CS680:
Radiometry

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(윤성의)

Course URL:
http://jupiter.kaist.ac.kr/~sungeui/SGA/

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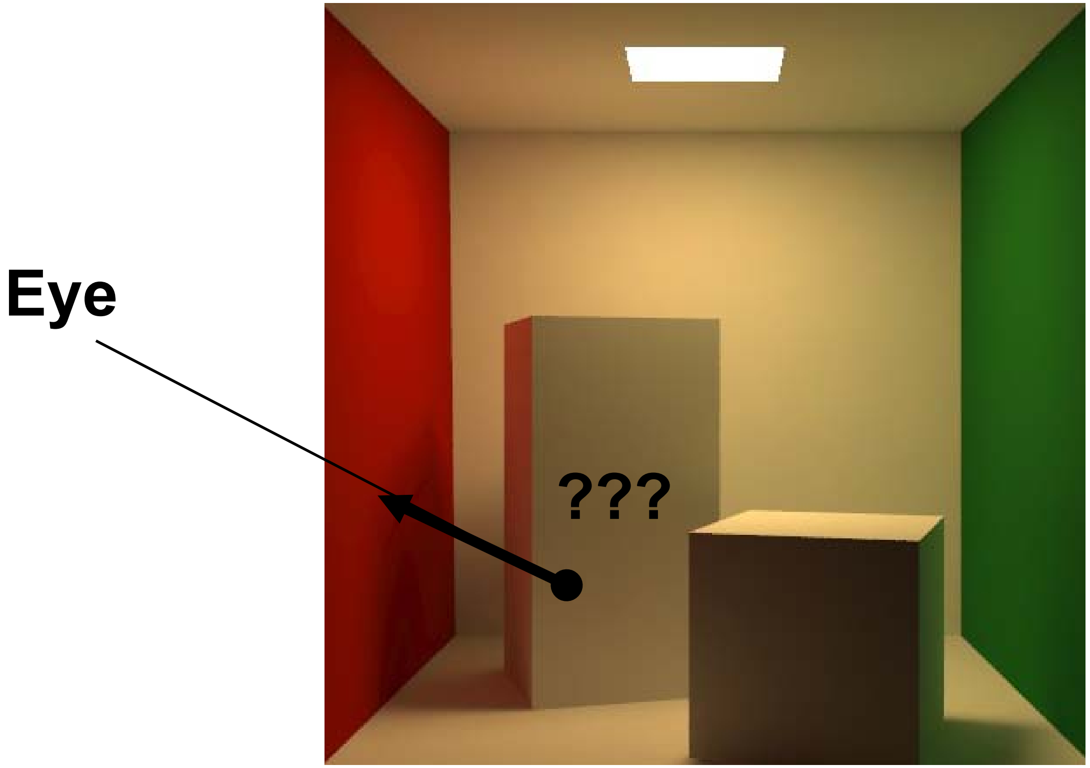
Announcements

- 2 papers for each student
  - Choose 4 papers from the paper list
  - Send them (titles of 4 papers) to TA (Bochang Moon) by Oct-11 (Mon)
  - Look at videos and talk files (captured talk video or presentation files)

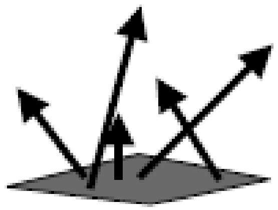
- Schedule of student presentations
  - Will be decided on Oct-12 (Tue)
  - Presentations will start after the mid-term

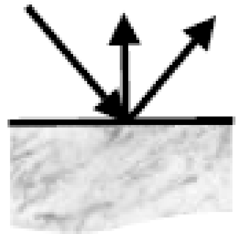
Motivation



Light and Material Interactions

- Physics of light





- Radiometry
- Material properties

From kavita's slides

- Rendering equation

Models of Light

- Quantum optics
  - Fundamental model of the light
  - Explain the dual wave-particle nature of light
- Wave model
  - Simplified quantum optics
  - Explains diffraction, interference, and polarization
- Geometric optics
  - Most commonly used model in CG
  - Size of objects >> wavelength of light
  - Light is emitted, reflected, and transmitted

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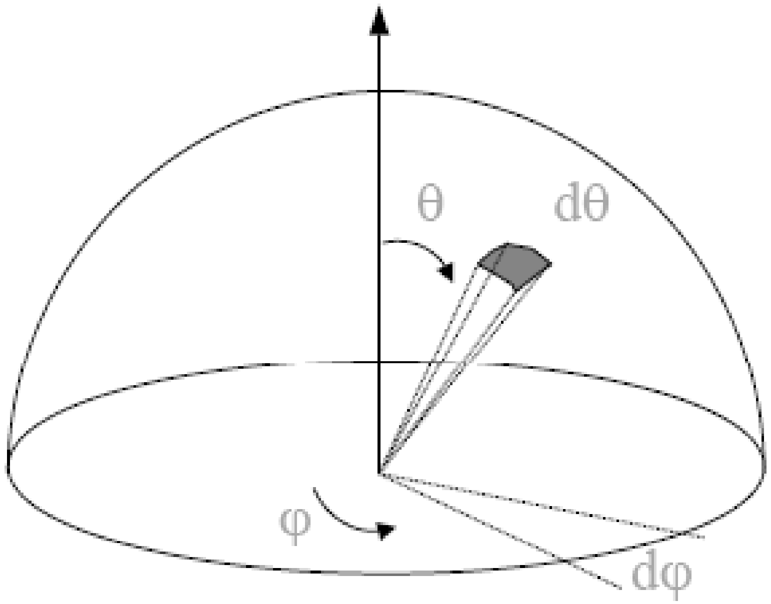
Radiometry

- **Measurement of light energy**
  - **Critical component for photo-realistic rendering**
- **Light energy flows through space**
  - **Varies with time, position, and direction**
- **Radiometric quantities**
  - **Densities of energy at particular places in time, space, and direction**
- **Photometry**
  - **Quantify the perception of light energy**

Hemispheres

- **Hemisphere**
  - **Two-dimensional surfaces**
- **Direction**
  - **Point on (unit) sphere**



