CS686: Configuration Space II

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(윤성의)

Course URL:
http://sglab.kaist.ac.kr/~sungeui/MPA

Coming Schedule and Homework

- **Browse recent papers (2014 ~ 2017)**
  - **You need to present two papers at the class**
- **Declare your chosen 2 papers at the KLMS by Apr-10 (Mon.)**
  - **First come, first served**
  - **Paper title, conf. name, publication year**
- **Decide our talk schedule on Apr.-11 (Tue.)**
- **Student presentations will start right after the mid-term exam**
  - **3 talks per each class; 15 min for each talk**

Class Objectives

- Configuration space
  - Definitions and examples
  - Obstacles
  - Paths
  - Metrics

Obstacles in the Configuration Space

- **A configuration q is collision-free, or free, if a moving object placed at q does not intersect any obstacles in the workspace**

- **The free space F is the set of free configurations**

- **A configuration space obstacle (C-obstacle) is the set of configurations where the moving object collides with workspace obstacles**

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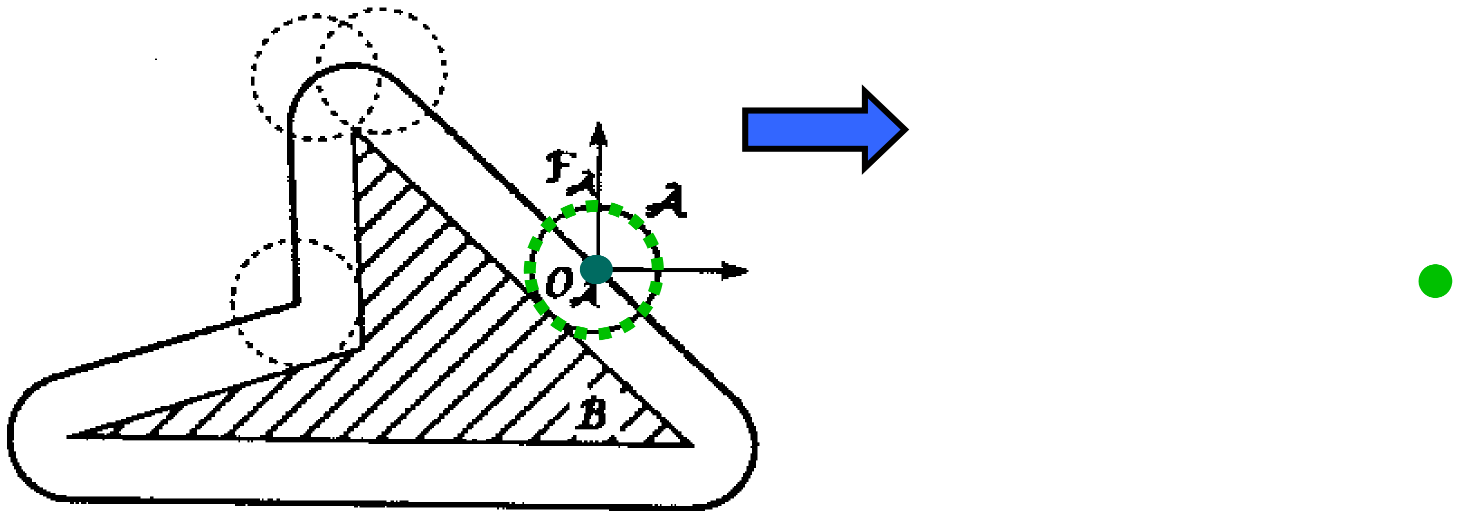
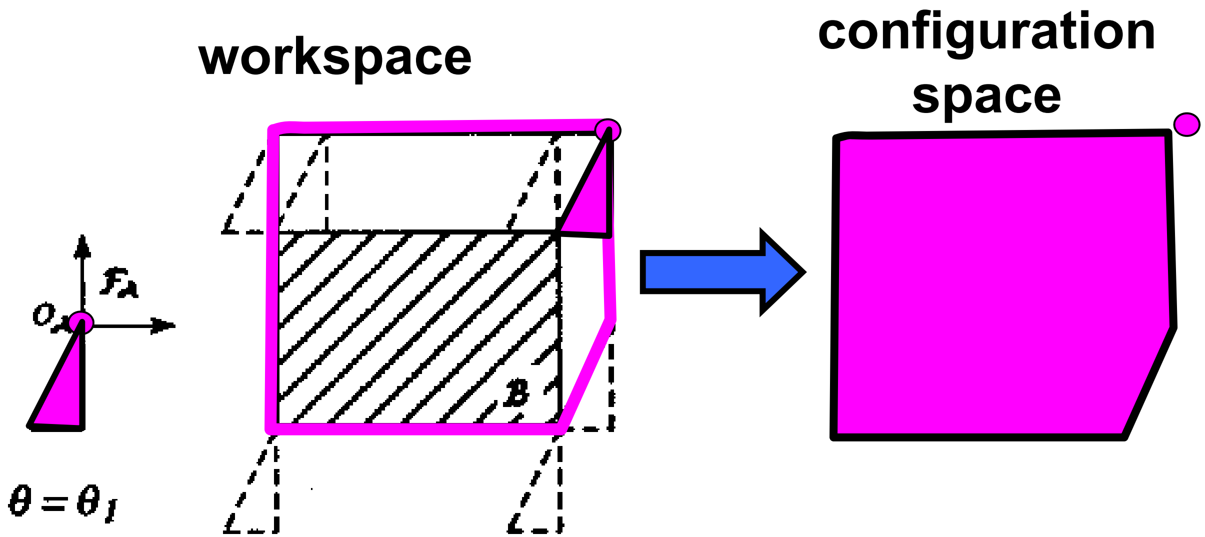
Disc in 2-D Workspace

