CS680:

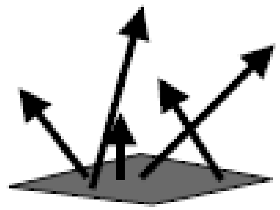
Rendering Equation

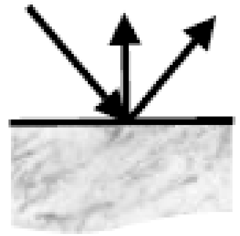
Sung-Eui Yoon
(윤성의)

Course URL:
http://jupiter.kaist.ac.kr/~sungeui/SGA/

Light and Material Interactions

- **Physics of light**
- **Radiometry**
- **Material properties**





From kavita's slides

- **Rendering equation**

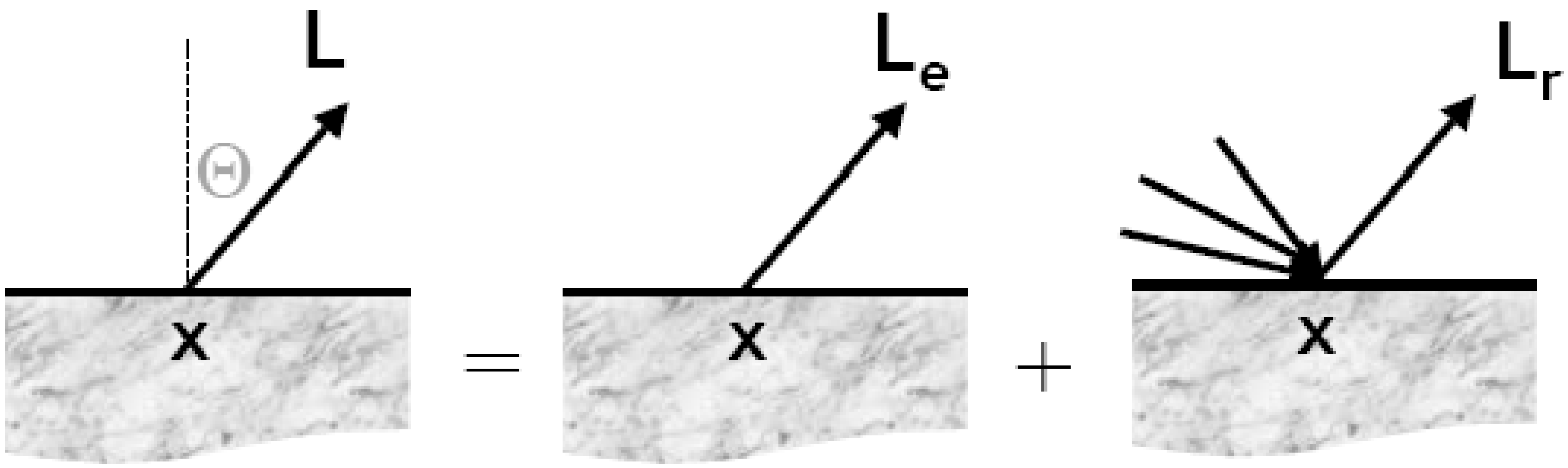
Light Transport

- Goal
  - Describe steady-state radiance distribution in scene
- Assumption
  - Geometric optics
  - Achieves steady state instantaneously

Rendering Equation

- Describes energy transport in scene
- Input
  - Light sources
  - Surface geometry
  - Reflectance characteristics of surfaces
- Output
  - Value of radiances at all surface points in all directions