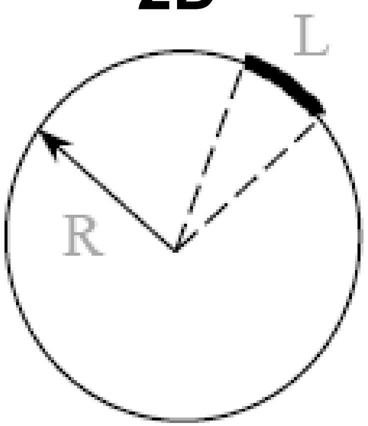
$$\theta \in [0, \frac{\pi}{2}]$$

$$\varphi \in [0, 2\pi]$$

From kavita's slides

Solid Angles

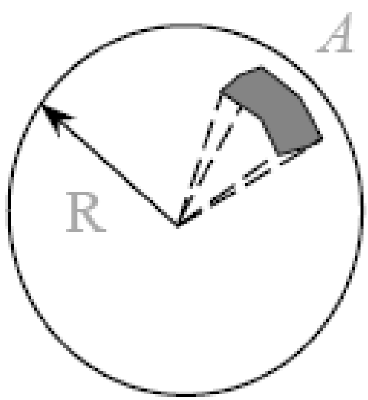
2D



$$\theta = \frac{L}{R}$$

Full circle = 2pi radians

3D

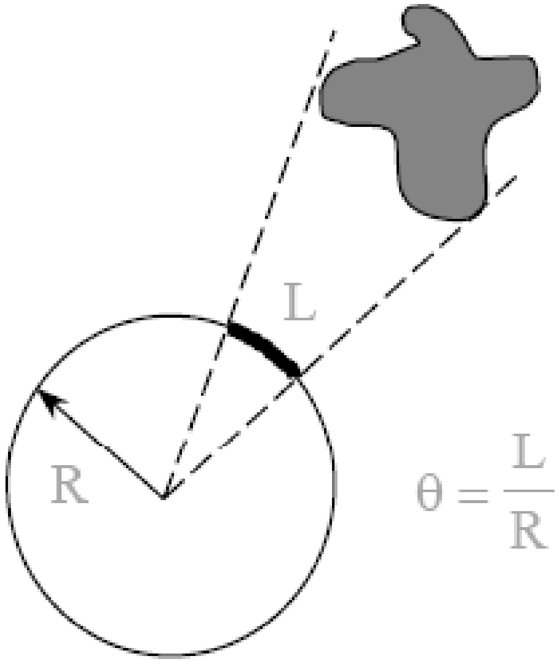


$$\Omega = \frac{A}{R^2}$$

Full sphere = 4pi steradians

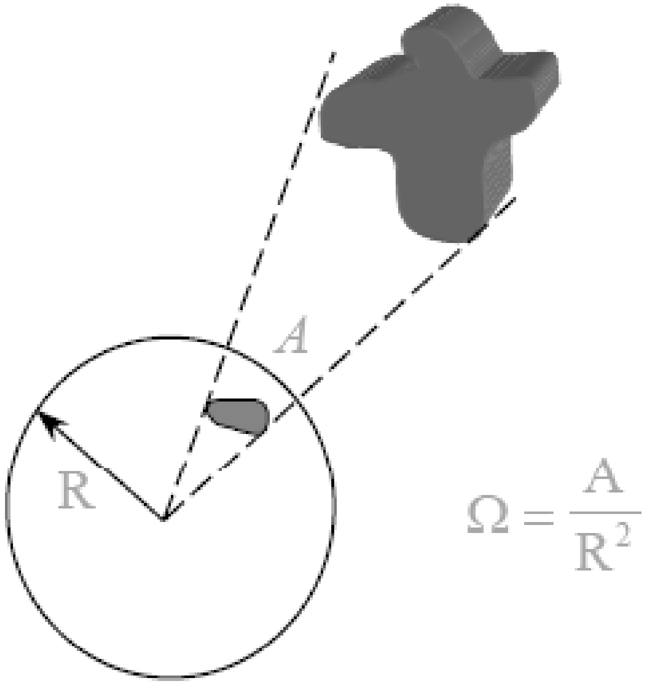
Solid Angles

2D



Full circle
= 2pi radians

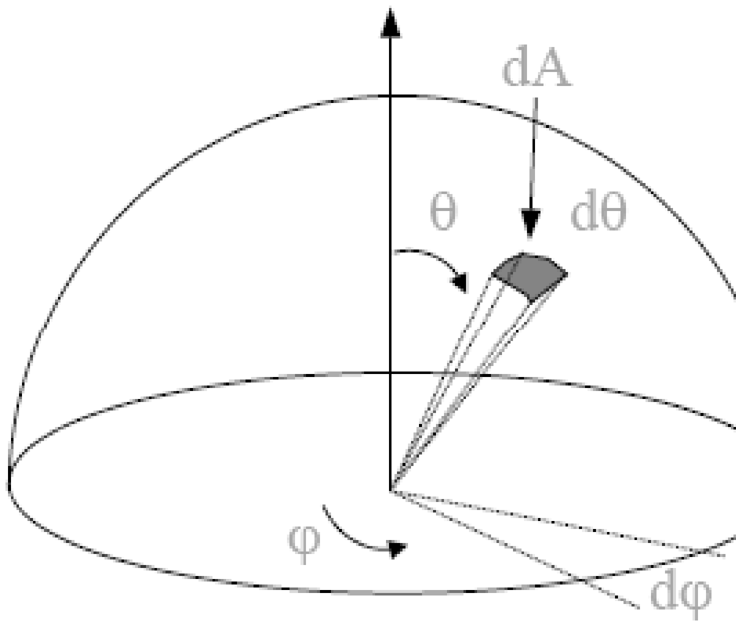
3D



Full sphere
= 4pi steradians

Hemispherical Coordinates

- **Direction, Θ**
  - **Point on (unit) sphere**



$$dA = (r \sin\theta d\varphi)(r d\theta)$$

From kavita's slides

Hemispherical Coordinates

- **Differential solid angle**

$$d\omega = \frac{dA}{r^2} = \sin\theta d\theta d\varphi$$

Hemispherical Integration

- **Area of hemispehre:**

$$\int_{\Omega_x} d\omega = \int_0^{2\pi} d\varphi \int_0^{\pi/2} \sin\theta d\theta$$