workspace

configuration space



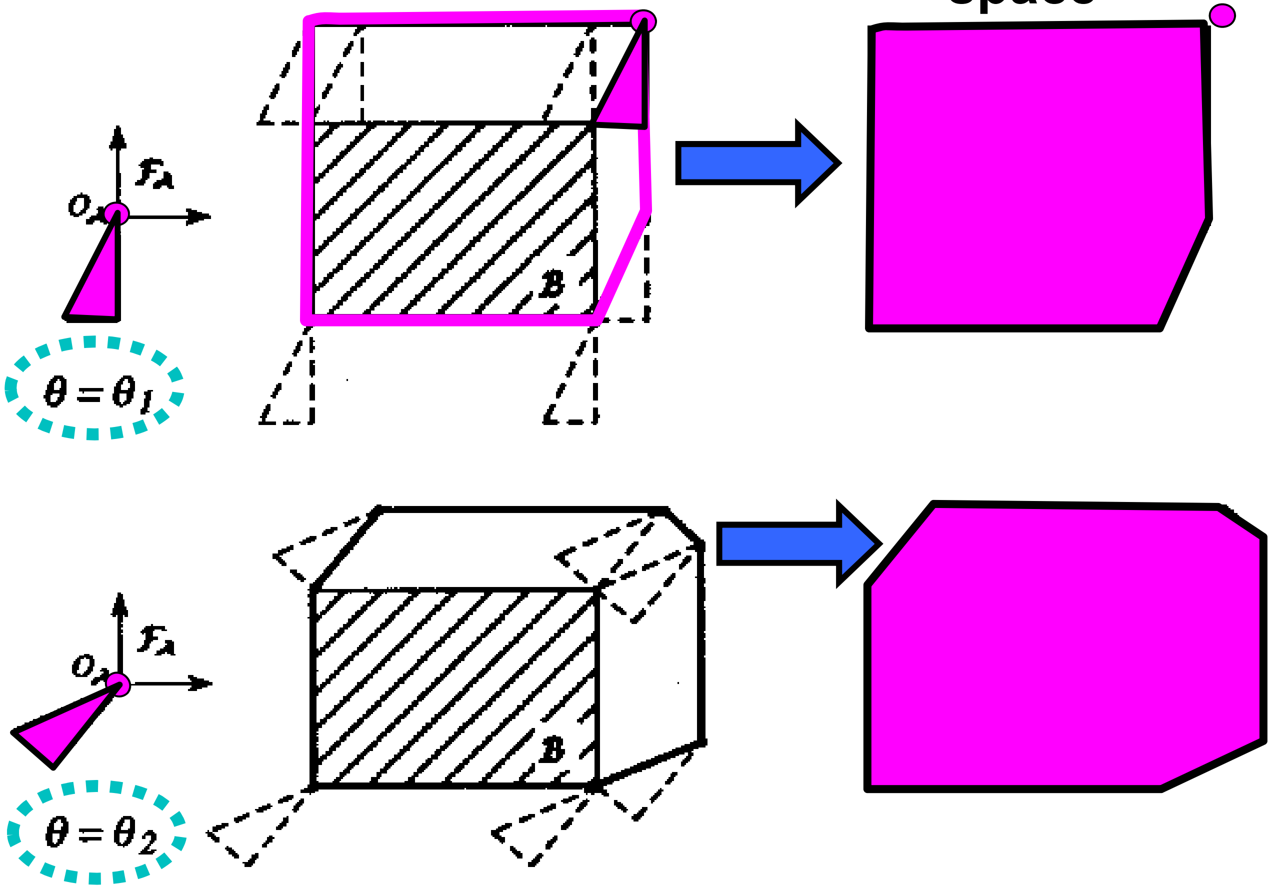
Polygonal Robot Translating in 2-D Workspace



Polygonal Robot Translating & Rotating in 2-D Workspace

workspace

configuration space



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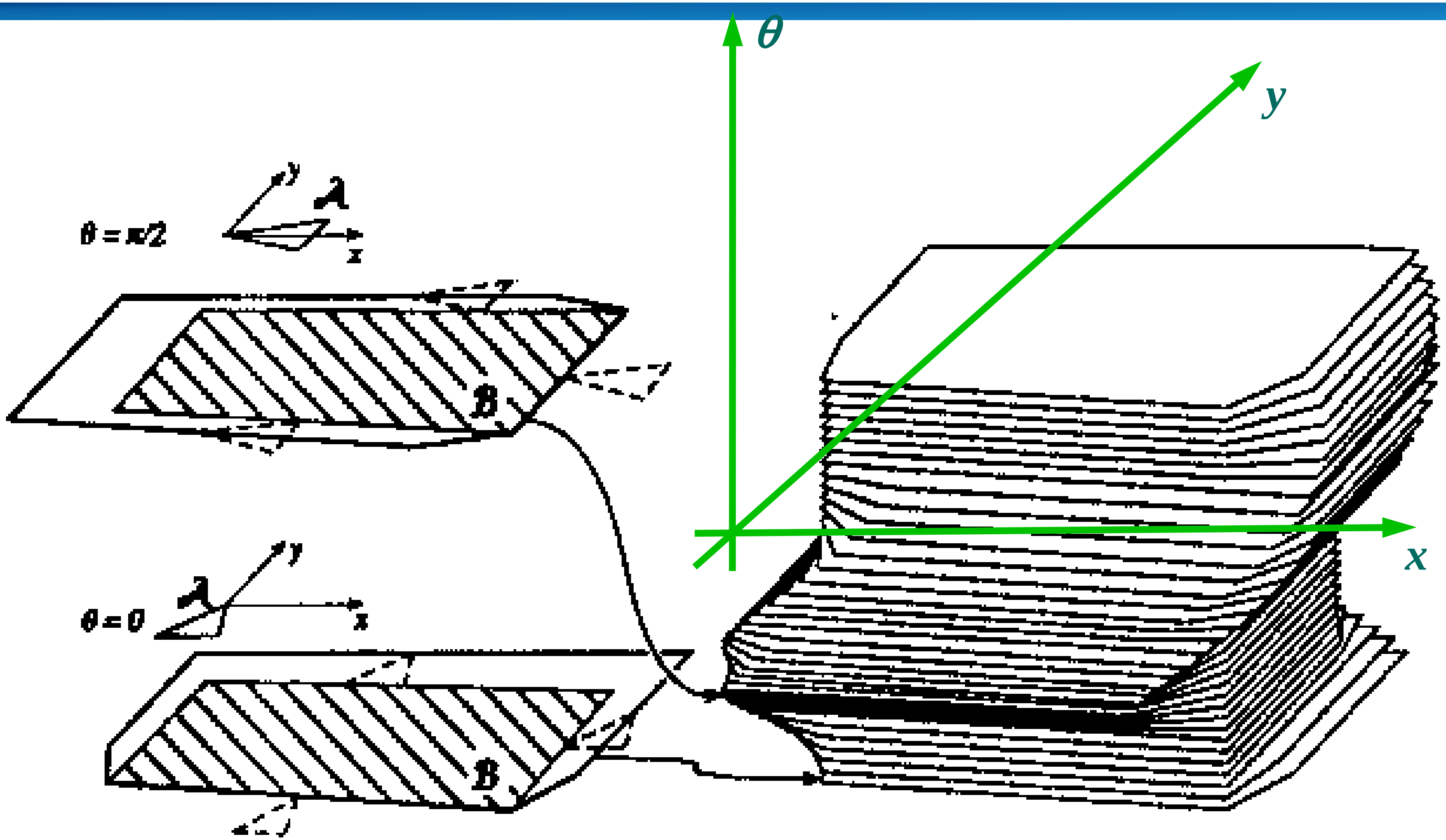
Polygonal Robot Translating & Rotating in 2-D Workspace