Rendering Equation



$$L(x \to \Theta) = L_e(x \to \Theta) + L_r(x \to \Theta)$$

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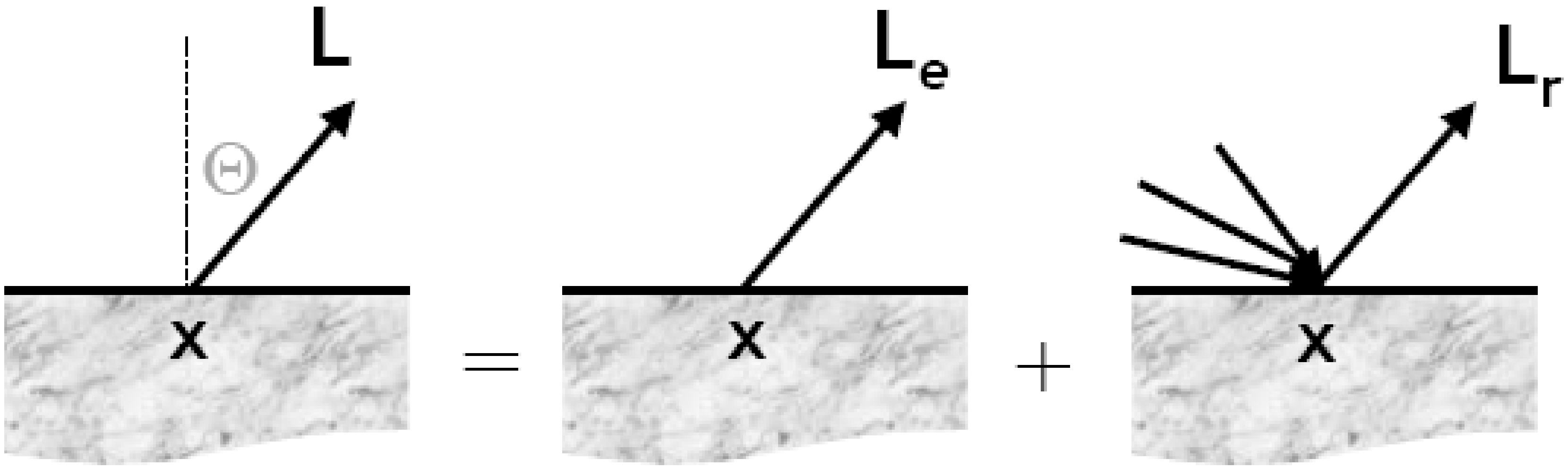
Rendering Equation



$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{hemisphere} L(x \leftarrow \Psi) \cdots$$

Rendering Equation

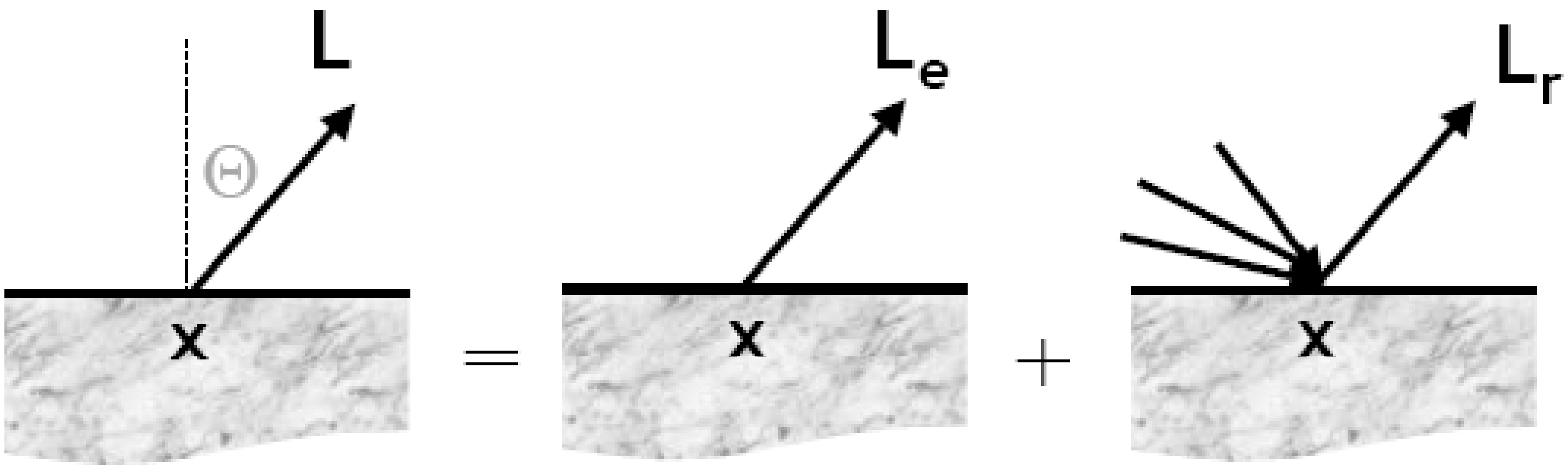
$$f_r(x, \Psi \leftrightarrow \Theta) = \frac{dL(x \to \Theta)}{dE(x \leftarrow \Psi)}$$