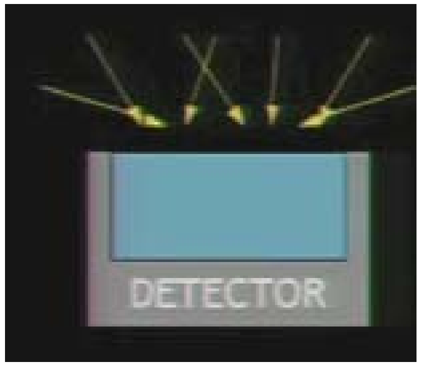
$$= \int_0^{2\pi} d\varphi \left[-\cos\theta\right]_0^{\pi/2}$$

$$= \int_0^{2\pi} d\varphi$$

$$= 2\pi$$

Energy

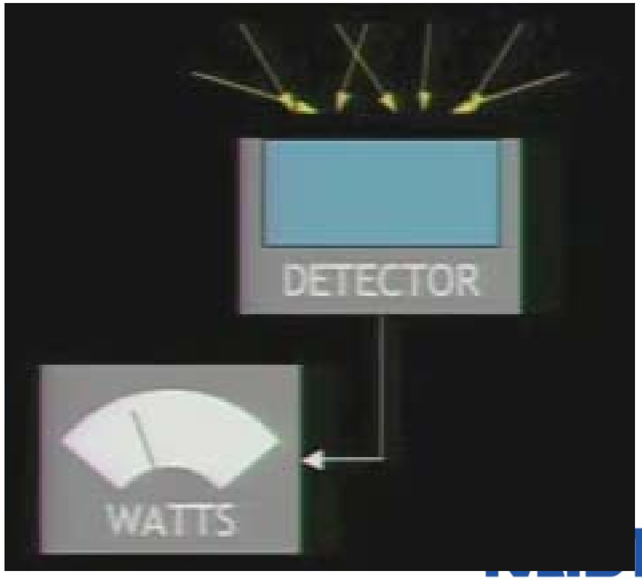
- **Symbol: Q**
  - **# of photons in this context**
  - **Unit: Joules**



From Steve Marschner's talk

Power (or Flux)

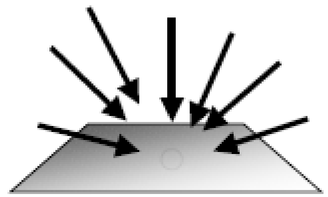
- **Symbol, P or Φ**
  - **Total amount of energy through a surface per unit time, dQ/dt**
  - **Radiant flux in this context**
  - **Unit: Watts (=Joules / sec.)**
  - **Other quantities are derivatives of P**

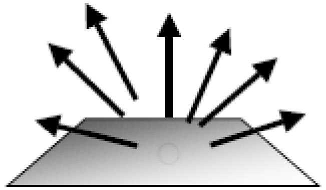


- **Example**
  - **A light source emits 50 watts of radiant power**
  - **20 watts of radiant power is incident on a table**

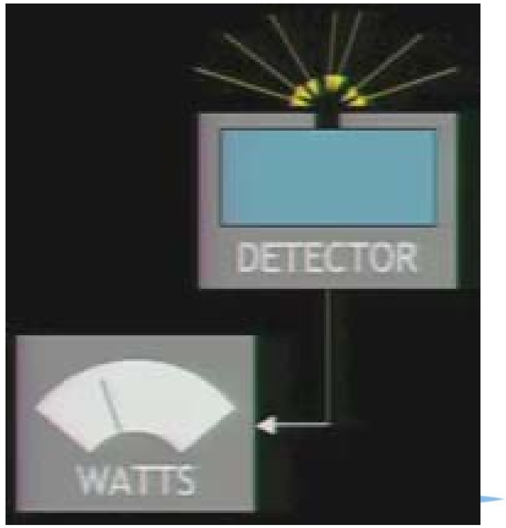
Irradiance

- **Incident radiant power per unit area (dP/dA)**
  - **Area density of power**





- **Symbol: E, unit: W/ m^2**
  - **Area power density existing a surface is called radiance exitance (M) or radiosity (B)**

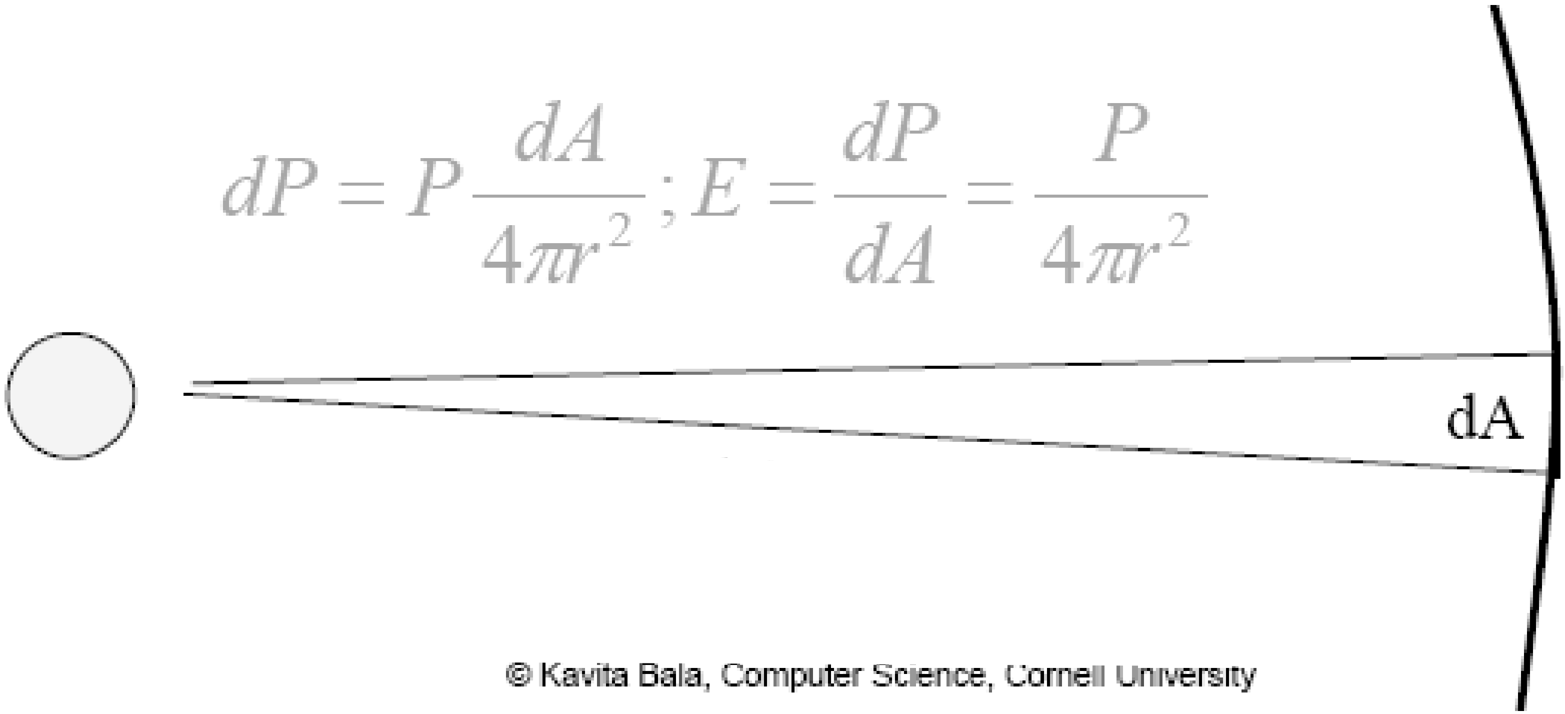


- **For example**
  - **A light source emitting 100 W of area 0.1 m^2**
  - **Its radiant exitance is 1000 W/ m^2**