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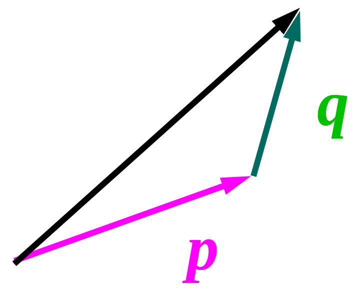
C-Obstacle Construction

- Input:
  - Polygonal moving object translating in 2-D workspace
  - Polygonal obstacles
- Output:
  - Configuration space obstacles represented as polygons

Minkowski Sum

- The **Minkowski sum** of two sets P and Q, denoted by $P \oplus Q$, is defined as

$$P \oplus Q = \{ p+q \mid p \in P, q \in Q \}$$



- Similarly, the **Minkowski difference** is defined as

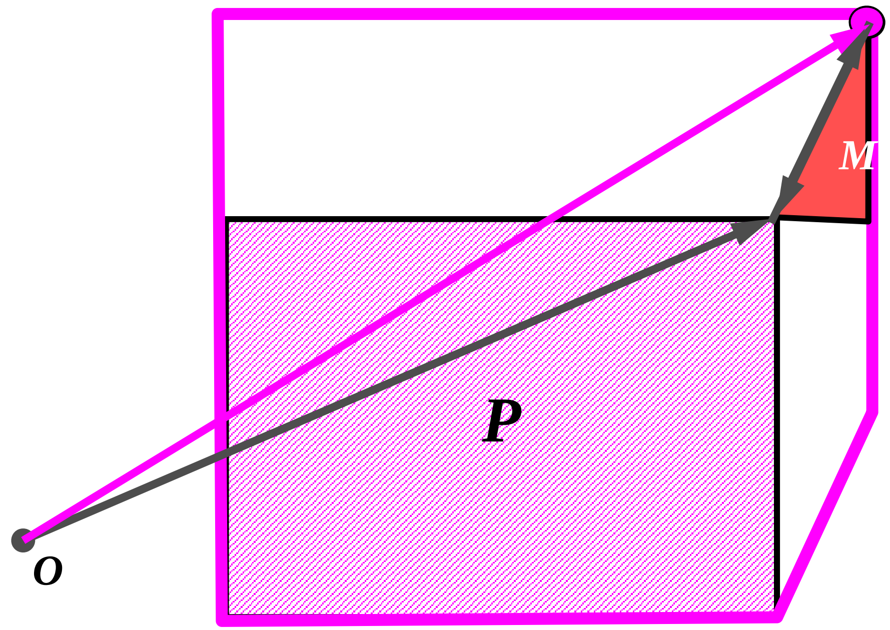
$$P \ominus Q = \{ p-q \mid p \in P, q \in Q \}$$
$$= P \oplus -Q$$

Minkowski Sum of Convex Polygons

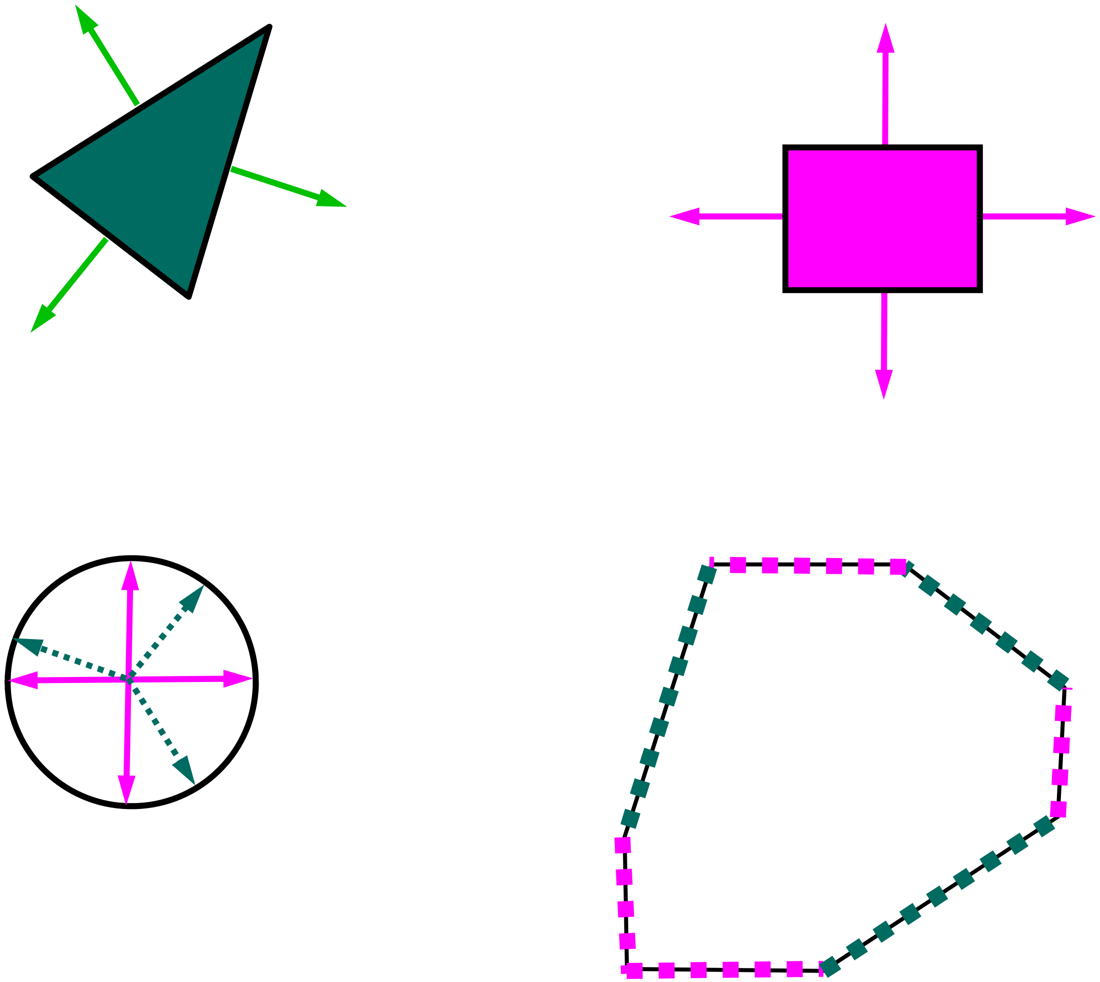
- **The Minkowski sum of two convex polygons P and Q of m and n vertices respectively is a convex polygon $P \oplus Q$ of $m + n$ vertices.**
  - **The vertices of $P \oplus Q$ are the "sums" of vertices of P and Q.**

Observation

- **If P is an obstacle in the workspace and M is a moving object. Then the C-space obstacle corresponding to P is $P \ominus M$.**



