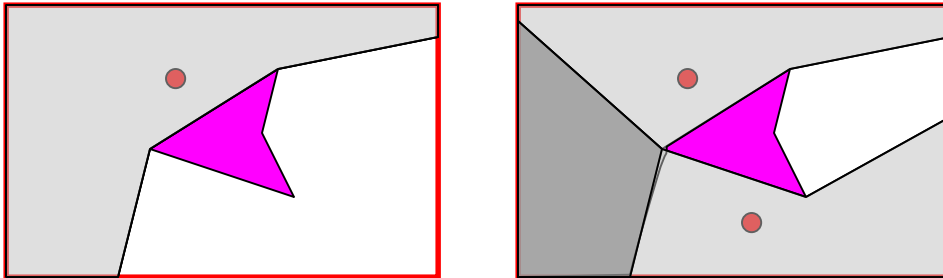
Guard Nodes

- The C-space fully captured by 'guard' nodes



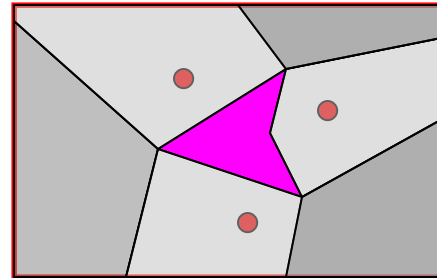
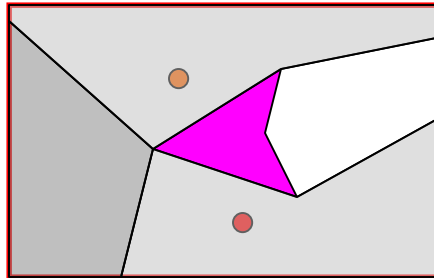
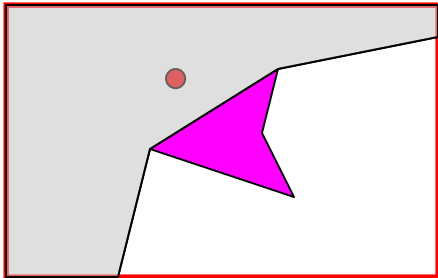
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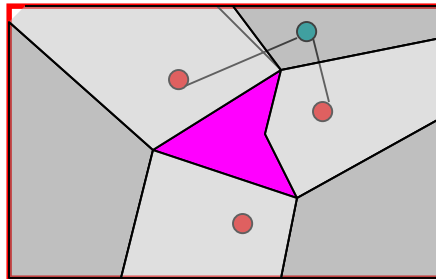
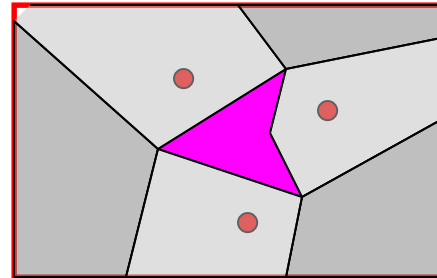
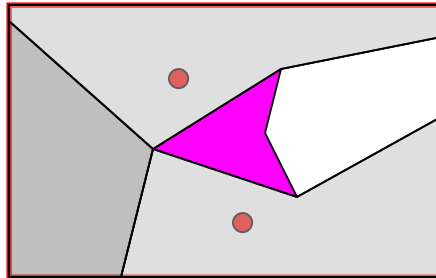
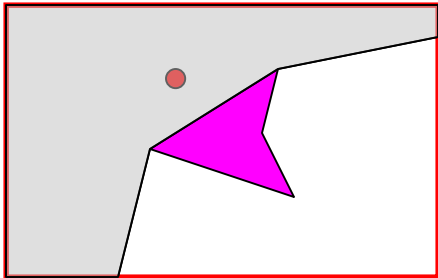
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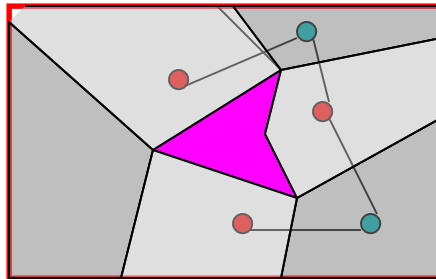
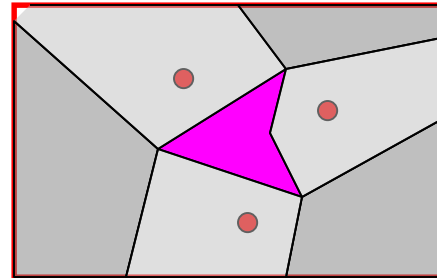
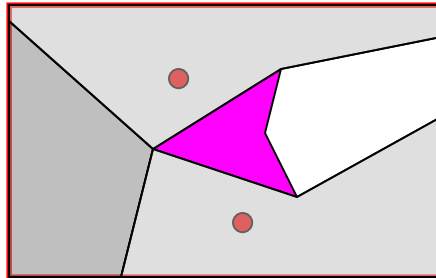
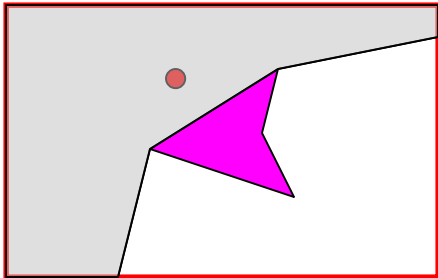
Connection Nodes

- The C-space being captured by 'guards' and 'connection' nodes.