Irradiance Example

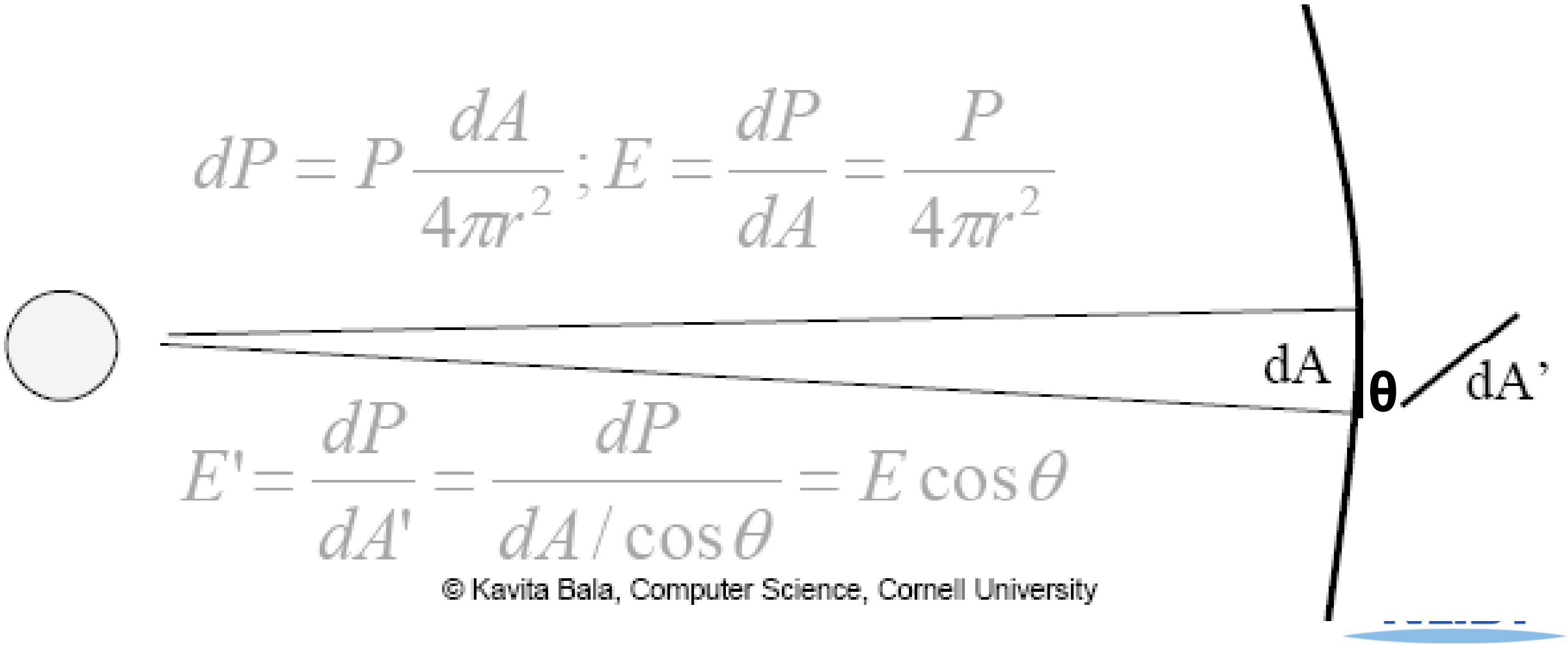
- **Uniform point source illuminates a small surface dA from a distance r**
  - **Power P is uniformly spread over the area of the sphere**

$$dP = P\frac{dA}{4\pi r^2}; E = \frac{dP}{dA} = \frac{P}{4\pi r^2}$$



Irradiance Example

- **Uniform point source illuminates a small surface dA from a distance r**
  - **Power P is uniformly spread over the area of the sphere**

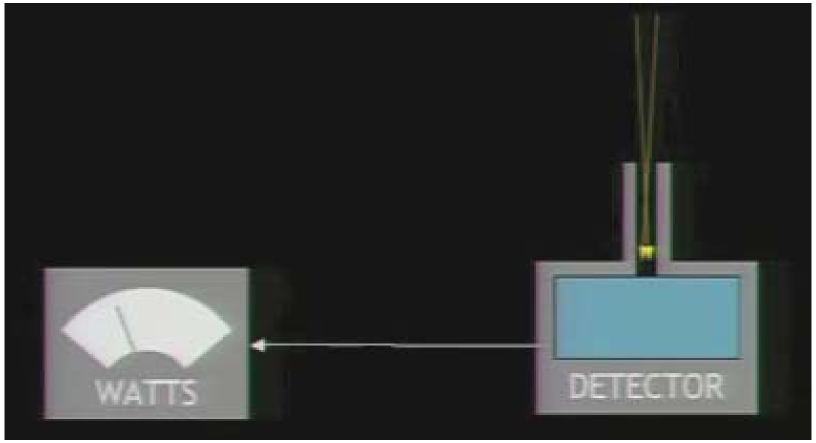


$$dP = P\frac{dA}{4\pi r^2}; E = \frac{dP}{dA} = \frac{P}{4\pi r^2}$$

$$E' = \frac{dP}{dA'} = \frac{dP}{dA/\cos\theta} = E\cos\theta$$

Radiance

- Radiant power at x in direction θ
  - $L(x \rightarrow \Theta)$: 5D function
    - Per unit area
    - Per unit solid angle