Computing C-obstacles



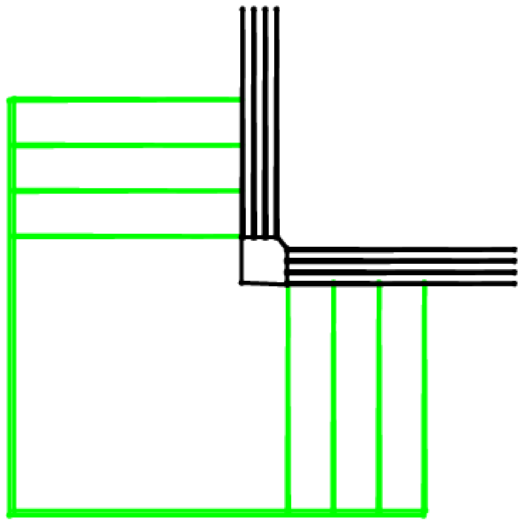
Computational efficiency

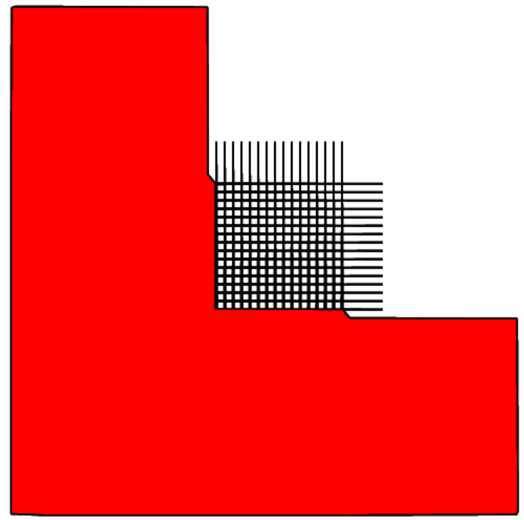
- **Running time $O(n+m)$**
- **Space $O(n+m)$**
- **Non-convex obstacles**
  - **Decompose into convex polygons (*e.g.,* triangles or trapezoids), compute the Minkowski sums, and take the union**
  - **Complexity of Minkowksi sum $O(n^2m^2)$**
- **3-D workspace**
  - **Convex case: $O(nm)$**
  - **Non-convex case: $O(n^3m^3)$**