$$dL(x \to \Theta) = f_r(x, \Psi \leftrightarrow \Theta) dE(x \leftarrow \Psi)$$

$$dL(x \to \Theta) = f_r(x, \Psi \leftrightarrow \Theta) L(x \leftarrow \Psi) \cos(N_x, \Psi) d\omega_\Psi$$

$$L_r(x \to \Theta) = \int_{hemisphere} f_r(x, \Psi \leftrightarrow \Theta) L(x \leftarrow \Psi) \cos(N_x, \Psi) d\omega_\Psi$$

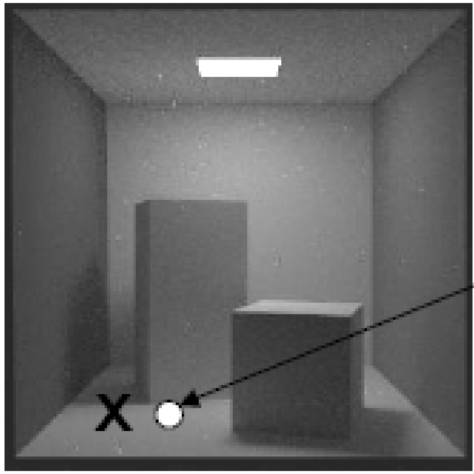
Rendering Equation



$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{hemisphere} L(x \leftarrow \Psi)\, f_r(x, \Psi \leftrightarrow \Theta) \cos(\mathbf{N}_x, \Psi)\, d\omega_\Psi$$

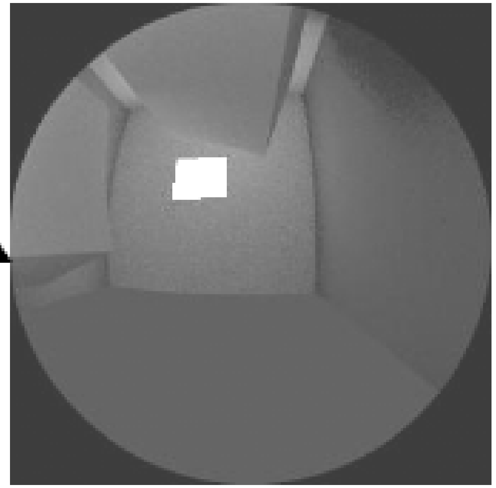
- Applicable for each wavelength

Rendering Equation



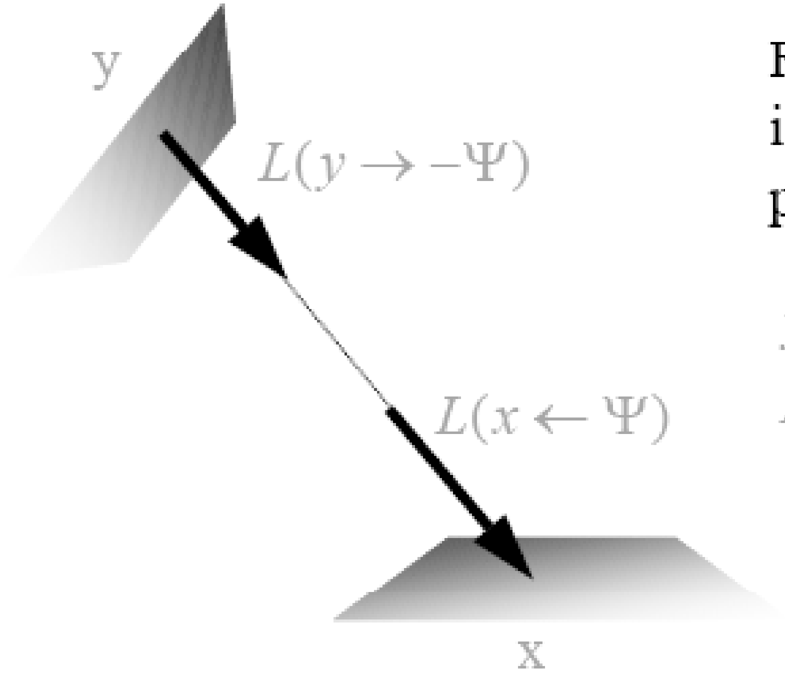
$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{hemisphere} L(x \leftarrow \Psi) f_r(x, \Psi \leftrightarrow \Theta) \cos(N_x, \Psi) d\omega_\Psi$$