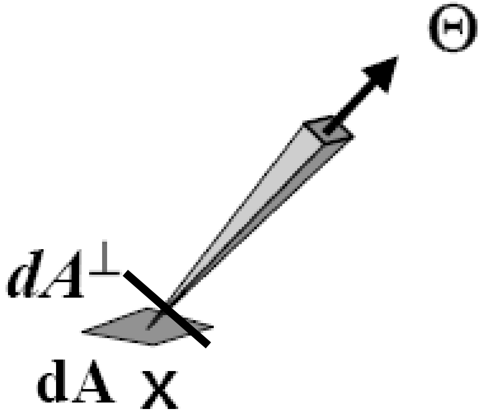


- Important quantity for rendering

Radiance

- **Radiant power at x in direction θ**
  - $L(x \rightarrow \Theta)$ **: 5D function**
    - **Per unit area**
    - **Per unit solid angle**

$$L(x \rightarrow \Theta) = \frac{d^2 P}{dA^{\perp} d\omega_{\Theta}}$$

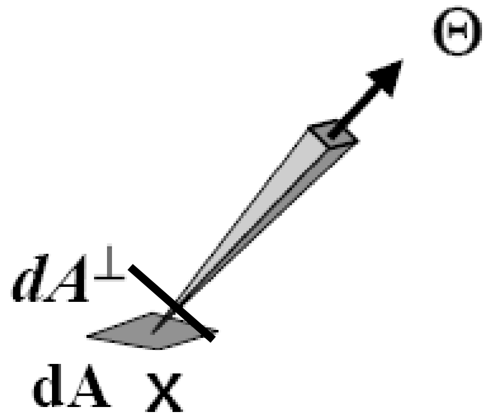


  - **Units: Watt / (m^2 sr)**
  - **Irradiance per unit solid angle**
  - **2^{nd} derivative of P**
  - **Most commonly used term**

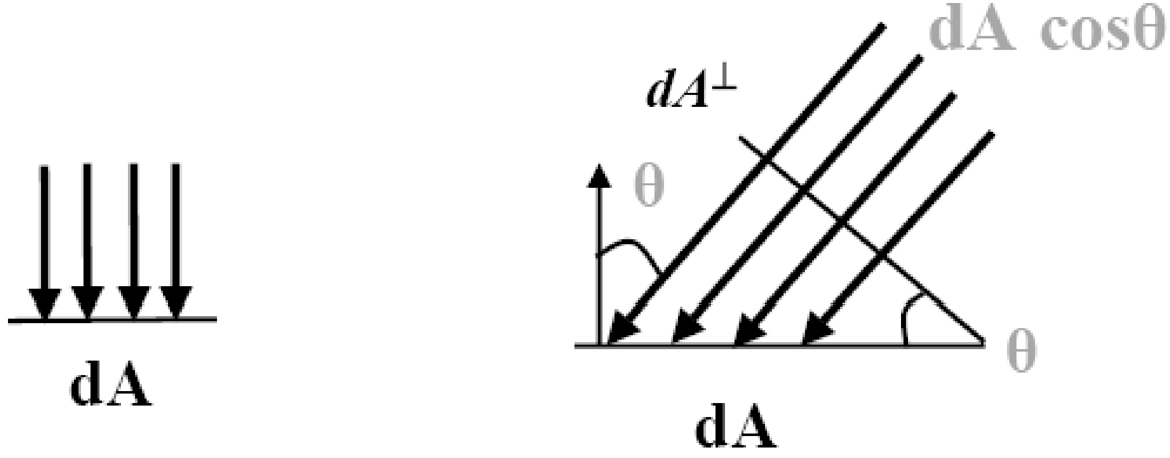
Radiance: Projected Area

$$L(x \to \Theta) = \frac{d^2P}{dA^{\perp} d\omega_{\Theta}}$$

$$= \frac{d^2 P}{d\omega_{\Theta}\, dA \cos\theta}$$

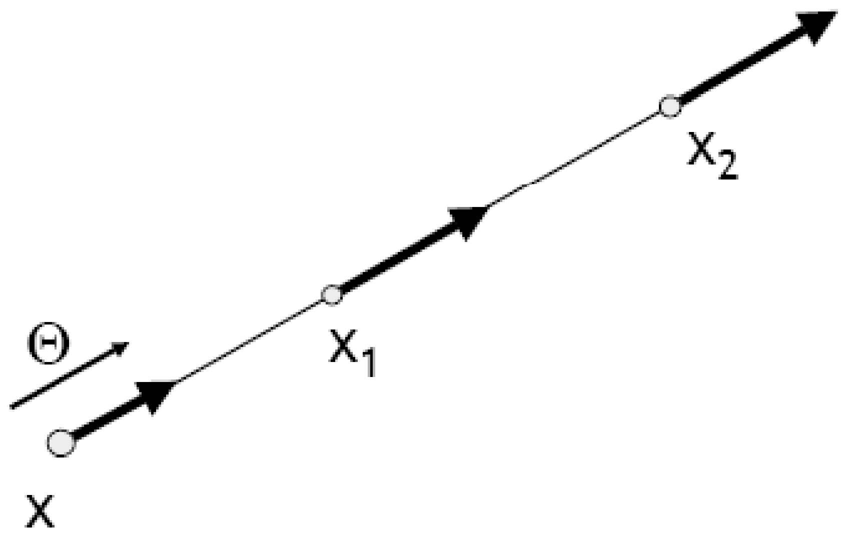


- **Why per unit projected surface area**



Properties of Radiance

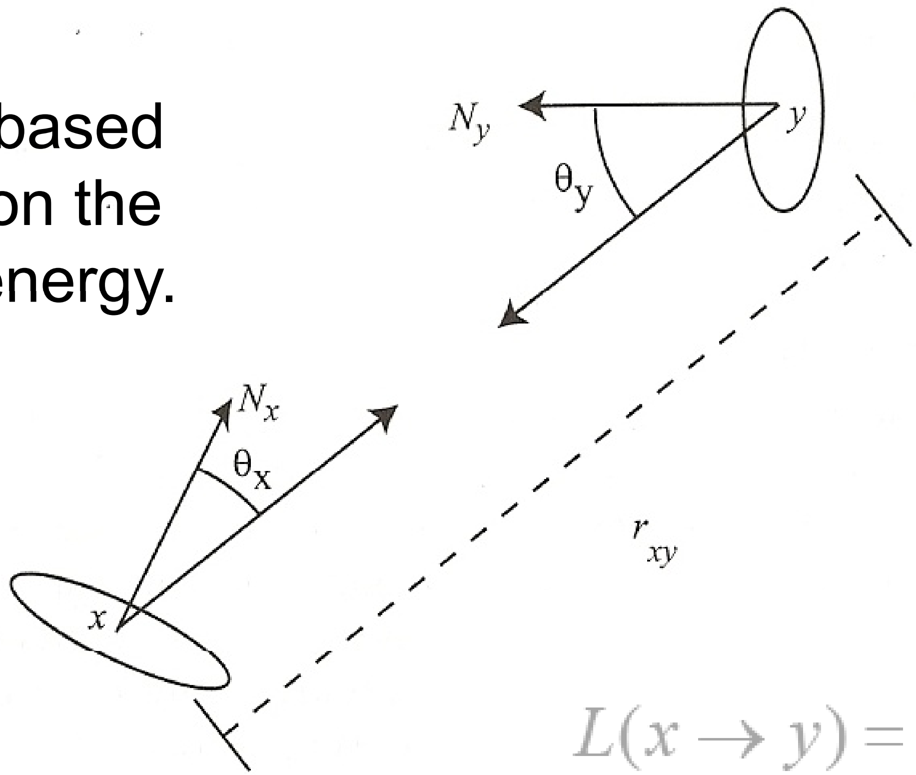
- **Invariant along a straight line (in vacuum)**



From kavita's slides

Invariance of Radiance

We can prove it based on the assumption the conservation of energy.