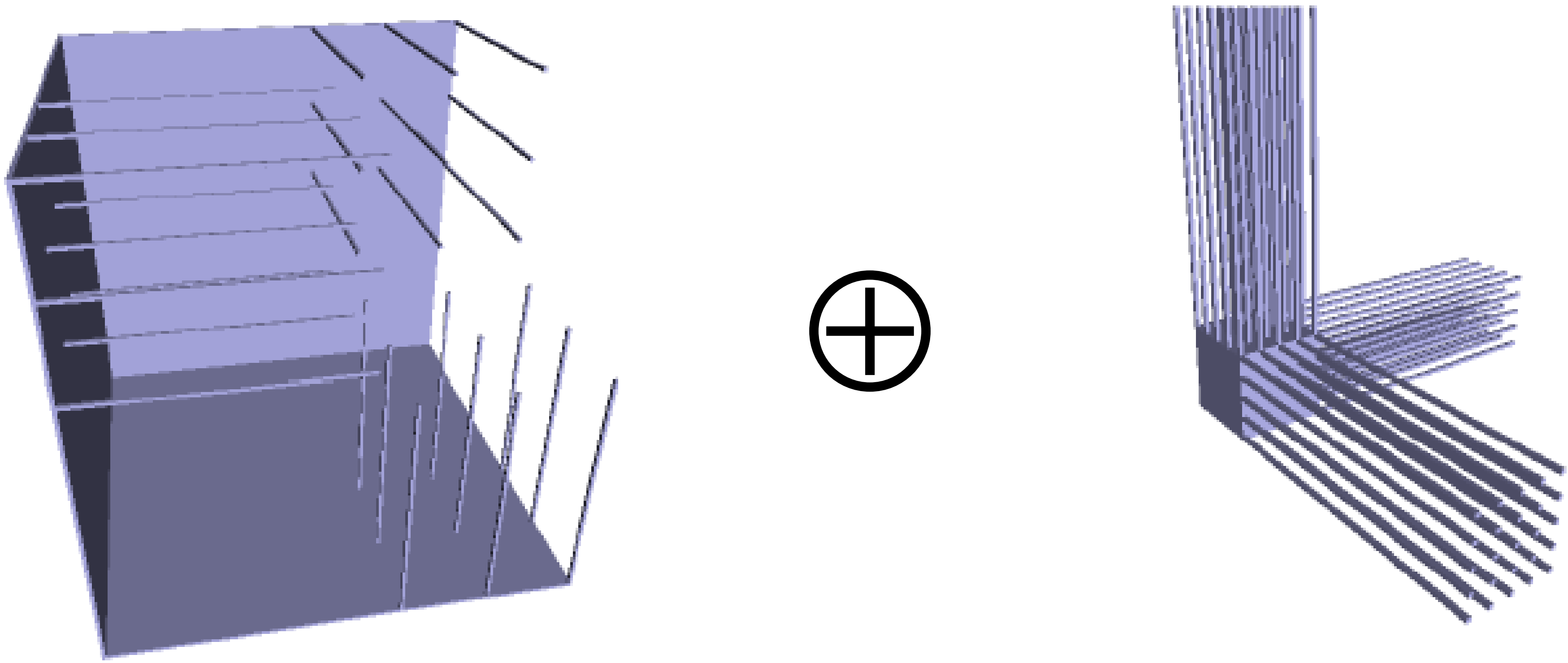
Worst case example

- $O(n^2m^2)$ **complexity**

2D example
Agarwal et al. 02

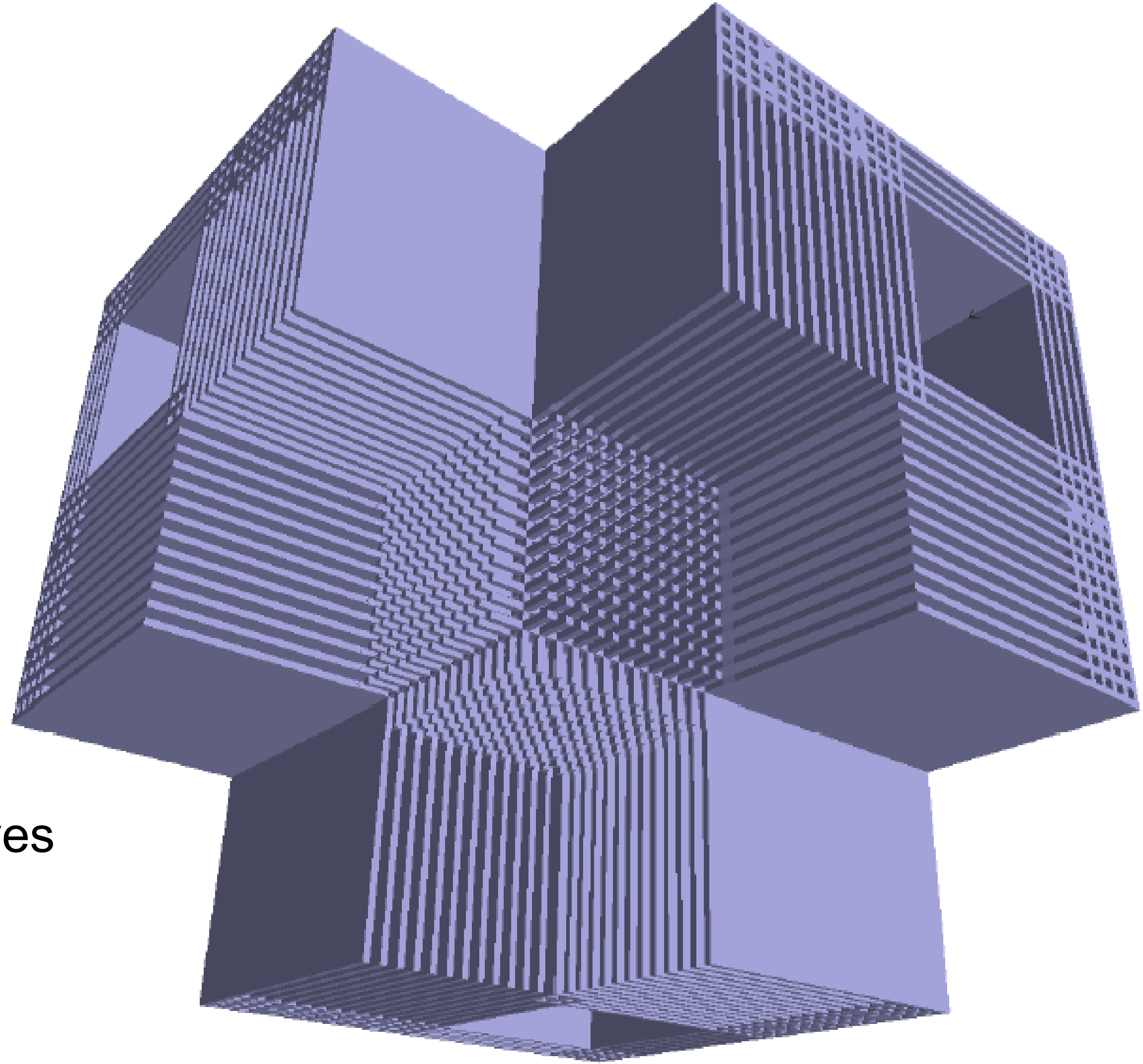






444 tris

1,134 tris



Union of
66,667 primitives

Main Message

- **Computing the free or obstacle space in an accurate way is an expensive and non-trivial problem**
- **Lead to many sampling based methods**
  - **Locally utilize many geometric concepts developed for designing complete planners**

Sensors!

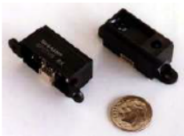
Robots' link to the external world...

sonar rangefinder

gyro

Sensors, sensors, sensors!
and tracking what is sensed: world models

compass



IR rangefinder

sonar rangefinder

CMU cam with on-board processing

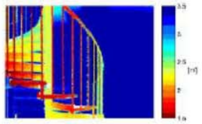
odometry…

16-735, Howie Choset with slides from G.D. Hager and Z. Dodds

Laser Ranging

LIDAR

Sick Laser

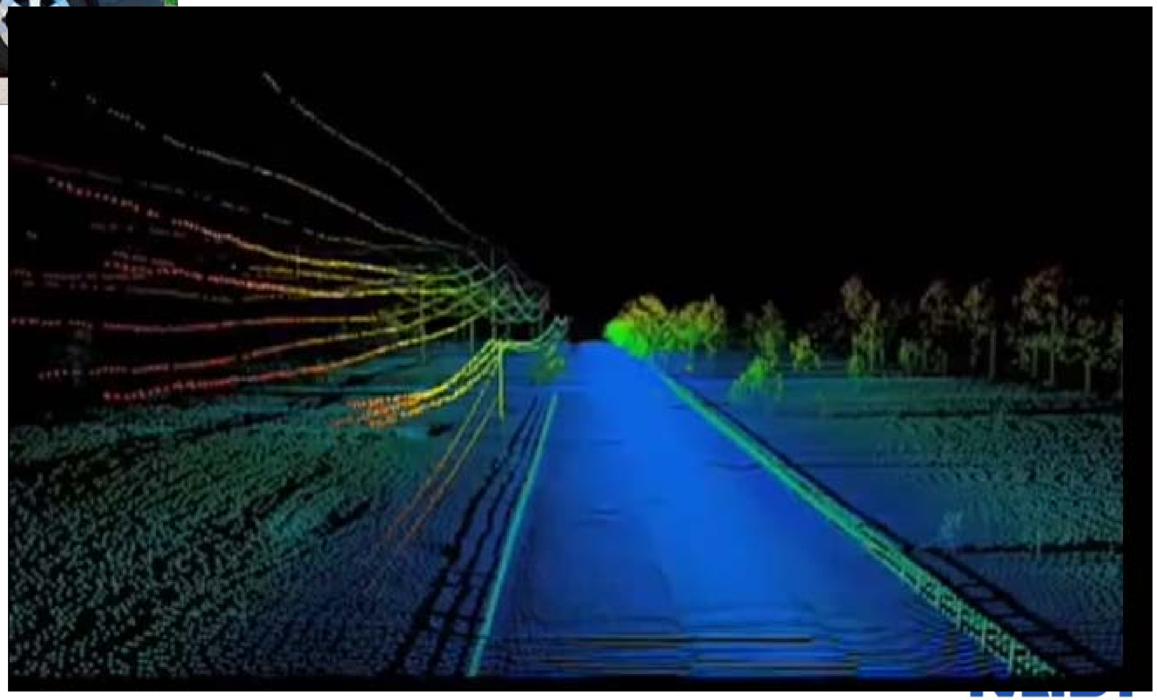


LIDAR map

Sick laser range finder

16-735, Howie Choset with slides from G.D. Hager and Z. Dodds

Velodyne

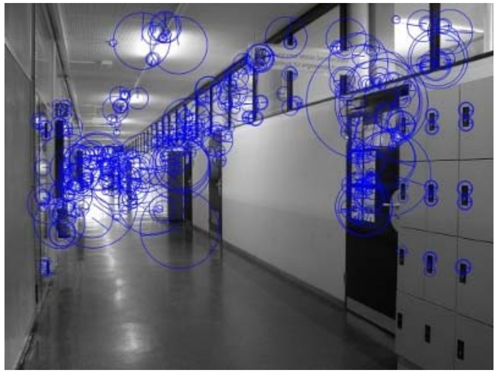


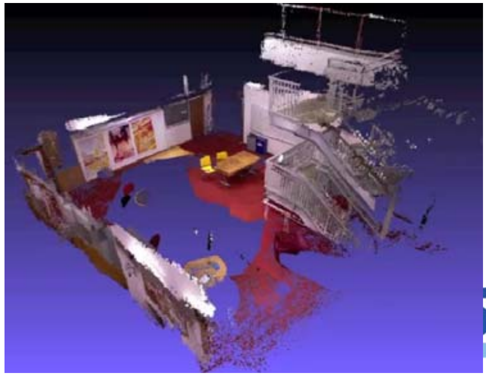
Kinect and Xtion

- **Kinect resolution**
  - **640×480 pixels @ 30 Hz (RGB camera)**
  - **640×480 pixels @ 30 Hz (IR depth-finding camera)**

