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RENDERING

FREELY AVAILABLE ON THE INTERNET

http://sglab.kaist.ac.kr/~sungeui/render

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13 Rendering Equation

In this chapter, we discuss the rendering equation that mathematically explains how the light is reflected given incoming lights. The radiosity equation (Ch. 11) is a simplified model of this rendering equation assuming diffuse reflectors and emitters.

Nonetheless, the rendering equation does not explain all the light and material interactions. Some aspects that the rendering equation does not capture include subsurface scattering and transmissions.

13.1 Rendering Equation

The rendering equation explains how the light interacts with materials. In particular, it assumes geometric optics (Sec. 12.1) and the light and material interaction in an equilibrium status.

The inputs to the rendering equation are scene geometry, light information, material appearance information (e.g., BRDF), and viewing information. The output of the rendering equation is radiance values transferred, i.e., reflected and emitted, from a location to a particular direction. Based on those radiance values for primary rays generated from the camera location, we can compute the final rendered image.

Suppose that we want to compute the radiance, $L(x \rightarrow \Theta)$, from a location x in the direction of Θ [1]. To compute the radiance, we need to sum the emitted radiance, $L_e(x \rightarrow \Theta)$, and the reflected radiance, $L_r(x \rightarrow \Theta)$ (Fig. 13.1). The emitted radiance can be easily given by the input light configurations. To compute the reflected radiance, we need to consider incoming radiance to the location x and the BRDF of the object at the location x. The incoming radiance can come to x in any possible directions, and thus we introduce an integration with the hemispherical coordinates. In other words, the reflected radiance is computed as the following:

$$L_r(x \rightarrow \Theta) = \int_{\Psi} L(x \leftarrow \Psi) f_r(x, \Psi \rightarrow \Theta) \cos\theta_x dw_{\Psi}, \qquad (13.1)$$

[1] For simplicity, we use a vector Θ for representing a direction based on the hemispherical coordinates.

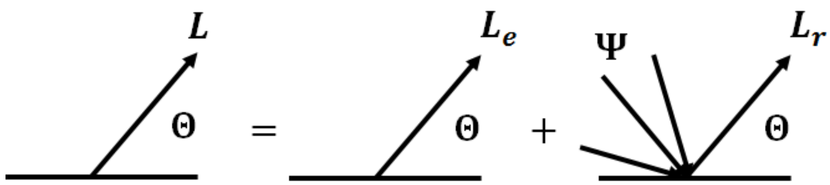

Figure 13.1: The radiance, $L(x \rightarrow \Theta)$, is computed by adding the emitted radiance, $L_e(x \rightarrow \Theta)$, and the reflected radiance, $L_r(x \rightarrow \Theta)$.

where $L(x \leftarrow \Psi)$ is a radiance arriving at x from the incoming direction, Ψ, $\cos\theta_x$ is used to consider the angle between the incoming direction and the surface normal, and the BRDF $f_r(\cdot)$ returns the outgoing radiance given its input.

We use the hemispherical coordinates to derive the rendering equation shown in Eq. 13.1, known as hemispherical integration. In some cases, a different form of the rendering equation, specifically area integration, is used. We consider the area integration of the rendering equation in the following section.

The rendering equation can be represented in different manners including hemispherical or area integration.

13.2 *Area Formulation*

To derive the hemispherical integration of the rendering equation, we used differential solid angles to consider all the possible incoming light direction to the location x. We now derive the area integration of the rendering equation by considering a differential area unit, in a similar manner using the differential solid angle unit.

Let us introduce a visible point, y, given the negated direction, $-\Psi$, of an incoming ray direction, Ψ, from the location x (Fig. 13.2). We can then have the following equation thanks to the invariance of radiance:

$$L(x \leftarrow \Psi) = L(y \rightarrow -\Psi). \tag{13.2}$$

Our intention is to integrate any incoming light directions based on y. To do this, we need to substitute the differential solid angle by the differential area. By the definition of the solid angle, we have the following equation:

$$dw_\Psi = \frac{dA \cos\theta_y}{r_{xy}^2}, \tag{13.3}$$

where θ_y is the angle between the differential area dA and the orthogonal area from the incoming ray direction, and r_{xy} is the distance between x and y.

When we plug the above two equations, we have the following equation:

$$L_r(x \rightarrow \Theta) = \int_y L(y \rightarrow -\Psi) f_r(x, \Psi \rightarrow \Theta) \frac{\cos\theta_x \cos\theta_y}{r_{xy}^2} dA, \tag{13.4}$$

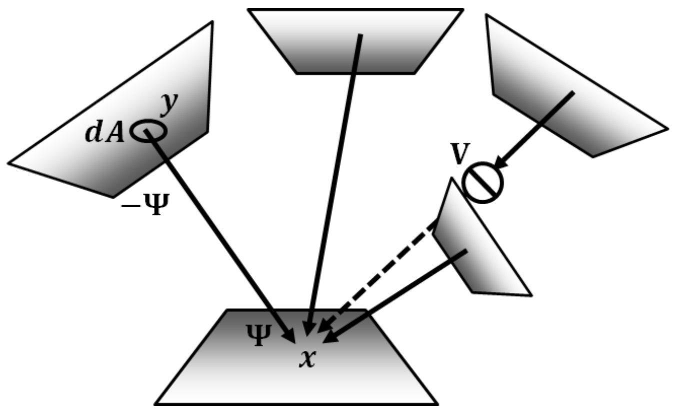

Figure 13.2: This figure shows a configuration for deriving the area formulation of the rendering equation.

where y is any visible area on triangles from x. In the above equation, we need to first compute visible areas from x on triangles. Instead, we would like to integrate the equation on any possible area while considering visibility, $V(x, y)$, which is 1 when y is visible from x, and 0 otherwise. We then have the following area integration of the rendering equation:

$$L_r(x \to \Theta) = \int_A L(y \to -\Psi) f_r(x, \Psi \to \Theta) \frac{\cos\theta_x \cos\theta_y}{r_{xy}^2} V(x, y) dA, \tag{13.5}$$

where A indicates any area on triangles.

Form factor. The radiosity algorithm requires to compute form factors that measure how much light from a patch is transferred to another patch (Sec. 11.2). The area integration of the rendering equation (Eq. 13.5) is equivalent to a form factor between a point on a surface and any points on another surface, while a diffuse BRDF is used in the equation. For the form factor between two surfaces, we simply perform one more integration over the surface.