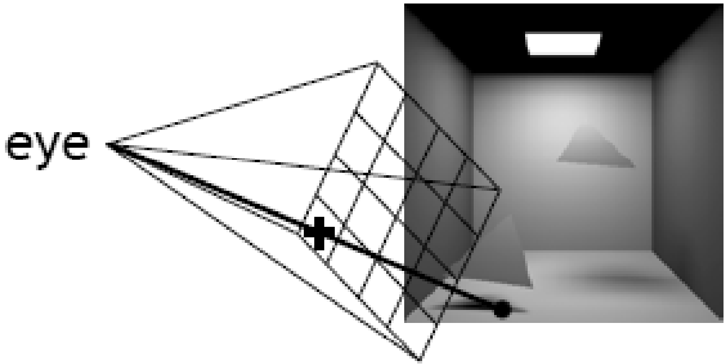


$$L(x \rightarrow y) = L(y \leftarrow x)$$

Figure 2.3. Invariance of radiance.

Sensitivity to Radiance

- **Responses of sensors (camera, human eye) is proportional to radiance**



From kavita's slides

- **Pixel values in image proportional to radiance received from that direction**

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Relationships

- **Radiance is the fundamental quantity**

$$L(x \rightarrow \Theta) = \frac{d^2 P}{dA^{\perp} d\omega_\Theta}$$

- **Power:**

$$P = \int_{Area} \int_{\substack{Solid\\Angle}} L(x \rightarrow \Theta) \cdot \cos\theta \cdot d\omega_\Theta \cdot dA$$

- **Radiosity:**

$$B = \int_{\substack{Solid\\Angle}} L(x \rightarrow \Theta) \cdot \cos\theta \cdot d\omega_\Theta$$

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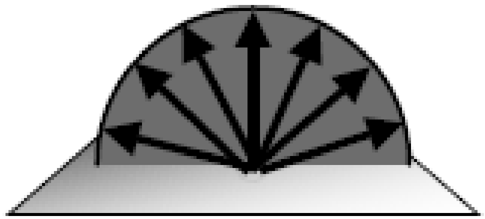
Example: Diffuse emitter

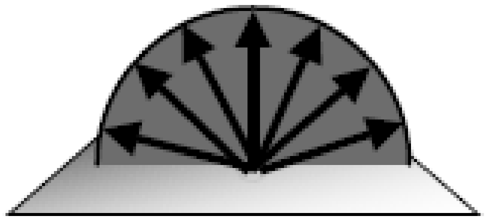
- Diffuse emitter: light source with equal radiance everywhere

$$L(x \rightarrow \Theta) = \frac{d^2 P}{dA^{\perp} d\omega_{\Theta}}$$



Example: Diffuse emitter

- Diffuse emitter: light source with equal radiance everywhere

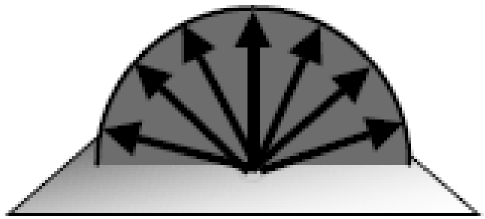


$$L(x \to \Theta) = \frac{d^2 P}{dA^{\perp} d\omega_{\Theta}}$$

$$P = \int_{Area} \int_{Solid\ Angle} L(x \to \Theta) \cdot \cos\theta \cdot d\omega_{\Theta} \cdot dA$$

Example: Diffuse emitter

- Diffuse emitter: light source with equal radiance everywhere



$$L(x \rightarrow \Theta) = \frac{d^2 P}{dA^{\perp} d\omega_{\Theta}}$$

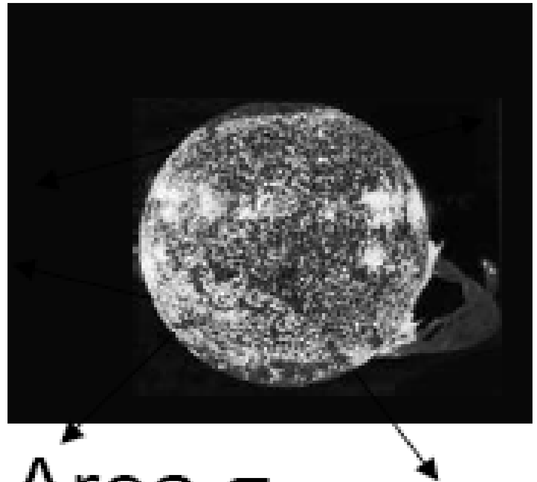
$$P = \int_{Area} \int_{Solid\ Angle} L(x \rightarrow \Theta) \cdot \cos\theta \cdot d\omega_{\Theta} \cdot dA$$