incoming radiance



Rendering Equation: Area Formulation

$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos\theta_x \cdot d\omega_\Psi$$

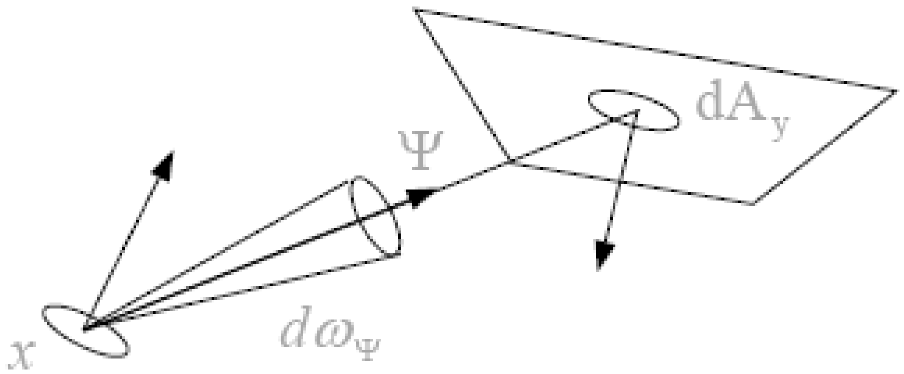


Ray-casting function: what is the nearest visible surface point seen from x in direction Ψ?

$$y = vp(x, \Psi)$$
$$L(x \leftarrow \Psi) = L(vp(x, \Psi) \to -\Psi)$$

Rendering Equation

$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos\theta_x \cdot d\omega_\Psi$$



$$y = vp(x, \Psi)$$

$$L(x \leftarrow \Psi) = L(vp(x, \Psi) \to -\Psi)$$

$$d\omega_\Psi = \frac{dA_y \cos\theta_y}{r_{xy}^2}$$

Rendering Equation: Visible Surfaces

$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos\theta_x \cdot d\omega_\Psi$$

Coordinate transform $\downarrow$

$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{\substack{y \text{ on} \\ \text{all surfaces}}} f_r(\Psi \leftrightarrow \Theta) \cdot L(y \to -\Psi) \ \cos\theta_x \cdot \frac{\cos\theta_y}{r_{xy}^2} \cdot dA_y$$