Whole Picture

- **Sensor**
  - **Point clouds as obstacle map**
- **Control**
  - **Compute force controls given a computed path**
- **SLAM (Simultaneous Localization and Mapping)**
- **Path / motion planner**

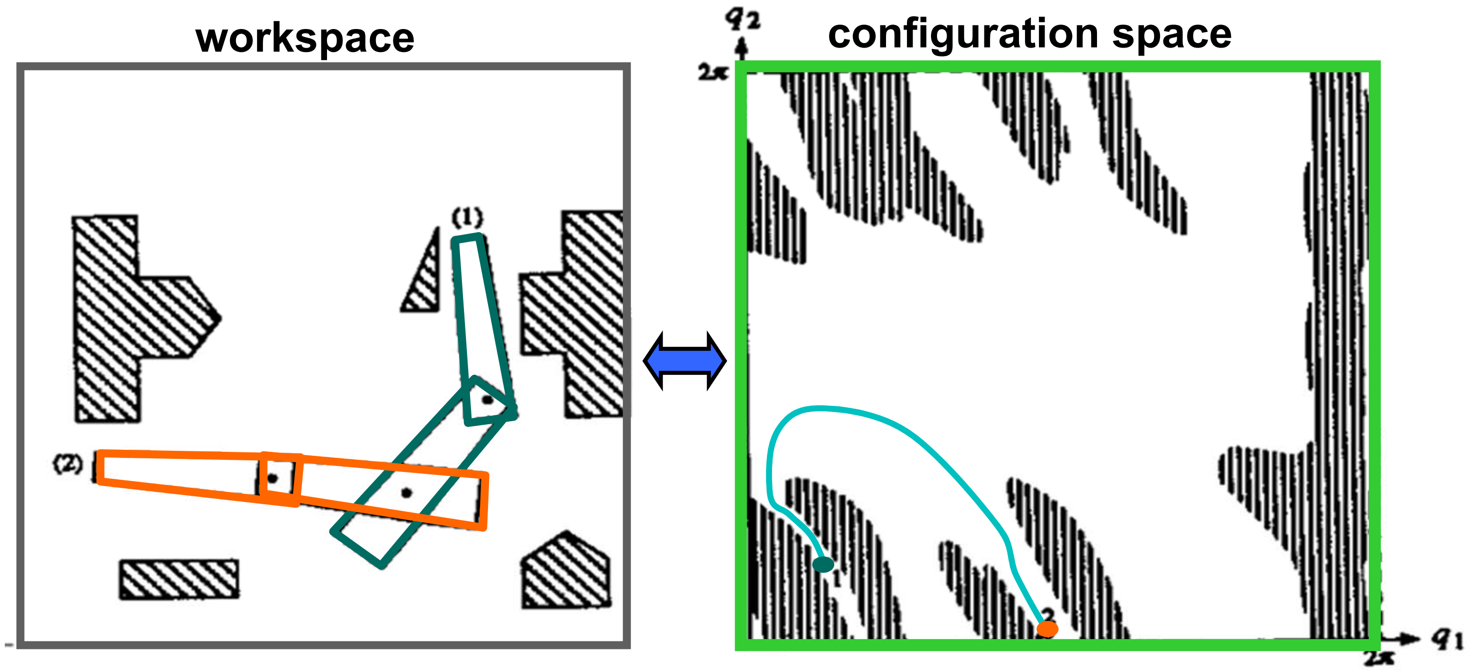




Configuration space

- Definitions and examples
- Obstacles
- Paths
- Metrics

Paths in the configuration space



- A **path** in C is a continuous curve connecting two configurations q and q':

$$\tau : s \in [0,1] \rightarrow \tau(s) \in C$$

such that $\tau(0) = q$ and $\tau(1) = q'$.

Constraints on paths

- A trajectory is a path parameterized by time:

$$\tau : t \in [0, T] \to \tau(t) \in C$$

- Constraints
  - Finite length
  - Bounded curvature
  - Smoothness
  - Minimum length
  - Minimum time
  - Minimum energy
  - ...

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Free Space Topology

- A **free** path lies entirely in the free space F
  - The moving object and the obstacles are modeled as closed subsets, meaning that they contain their boundaries.
  - One can show that the C-obstacles are closed subsets of the configuration space C as well
  - Consequently, the free space F is an open subset of C