$$= L \int_{Area} dA \int_{Solid\ Angle} \cos\theta \cdot d\omega_{\Theta}$$

$$= L \cdot Area \cdot \pi$$

Sun Example: radiance

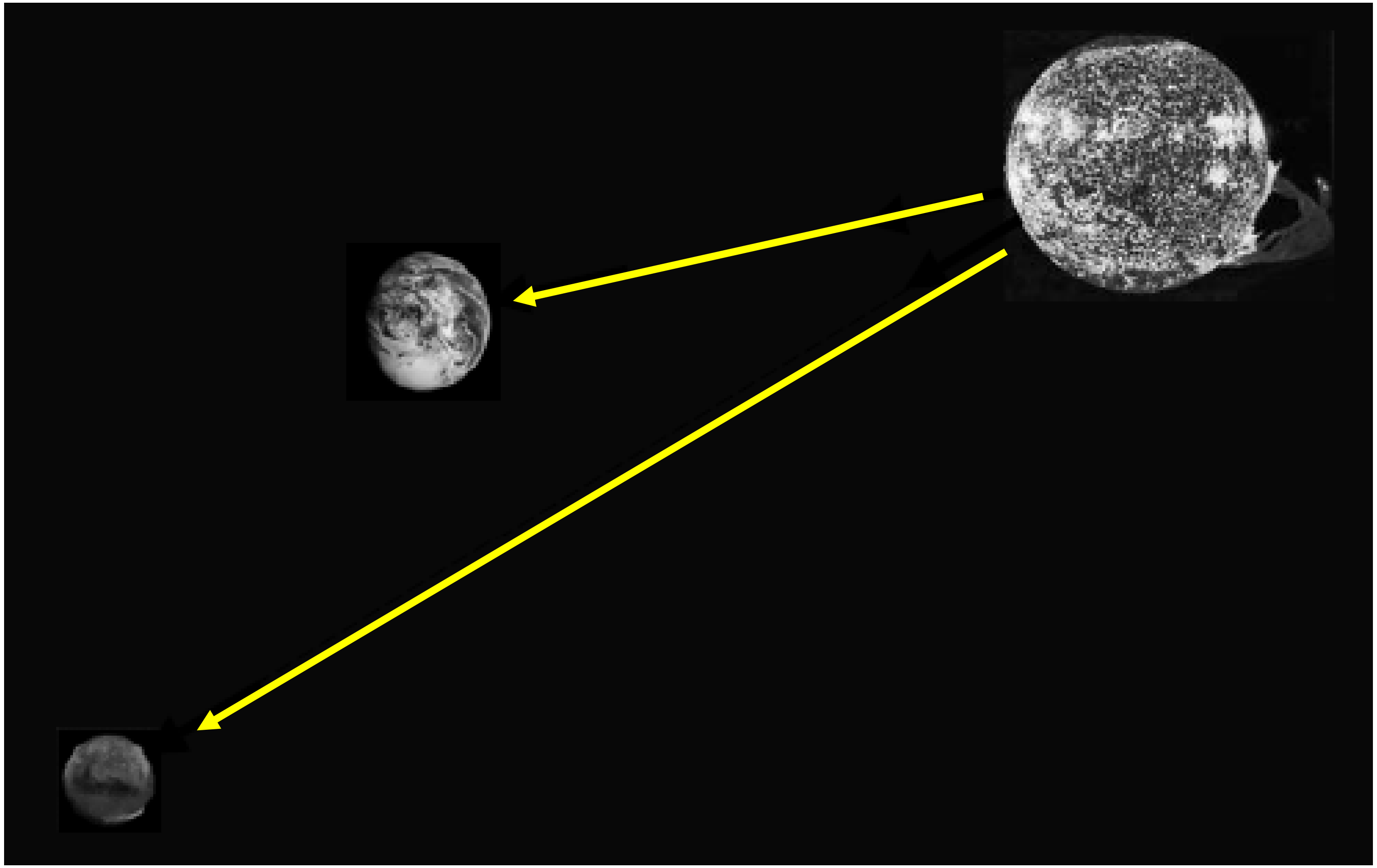
- Power: 3.91×10^{26} W
- Surface Area: 6.07×10^{18} m^2



- Power = Radiance.Surface Area.π
- Radiance = Power/(Surface Area.π)

- Radiance = 2.05×10^7 W/ m^2.sr

Sun Example

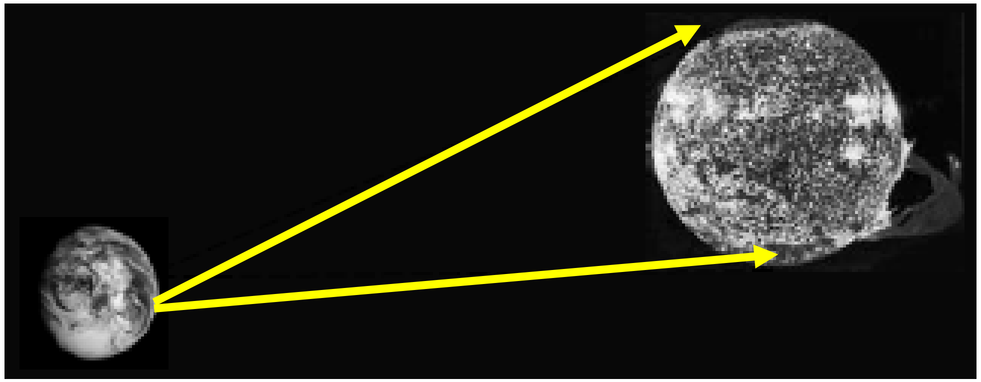


Same radiance on Earth and Mars?

Sun Example: Power on Earth

- Power reaching earth on a $1m^2$ square:

$$P = L \int_{Area} dA \int_{Solid\ Angle} \cos\theta \cdot d\omega_{\Theta}$$

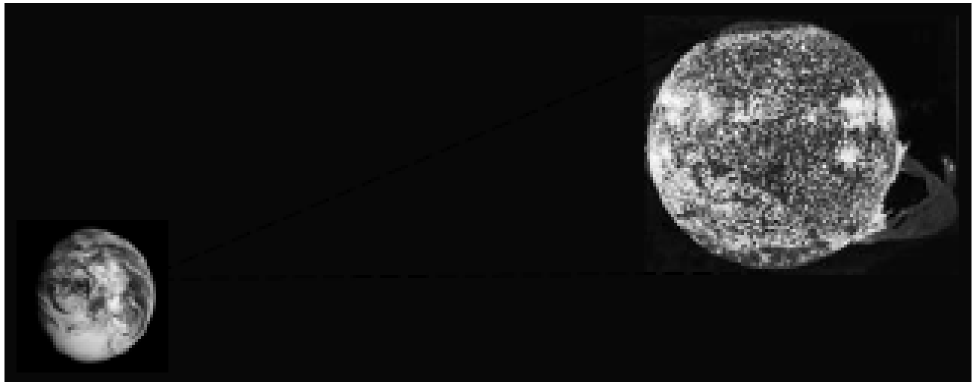


- Assume $\cos\theta = 1$ (sun in zenith)

$$P = L \int_{Area} dA \int_{Solid\ Angle} d\omega_{\Theta}$$

Sun Example: Power on Earth

Power = Radiance.Area.Solid Angle



Solid Angle = Projected $\text{Area}_{\text{Sun}}/(\text{distance}_{\text{earth\_sun}})^2$
$= 6.7\ 10^{-5}$ sr