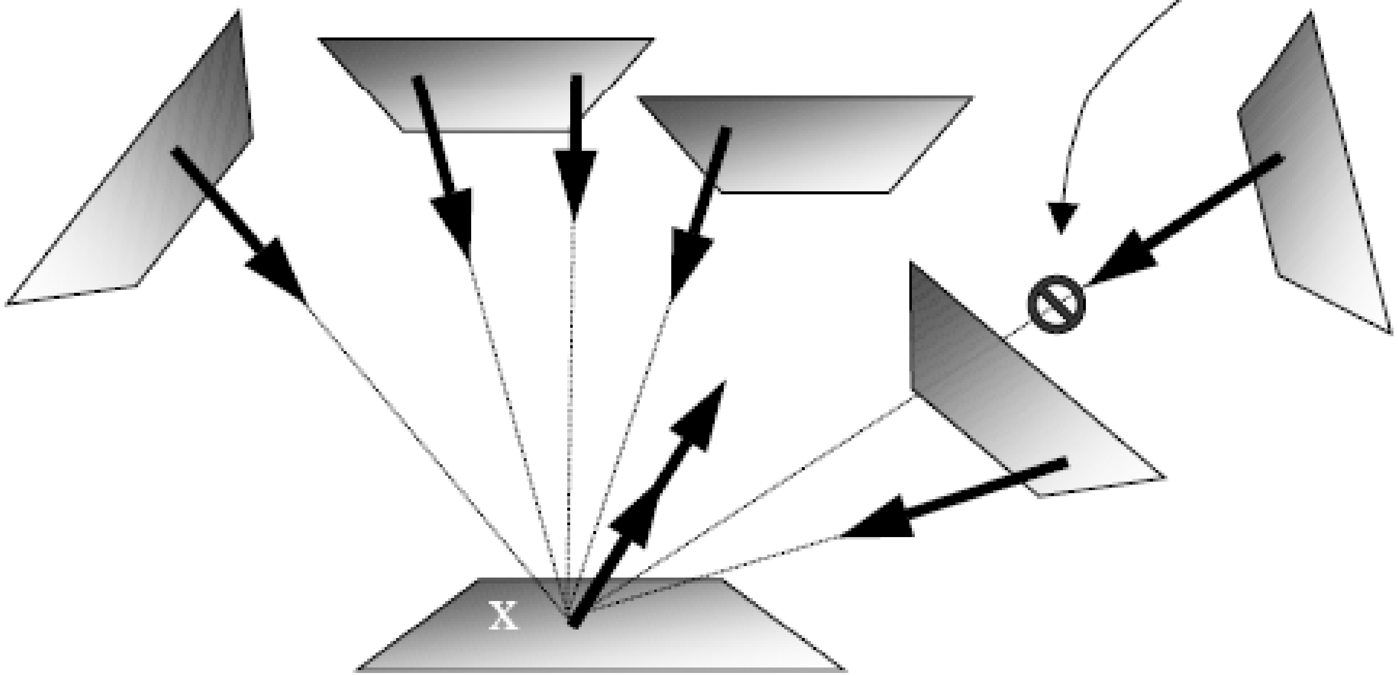
$$y = vp(x, \Psi)$$

Integration domain = visible surface points y

- Integration domain extended to ALL surface points by including visibility function

Rendering Equation: All Surfaces

$$L(x \to \Theta) = L_e(\ldots) + \int_A f_r(\ldots) \cdot L(y \to -\Psi) \cdot \frac{\cos\theta_x \cdot \cos\theta_y}{r_{xy}^2} \cdot \boxed{V(x,y)}\, dA_y$$



Two Forms of the Rendering Equation

- **Hemisphere integration**

$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos\theta_x \cdot d\omega_\Psi$$

- **Area integration**

$$L(x \to \Theta) = L_e(x \to \Theta) + \int_A f_r(\Psi \leftrightarrow \Theta) \cdot L(y \to -\Psi) \cdot \frac{\cos\theta_x \cdot \cos\theta_y}{r_{xy}^2} \cdot V(x,y) \cdot dA_y$$

Summary

- Geometric optics
- Radiometry
- Rendering equation
  - Mathematical formulation that global illumination algorithms must solve

Next Time

- Monte Carlo ray tracing

One-on-One Meetings

- Start from next week
  - Right after the class

- Discuss the scope of your project and feasibility of your idea

Announcements

- 2 papers for each student
  - Choose 4 papers from the paper list
  - Send them (titles of 4 papers) to TA (Bochang Moon) by Oct-11 (Mon)
  - Look at videos and talk files (captured talk video or presentation files)

- Schedule of student presentations
  - Will be decided on Oct-12 (Tue)
  - Presentations will start after the mid-term

Exams

- **Consists of two parts**
  - **Review of materials that I lectured (Oct – 21)**
  - **Review of materials that each student lectured (Dec-16)**

- **Each student will make two simple questions for the final-term exam**
  - **Questions should be about the core concepts of their presentations**
  - **Provide answers to the TA**

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Project Presentations

- Mid
  - Nov-18 (Thu.)
- Final
  - Dec-2 (Tue.)
    - Can be adjusted