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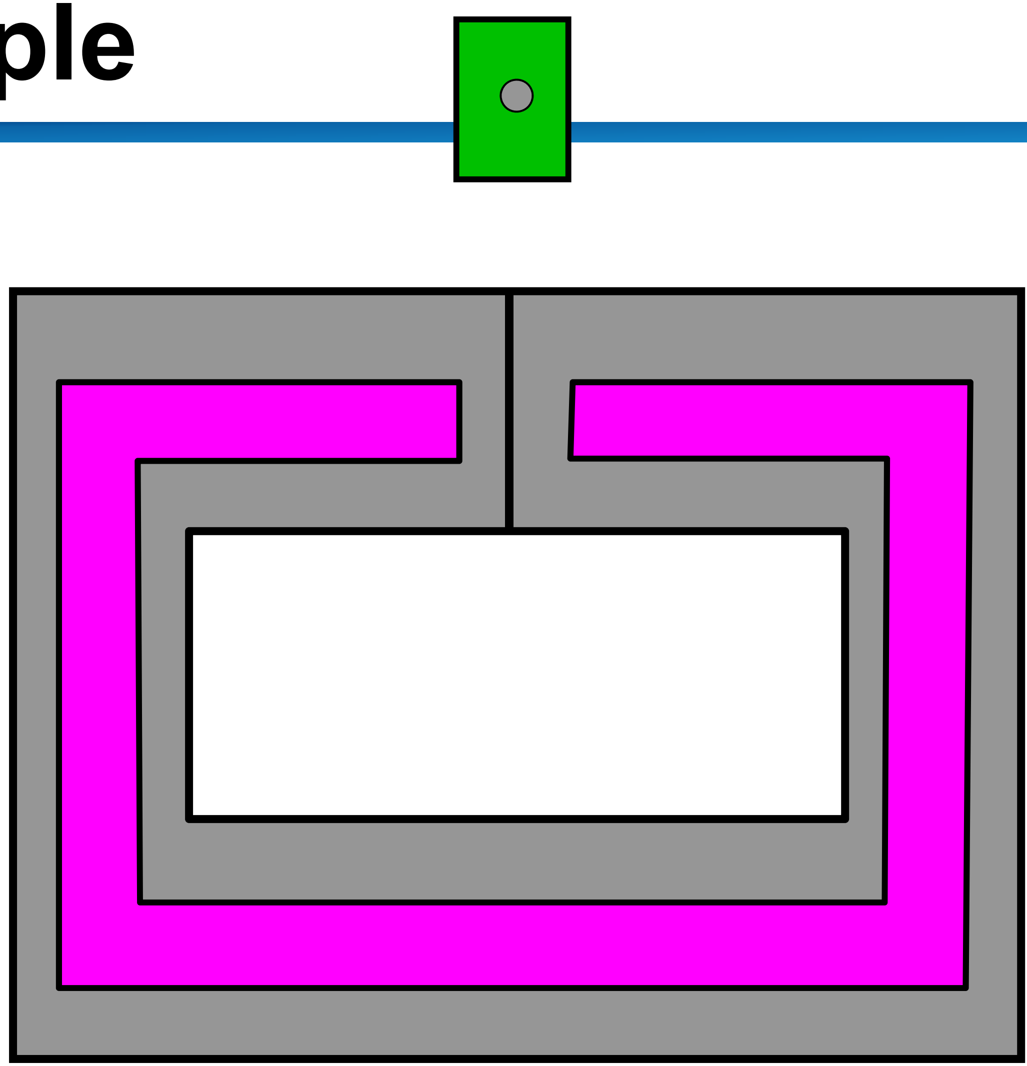
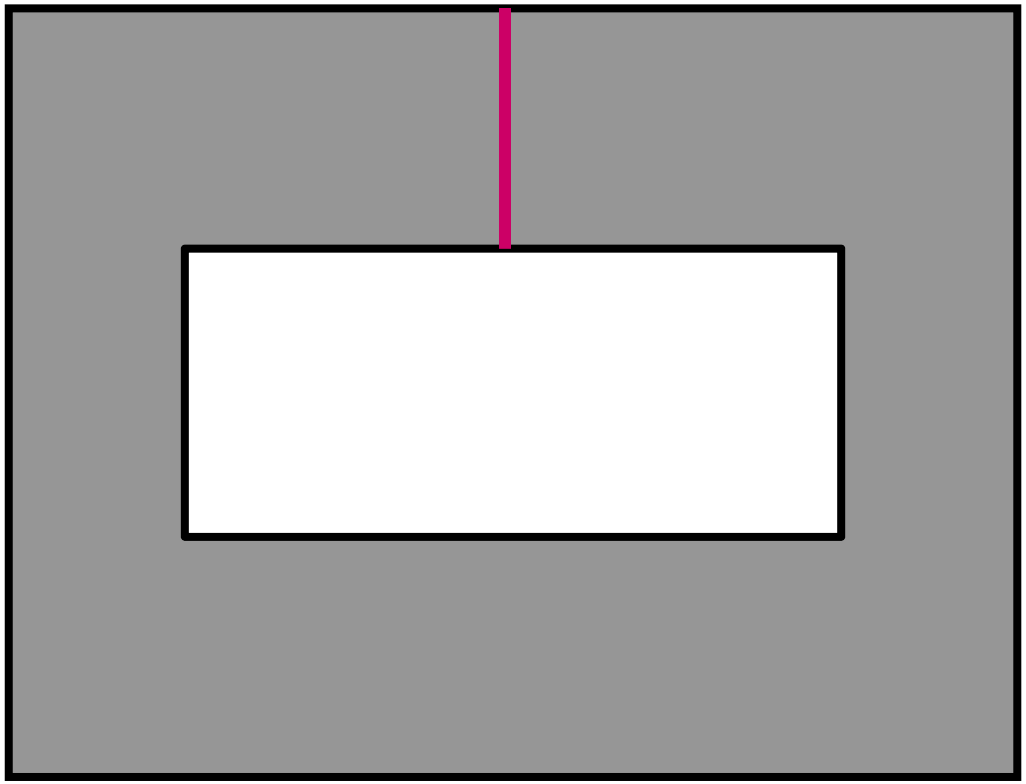
Semi-Free Space

- A configuration q is **semi-free** if the moving object placed q touches the boundary, but not the interior of obstacles.
  - Free, or
  - In contact
- The semi-free space is a closed subset of C

Example



Example



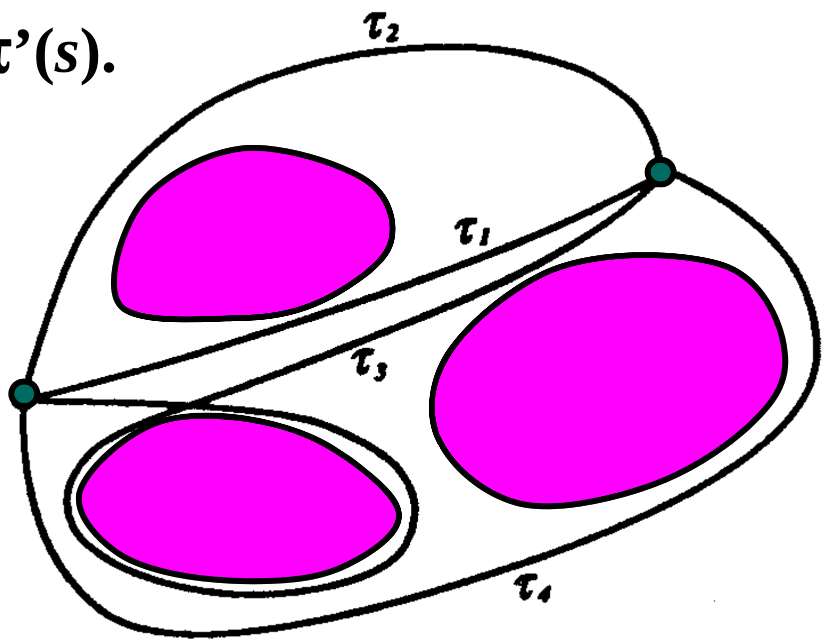
Homotopic Paths

- **Two paths τ and τ' (that map from U to V) with the same endpoints are homotopic if one can be continuously deformed into the other:**

$$h : U \times [0,1] \to V$$

with $h(s,0) = \tau(s)$ and $h(s,1) = \tau'(s)$.