P = $(2.05 \times 10^7$ W/ m^2.sr) x (1 m^2) x $(6.7\ 10^{-5}$ sr)
= 1373.5 Watt

Sun Example: Power on Mars

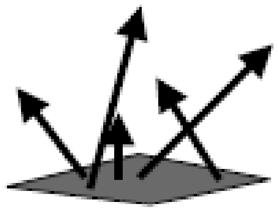
Power = Radiance.Area.Solid Angle

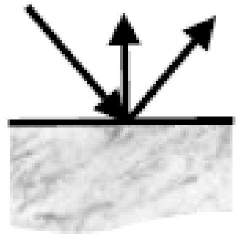
Solid Angle = Projected $Area_{Sun}$/$(distance_{mars\_sun})^2$
= $2.92\ 10^{-5}$ sr

P = (2.05×10^7 W/ m^2.sr) x (1 m^2) x ($2.92\ 10^{-5}$ sr)
= 598.6 Watt

Light and Material Interactions

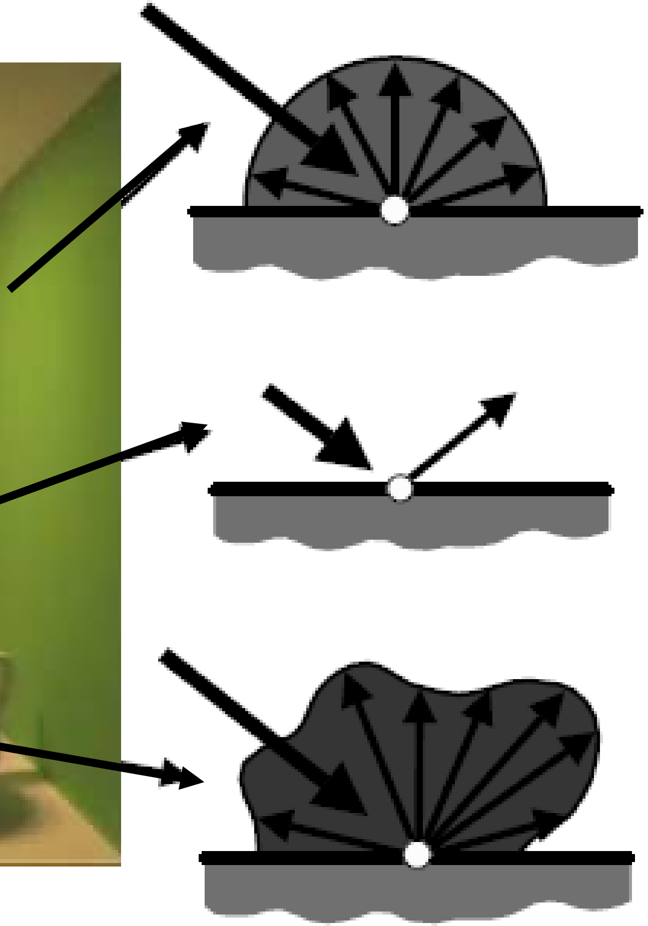
- Physics of light
- Radiometry
- **Material properties**
- Rendering equation





From kavita's slides

Materials



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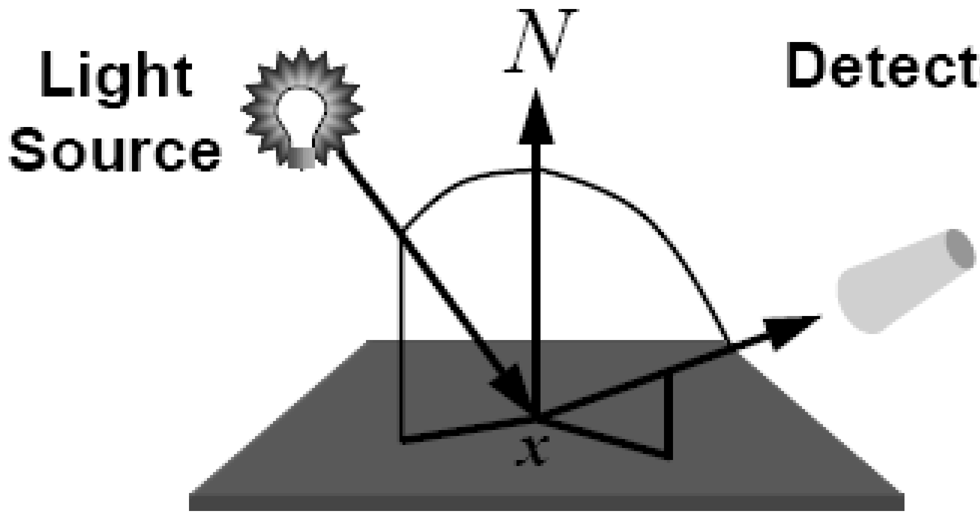
Bidirectional Reflectance Distribution Function (BRDF)

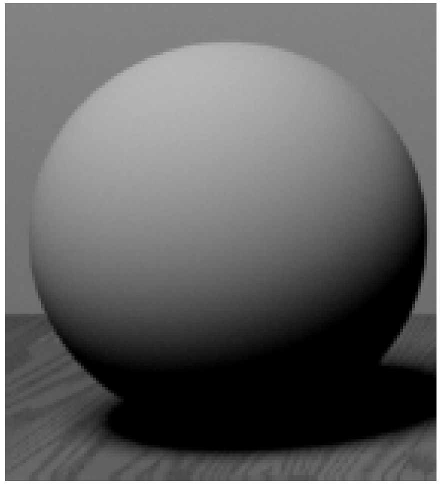


$$f_r(x, \Psi \to \Theta) = \frac{dL(x \to \Theta)}{dE(x \leftarrow \Psi)} = \frac{dL(x \to \Theta)}{L(x \leftarrow \Psi)\cos(N_x, \Psi)d\omega_\Psi}$$

BRDF special case: ideal diffuse

Pure Lambertian

$$f_r(x, \Psi \rightarrow \Theta) = \frac{\rho_d}{\pi}$$



$$\rho_d = \frac{Energy_{out}}{Energy_{in}} \qquad 0 \leq \rho_d \leq 1$$

Properties of the BRDF

- Reciprocity:

$$f_r(x, \Psi \to \Theta) = f_r(x, \Theta \to \Psi)$$

- Therefore, notation: $f_r(x, \Psi \leftrightarrow \Theta)$

- Important for bidirectional tracing

Properties of the BRDF

- Bounds:

$$0 \leq f_r(x, \Psi \leftrightarrow \Theta) \leq \infty$$

- Energy conservation:

$$\forall \Psi \quad \int_{\Theta} f_r(x, \Psi \leftrightarrow \Theta) \cos(N_x, \Theta) d\omega_{\Theta} \leq 1$$

Homework

Next Time

- **Rendering equation**