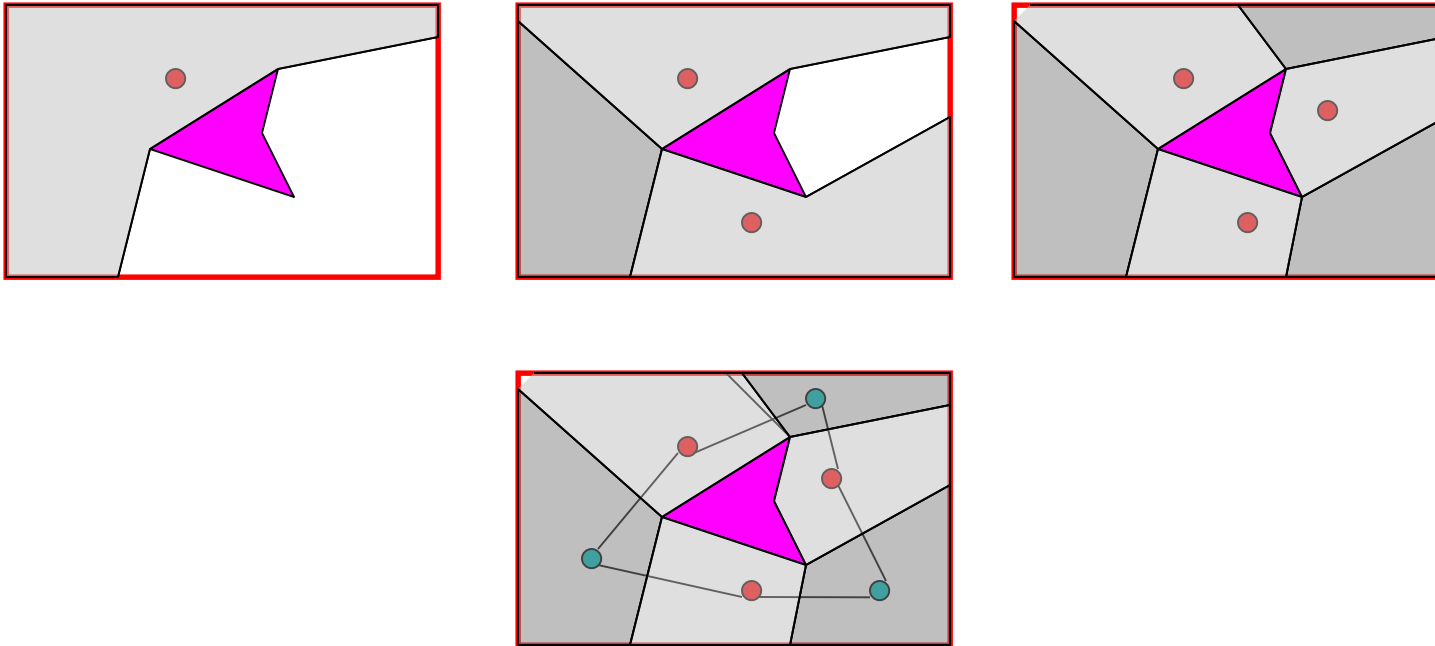
Connection Nodes

- The C-space being captured by 'guards' and 'connection' nodes.



Connection Nodes

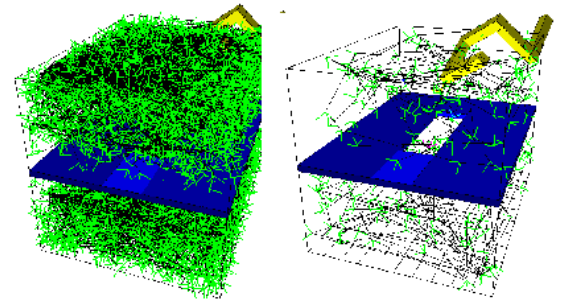
- The C-space fully captured by 'guards' and 'connection' nodes.



- We do not need any other additional node in the roadmap

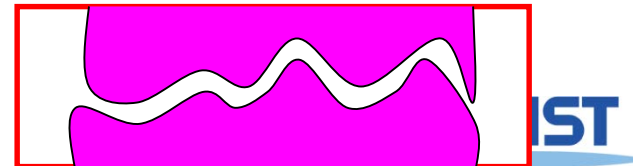
Remarks

- Maintains a very compact roadmap, resulting in faster query time



- But:

- There is a tradeoff with high cost of processing each new milestone
- How many iterations needed to capture the full connectivity?
- The problem of capturing the narrow passage effectively is still the same as in the basic PRM.



Summary

- **What probability distribution should be used for sampling milestones?**
- **How should milestones be connected?**
- **A path generated by a randomized algorithm is usually jerky. How can a path be smoothed?**

- **Single-query PRMs were proposed, but RRT techniques are more widely used**

Class Objectives were:

- **Understand probabilistic roadmap (PRM) approaches**
 - **Multi-query PRMs**

Next Time..

- **RRT techniques and their recent advancements**

Homework for Every Class

- **Submit summaries of 2 ICRA/IROS/RSS/CoRL/TRO/IJRR papers**
- **Go over the next lecture slides**
- **Come up with three question before the mid-term exam**