- **A homotopic class of paths contains all paths that are homotopic to one another**



Connectedness of C-Space

- C is **connected** if every two configurations can be connected by a path.

- C is **simply-connected** if any two paths connecting the same endpoints are homotopic.
  Examples: R^2 or R^3

- Otherwise C is multiply-connected.

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Configuration space

- Definitions and examples
- Obstacles
- Paths
- Metrics

Metric in Configuration Space

- A **metric** or **distance** function d in a configuration space C is a function
  $$d : (q, q') \in C^2 \rightarrow d(q, q') \geq 0$$
  such that
  - $d(q, q') = 0$ if and only if $q = q'$,
  - $d(q, q') = d(q', q)$,
  - $d(q, q') \leq d(q, q'') + d(q'', q')$.

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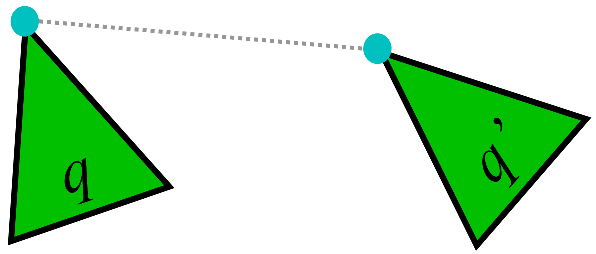
Example

- Robot A and a point x on A
- $x(q)$: position of x in the workspace when A is at configuration q
- A distance d in C is defined by

$$d(q, q') = \max_{x \in A} \| x(q) - x(q') \|,$$

where $||x - y||$ denotes the Euclidean distance between points x and y in the workspace.



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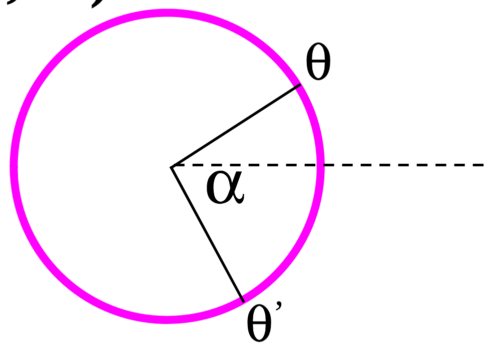
L_p Metrics

$$d(x, x') = \left(\sum_{i=1}^{n} | x_i - x_i' |^p \right)^{1/p}$$

- L_2: Euclidean metric
- L_1: Manhattan metric
- L_∞: Max $(| x_i - x_i' |)$

Examples in R^2 x S^1

- **Consider R^2 x S^1**
  - $q = (x, y, \theta)$, $q' = (x', y', \theta')$ **with** $\theta, \theta' \in [0, 2\pi)$
  - $\alpha = \min \{ |\theta - \theta'|, 2\pi - |\theta - \theta'| \}$

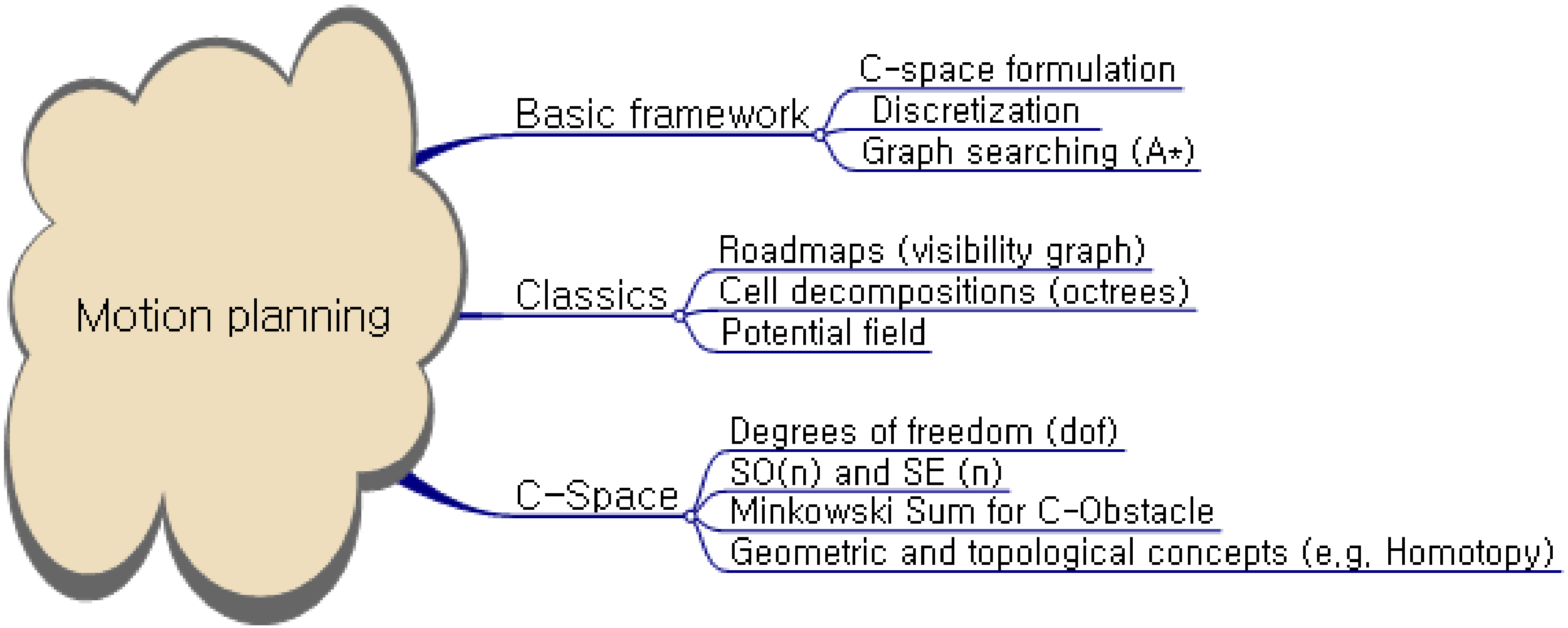


- $d(q, q') = \text{sqrt}((x-x')^2 + (y-y')^2 + \alpha^2))$

Class Objectives were:

- **Configuration space**
  - **Definitions and examples**
  - **Obstacles**
  - **Paths**
  - **Metrics**

Summary



Motion planning
Basic framework
C-space formulation
Discretization
Graph searching (A*)
Classics
Roadmaps (visibility graph)
Cell decompositions (octrees)
Potential field
C-Space
Degrees of freedom (dof)
SO(n) and SE (n)
Minkowski Sum for C-Obstacle
Geometric and topological concepts (e.g. Homotopy)

Next Time….

- **Collision detection and distance computation**

Homework

- **Submit summaries of 2 ICRA/IROS/RSS/WAFR/TRO/IJRR papers**
- **Go over the next lecture slides**
- **Come up with one question on what we have discussed today and submit at the end of the class**

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